Handout 16: Nature of a Star

R/Rsun	<T>	t	
10	10^6 K	$t_{K-H} \sim 10^4$ y	
5	2 10^6 K	$t_{K-H} \sim 10^5$ y	Nucl. Reactions start
2	5 10^6 K	$t_{K-H} \sim 10^6$ y	Nucl. Reactions dominate
1	10^7 K	$t_{nucl} \sim 10^{10}$ y	Main Sequence

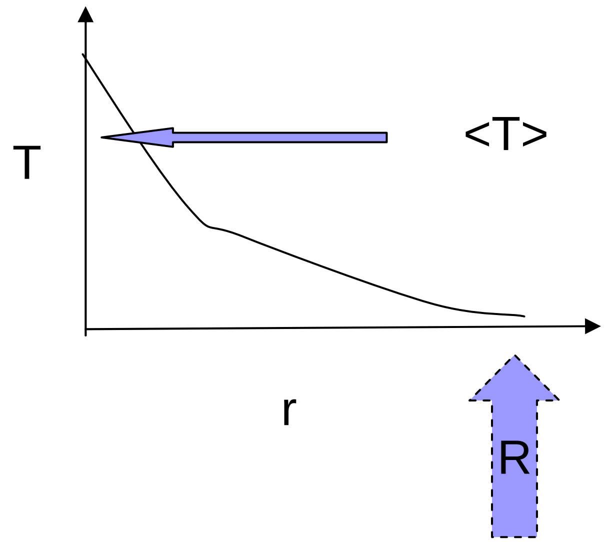
The contraction stops when d(Enucl)/dt in the interior exactly equals the Luminosity radiated into outer space. The $64 dollar question, then, is what determines the Luminosity? From the perspective of the stellar atmosphere, it is given by the T at the $\tau = 1$ level in the atmosphere. The question is, what determines this?

Luminosity governed by Thermodynamics

- Consider the T structure of a star
    - <T> = 10^7 K
    - $T_{surface} = 10^4$ K, i.e. nearly zero
  - How does T depend on r?
- 2^{nd} law of Thermo: energy always flows from hotter region to cooler
  - Since L(r) is always outwards
    - ➔ T increases inwards
    - $T_{center} > \langle T \rangle$, i.e. 1.5 10^7 K for the sun

T structure, energy flow

- Consider T vs. r inside star



  - Think in terms of a voltage potential, and the flow of electrical current

Flow of energy ~ electrical current

- Break up interior into 3 regions
  - Electrical resistances R1, R2, R3
  - Voltage drop from V_{center} to zero
  - Current flow governed by **biggest** resistor
    - i.e. if $R1 = 1\ \Omega = R3$
    - But $R2 = 100\ \Omega$
    - $I \sim V_{center} / R2$
- Therefore we seek the biggest roadblock to E-flow in the interior regions

Electrical current analogy, parallel and series resistors

- Also note
  - If in one interior zone there are 2 competing modes of Energy transport
    - Like 2 parallel resistors R1 and R2
    - Net resistance $1/R = 1/R1 + 1/R2$
    - If $R1 \ll R2$, $R \sim R1$
  - i.e. the **least-resistive** mode dominates

Key to Luminosity is energy transport

- There are 3 ways to transport energy
  - Radiation
    - Net flow of photons from hotter region to cooler
  - Convection
    - Gas flows from hotter to cooler, carrying heat energy
  - Conduction
    - Neighboring atoms bump into each other, transferring some of their energy like a bucket-brigade
    - Only important in dense stars (i.e. White Dwarf)

Energy transport via radiation

- We've already done this for the atmosphere
  - Intensity given by integral of source function
    - "see" to optical depth 1
  - Flux given by integral of Intensity over angle
    - Average of vertical optical depths is 2/3
  - The interior is optically thick in both directions

Exact, radiative luminosity

$$F_{down} = \pi \cdot B\left(\tau_v - \frac{2}{3}\right) \qquad F_{up} = \pi \cdot B\left(\tau_v + \frac{2}{3}\right)$$

$$F = F_{up} - F_{down} = \pi \cdot \frac{dB}{d\tau} \cdot \frac{4}{3}$$

net flux upward = outward

$$B = \frac{\sigma}{\pi} \cdot T^4$$

$$\frac{dB}{d\tau} = \frac{-1}{\kappa \cdot \rho} \cdot \frac{dB}{dr} = \frac{-1}{\kappa \cdot \rho} \cdot \frac{\sigma}{\pi} \cdot 4 \cdot T^3 \cdot \frac{dT}{dr}$$

$$F = \frac{-16}{3} \cdot \frac{\sigma}{\kappa \cdot \rho} \cdot T^3 \cdot \frac{dT}{dr}$$

radiative flux

$$L_r = 4 \cdot \pi \cdot r^2 \cdot F = \frac{-64 \cdot \pi}{3} \cdot \frac{\sigma}{\kappa \cdot \rho} \cdot r^2 \cdot T^3 \cdot \frac{dT}{dr}$$

radiative luminosity

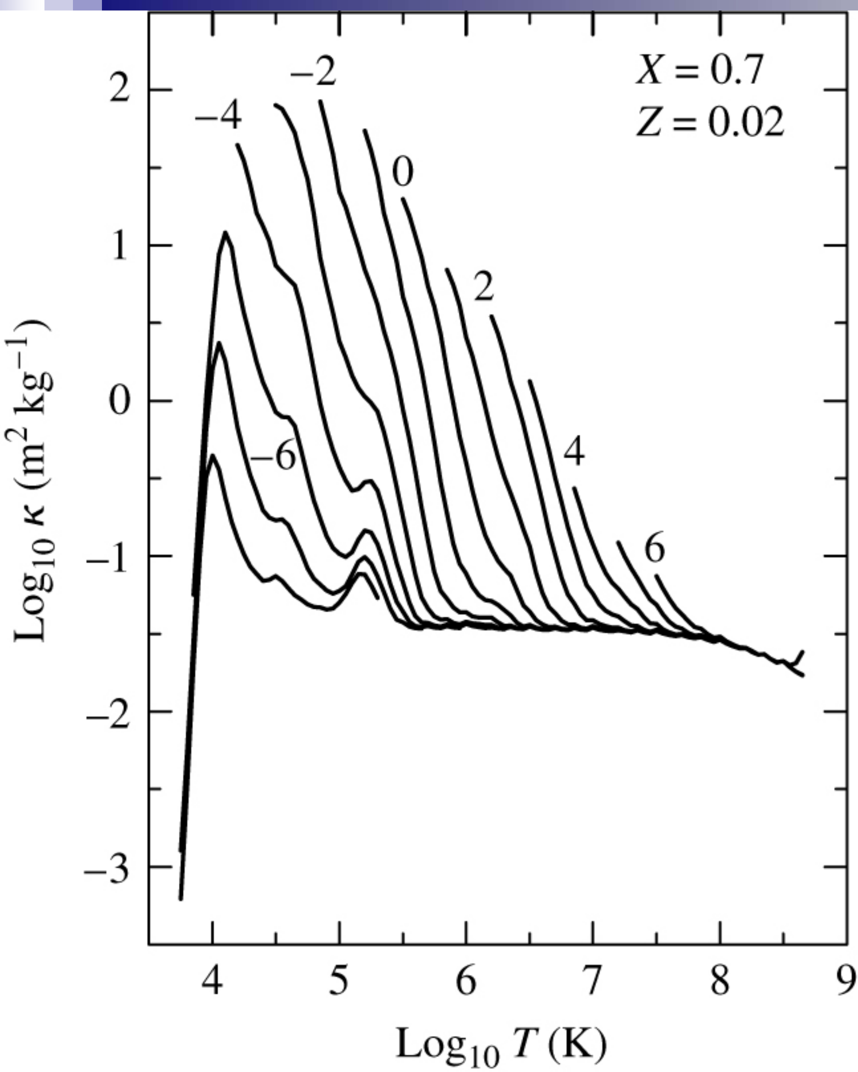
Notes on radiative luminosity

- Note
  - L ~ |dT/dr|, the temperature gradient
    - And
  - L ~ 1/κ, opacitive blockage of energy flow
    - κ is analogous to electrical resistivity
- Also, if energy x-port is radiative
  - The L_r equation can be inverted to give the T gradiant dT/dr
    - i.e. a Luminosity implies a temperature gradiant
    - Larger κ ➔ larger temperature gradiant |dT/dr|

Luminosity governed by radiation transport, not nuclear reactions

- Very important,
  - The typical temperature gradient is governed by the Virial Theorem
    - i.e. $\langle T \rangle \sim M/R$, about 10^7 K for sun
    - $dT/dr \sim -\langle T \rangle/R \sim M/R^2$
  - The opacity κ comes from atomic physics
    - See Fig. 9.10
    - Sum of free-free, free-bound, bound-bound, and e-scattering for all species
      - Averaged over the Planck function

Fig. 9.10 C&O Rosseland mean opacity vs. T and ρ (curves labelled with log(ρ)) for the Sun



For interiors, T above log(T) = 5 are appropriate

Note the steep drop-off in opacity with T
Caused by ionization of trace 'metals'
They are supplying opacity through b-free and free-free (**Kramer's** opacity law)

Note the opacity bottoms out at about 0.4 cm^2/gm
When all atoms are totally ionized, no more bound-free opacity.
Electron scattering opacity dominates
Independent of wavelength and T

The steep rise with T below 10,000 K leads to vertical convective Hayashi tracks describing the evolution of young stars on the H-R diagram