Handout 14: Stellar interiors

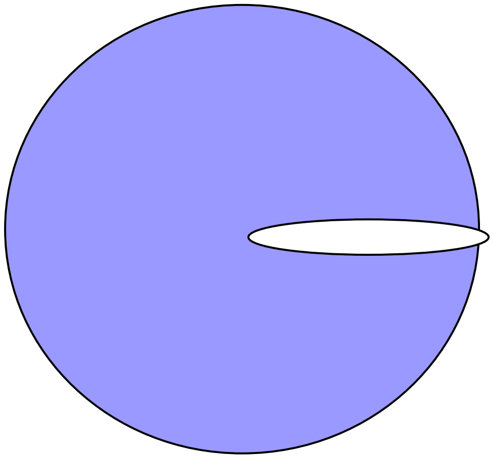
- The theory of stellar interiors has been a triumph of 20th century Astrophysics
  - It explains the basic nature of stars, including the sun
  - It shows how such stars came to be
    - Stellar evolution
  - It quantifies how long stars can live
    - Fuel supply
  - It explains what happens after fuel runs out
    - Stellar evolution
- Theory is guided by accurate observations

Start by explaining the sun

- Facts
  - Mass $2\ 10^{33}$ gm
    - 70% H, 28% He, 2% “metals” by mass
  - Radius $7\ 10^{10}$ cm
    - ~ 2 light seconds
    - Subtends 0.5 degrees at 1 A.U.
  - Shape = nearly perfect sphere
  - Luminosity $4\ 10^{33}$ erg/s
    - ➔ effective T 5800 K
    - High temperature demands **<u>gas</u>**eous sun

Free-fall timescale

- Spherical shape and large mass ➔ **gravity** hold sun together
  - Need an equal opposing force, otherwise sun would collapse on free-fall timescale.
    - Consider test-mass orbit
      - Very eccentric
      - Start at aphelion = solar radius
      - End at perhelion = center of sun
      - Free-fall time = P/2
      - Rest of sun collapses to center



Sun's free-fall timescale = 1/2 hr

Kepler/Newton III:

$$\frac{G}{4\cdot\pi^2}\cdot M\cdot P^2 = a^3$$

$$a = \frac{1}{2}\cdot R$$

$$t = \frac{P}{2} = \frac{1}{2}\cdot\sqrt{\frac{4\cdot\pi^2\cdot a^3}{G\cdot M}}$$

$$\frac{M}{a^3} = \frac{M}{\left(\frac{1}{2}\right)^3\cdot R^3} = \frac{4}{3}\cdot\pi\cdot 2^3\cdot\rho$$

$$\rho = \text{average\_density}$$

$$t := \sqrt{\frac{3\cdot\pi}{32}}\cdot\frac{1}{\sqrt{G\cdot\rho}}$$

$$\rho \equiv 1.4\cdot\frac{gm}{cm^3}$$

$$t = 0.493hr$$

Hydrostatic equilibrium

- Watch the sun for 30 minutes
  - It doesn’t contract (or expand)
  - There must be an **outward** force **exactly** balancing gravity
    - ➔pressure gradient in sun
    - ➔hydrostatic equilibrium
      - Same force keeps earth’s atmosphere from collapsing
      - Given the scale height of 9 km, what is the free-fall timescale for our atmosphere?

dP/dr = -gρ (hydrostatic equil.)

define $M_r = \int_0^r 4\cdot\pi\cdot r^2\cdot\rho \, dr$ mass interior to r

Using the calculus, Newton showed that the mass interior to r acts as if all the mass were at the center. Likewise, the mass exterior to r exerts zero net force.

Consider a small element of mass dm in the form of a disk of area A and thickness dr. By symmetry, the net force will be in the radial direction. There are 3 forces on the disk:

$$F_{graviy} = -dm\cdot\frac{G\cdot M_r}{r^2} \qquad \frac{G\cdot M_r}{r^2} = g(r)$$

pressure on top $-A\cdot P(r + dr)$

pressure on bottom $A\cdot P(r)$

$$net\_force = -A\cdot\rho dr\cdot g - A\cdot P(r + dr) + A\cdot P(r) = 0$$

$$\frac{P(r + dr) - P(r)}{dr} = -\rho\cdot g \qquad \frac{d}{dr}P(r) = -\rho(r)\cdot g(r)$$

Pressure

- Must hold throughout interior of star
- Pressure is force/unit area
  - Force is d(momentum)/dt
  - Pressure is **momentum flux** d(momentum)/dAdt
- Two sources, gas and photons
  - $P_{rad} = (1/3)u = (1/3)aT^4$
    - Only important in hottest stars
    - Concentrate on gas pressure

Gas pressure

- To calculate momentum transport (flux)
  - Need distribution n(p) in momentum space
  - Certainly we can assume T.E.
    - Collisional m.f.p. very small
    - Velocities equilibrated very quickly
  - In general, distribution will be
    - Fermi-Dirac for fermions
    - Bose-Einstein for bosons
  - At high T and low density, these both ➔Maxwell-Boltzman
    - **P = nkT**, ideal gas pressure

Mean molecular weight μ

- The connection between mass density ρ and number density n is the mean molecular weight μ

definition of μ:

$$\mu \cdot m_H \cdot n = \rho \qquad m_H = \frac{1 \cdot gm}{A} = 1.67 \cdot 10^{-24} \cdot gm$$

consider all species "j"

$$n = \sum_j n_j \qquad \rho = \sum_j \rho_j$$

$$\mu = \frac{1}{m_H} \cdot \left(\frac{\sum_j \rho_j}{\sum_j n_j} \right) = \frac{1}{m_H} \cdot \frac{\sum_j m_j \cdot n_j}{\sum_j n_j}$$

μ of order 1 for normal stars

- Neutral H
  - $\mu = 1$
- Ionized H
  - $\mu = ½$
- He atoms 10% of H atoms, neutral
  - $\mu = (1+0.4)/(1+0.1) \sim 1.3$
- Stellar interior strongly depends on μ
  - $P \sim T/\mu$, a small μ requires a large T
  - Energy generation rate $\sim T^{4-10}$