Handout 20: Evolution and Nature of stars

- To recap
  - Low T stars are fully convective
    - High Kramers’ opacity in the interior
    - High heat capacity just below the photosphere
      - H and He ionization
  - Luminosity determined by matching convective interior to radiative photosphere
    - See next slide, 2 equations, 2 unknowns, solve for effective $T \sim R^{0.06}$.
      - Nearly vertical track (Hayashi) on the H-R diagram

Integrate from the surface in: $$P_{ph} = \left[\frac{2}{3}\cdot(a+1)\cdot\frac{g}{\kappa_0\cdot T_e{}^{b}}\right]^{\frac{1}{a+1}}$$

From the center out: $$P_{ph} = K'\cdot T_e{}^{\frac{\gamma}{\gamma-1}}$$ where $$K' = \frac{P_c}{T_c{}^{\frac{\gamma}{\gamma-1}}}$$

2 Equations, 2 unknowns, solve for effective T

$$T_e = \left[\frac{2}{3}\cdot(1+a)\cdot\frac{G\cdot M}{\kappa_0\cdot R^2}\cdot K'^{-(1+a)}\right]^{\frac{1}{b+\frac{\gamma}{\gamma-1}\cdot(1+a)}}$$

Opacity $a = 0.7$ $b = 5.3$ Ideal gas $\gamma = \frac{5}{3}$

Virial theorem $T_c = \text{const}\cdot\frac{M}{R}$ $P_c = \text{const}\cdot\frac{M}{R^3}\cdot T_c$

Gives $$T_e = \text{const}\cdot\left(R^{-2}\cdot R^{1.7\cdot 1.5}\right)^{\frac{1}{9.55}} = \text{const}\cdot R^{0.06}$$

i.e. nearly vertical tracks on H-R diagram

Stars start large, therefore cool, therefore convective

- Star contracts
  - Half the P.E. used to heat the interior
  - Half released as luminosity
    - Kelvin-Helmholtz timescale
- T high enough to develop radiative core
  - Luminosity determined by opacity
  - $L \sim M^3/\kappa$
  - For f-f + f-b (Kramers) $\kappa \sim \rho T^{-3.5}$
  - $L \sim R^{-0.5}$
    - L and effective T increase as star contracts

Radiative track, nearly horizontal

- Track moves to left and up on H-R diagram
  - Contraction continues till nuclear energy generated in the center exactly equals luminosity emitted from the surface ("Thermal equilibrium")
    - Nuclear time scale ~ 10 Gy for sun
- More massive stars heat up enough to be dominated by e-scattering opacity throughout the interior
  - L independent of radius
  - Tracks exactly horizontal on H-R diagram
    - Till "Thermal Equilibrium"

Less massive stars stay convective

- For M < 0.4 solar
  - T never high enough to become radiative
  - Track stays vertical right down to M.S.
    - Nuclear energy = Luminosity
- M.S. lifetime ~ M/L ~ $M^{-2 \text{ to } -3}$
- For M < 0.1 solar – Brown dwarf
  - Electron degeneracy pressure stops contraction
    - About 1 Jupiter radius
  - Central temperature never reaches 3 million K
  - H-burning not possible
  - Cools eternally