Handout 19: Evolution of stars on the Hertzsprung-Russell Diagram

- From H.W. #7:

Adiabatic T gradient is : $\frac{d\ln(T)}{d\ln(P)} = \frac{\gamma - 1}{\gamma}$

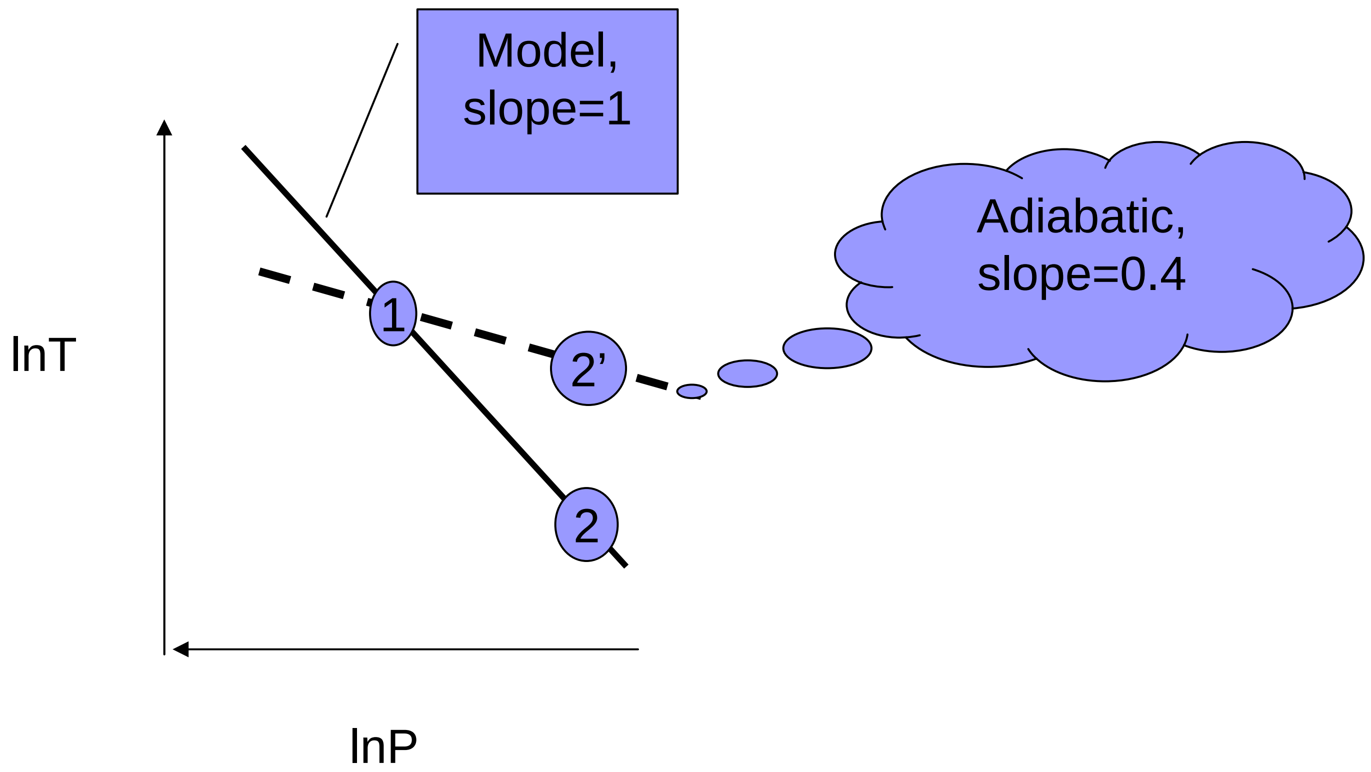
$\gamma = \frac{c_P}{c_V}$ so $\frac{d\ln(T)}{d\ln(P)} = \frac{c_P - c_V}{c_P}$

- If the T-gradient required by rad'n xfer in order to transport the L outward is steeper than this
  - $(d\ln T/d\ln P)_{rad} > (d\ln T/d\ln P)_{adiabatic}$
- Convectively unstable
  - Energy carried by convection rather than rad'n
    - Only slightly superadiabatic gradient is needed
    - $(d\ln T/d\ln P)_{actual} \sim (d\ln T/d\ln P)_{adiabatic}$

Uniform density stellar model is convectively unstable

- $P = nkT$
  - number density n = constant
  - $dT/dP = 1/nk$
  - $(d\ln T/d\ln P) = (P/T)(dT/dP) = 1$
  - 1 > 0.4, model is convectively unstable everywhere!
    - Balloon moved from 1 to 2' (figure) adiabatically
      - i.e. no heat added
    - Hotter than surroundings (i.e. 2)
    - Less dense → will rise further (convection)
    - Carries heat energy to higher levels!

Figure: Unstable T-gradient



Uniform stellar model

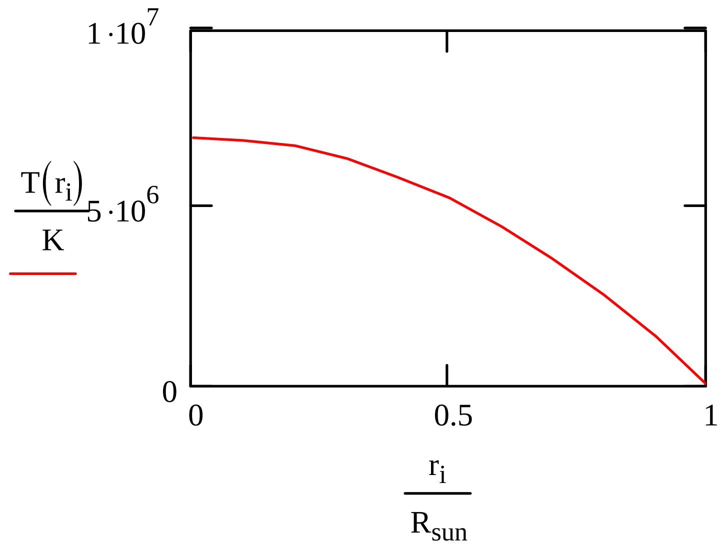
$$\rho := \frac{M_{sun}}{\frac{4}{3}\cdot\pi\cdot R_{sun}^{3}}$$ constant $$g(r) := \frac{G\cdot\rho\cdot\frac{4}{3}\cdot\pi\cdot r^{3}}{r^{2}}$$

$$\frac{dP}{dr} = -g\cdot\rho = \frac{-4}{3}\cdot\pi\cdot G\cdot\rho^{2}\cdot r \qquad P(R_{sun}) = 0$$

$$P(r) = \int_{r}^{R_{sun}} \frac{-4}{3}\cdot\pi\cdot G\cdot\rho^{2}\cdot r\,dr \qquad P(r) := \frac{2}{3}\cdot\pi\cdot G\cdot\rho^{2}\cdot\left(R_{sun}^{2} - r^{2}\right)$$

$$\mu := \frac{10+4}{20+3} \qquad \mu = 0.609 \qquad n := \frac{\rho}{\mu\cdot m_H}$$

$$T(r) := \frac{1}{n\cdot k}\cdot P(r) \qquad i := 0..10 \qquad r_i := \frac{i}{10}\cdot R_{sun}$$



Convection continued

- The **maximum** adiabatic gradient is 0.4
  - **Any** additional heat capacity will reduce this
    - Reducing the gradient toward zero
    - Forcing convection
      - Zero gradient means **no** radiative flux
  - Sources of heat capacity
      - H ionization 6-10,000 K
      - He ionization 20-50,000 K
    - 13.6 eV needed to ionize
    - Plus (3/2)kT = 1.5 eV for the K.E. of the electron
  - Adiabatic gradient reduced to 0.1

Convective zones in Main Sequence stars

- H, He ionization create convective zone just below the photosphere for sun
  - The convection generates magnetic fields
    - Leading to sunspots, flares, active chromosphere, and x-rays
  - Extends down to 0.8 of solar radius
- Hotter stars, convection zone shrinks
  - Above 8000 K (F-stars) fully radiative
- cooler stars, the convection zone deepens
  - Stars below 0.3 solar masses fully convective
- Above 0.3 solar masses, Luminosity determined by radiation

Convection crucial in early stellar evolution

- Figure 9.10 shows interior $\kappa \sim T^{-3.5}$
  - But <T> ~ M/R
    - Virial theorem
  - Increasingly large |dT/dr| for larger stars
  - Convection sets in
- Stars < 5 solar mass start fully convective
  - Sun at 30 solar radii was "fully convective"
  - Major blockage to luminosity is **photosphere**
    - $L = 4\pi R^2 \sigma T(\tau=2/3)^4$

Fully convective stars nearly constant effective T, H- opacity

- The dominant opacity at low T's is H-
  - Free-free and free-bound
- Very strong T-dependence (Fig. 9.10)
  - H- needs electrons
  - Electrons come from ionized metals
  - Saha dictates

$$\kappa = \kappa_0 \cdot P^{0.7} \cdot T^{5.3} = \kappa_0 \cdot P^a \cdot T^b$$

- Luminosity determined by transition between convection and radiative photosphere
  - i.e. T_e = T at τ = 2/3 level

Solve for P at τ = 2/3

- As in homework, solve for P(τ=2/3)
  - κ not constant, need to do integral
  - Weak dependence of T on τ,
    - T can be taken out of integral
    - Solve for P (see next slide):

$$P_{ph} = \left[\frac{2}{3}\cdot(a+1)\cdot\frac{g}{\kappa_0\cdot T_e^{\,b}}\right]^{\frac{1}{a+1}}$$

Photospheric P (*i.e.* $\tau = 2/3$)

$$\frac{2}{3} = \tau = \int_{r_{photosphere}}^{R} \kappa \cdot \rho \, dr = \kappa_0 \cdot T_e{}^b \cdot \int_{r_{photosphere}}^{R} P^a \cdot \rho \, dr$$

Hydrostatic equil. implies $\rho = \frac{-1}{g} \cdot \frac{dP}{dr}$ g = const.

$$\frac{2}{3} = \kappa_0 \cdot T_e{}^b \cdot \left(\frac{-1}{g}\right) \cdot \int_{r_{photosphere}}^{R} P^a \cdot \frac{dP}{dr} \, dr = \kappa_0 \cdot T_e{}^a \cdot \left(\frac{-1}{g}\right) \cdot \frac{1}{a+1} \cdot -\left(P_{photosphere}{}^{a+1}\right)$$

since $\frac{dP}{dr} \cdot dr = dP$ and $P(r = R) = 0$

so $P_{ph} = \left[\frac{2}{3} \cdot (a+1) \cdot \frac{g}{\kappa_0 \cdot T_e{}^b}\right]^{\frac{1}{a+1}}$

Integrate adiabatic gradient from center of star to photosphere

$$\frac{d\ln(T)}{d\ln(P)} = \frac{\gamma - 1}{\gamma}$$ Eq. (10.75)

$$\frac{\gamma}{\gamma - 1} \cdot \int_{T_e}^{T_c} 1 \, d\ln(T) = \int_{P_{photosphere}}^{P_c} 1 \, d\ln(P)$$

$$\frac{\gamma}{\gamma - 1} \cdot \left(\ln\left(T_c\right) - \ln\left(T_e\right)\right) = \left(\ln\left(P_c\right) - \ln\left(P_{ph}\right)\right)$$

exponentiate, and solve for P at the tau = 2/3 level

$$P_{ph} = K' \cdot T_e^{\frac{\gamma}{\gamma-1}}$$ where $$K' = \frac{P_c}{T_c^{\frac{\gamma}{\gamma-1}}}$$

solve for eff. T

$$T_e = \left[\frac{2}{3} \cdot (1 + a) \cdot \frac{G \cdot M}{\kappa_0 \cdot R^2} \cdot K'^{-(1+a)} \right]^{\frac{1}{b + \frac{\gamma}{\gamma-1} \cdot (1+a)}}$$

Vertical, Hayashi tracks on the H-R diagram

- Insert appropriate values
  - Note: $T_c \sim M/R$, $P_c \sim (M/R^3)T_c$
  - $T_e \sim R^{0.06}$
    - $L \sim R^2 T_e^{\,4} \sim T_e^{36}$,
    - $T_e \sim L^{0.03}$
    - Tracks nearly vertical on H-R diagram (draw)
    - Hayashi figured this out in 1961

$$\frac{\gamma}{\gamma - 1} = 2.5 \qquad K' = \text{const} \cdot M^{-0.5} \cdot R^{-1.5}$$

$$T_e = \text{const} \cdot \left(R^{-2} \cdot R^{1.7 \cdot 1.5}\right)^{\frac{1}{9.55}} = \text{const} \cdot R^{0.06}$$