Handout 12: Limb darkening, absorption lines

- When $\tau_v = 2/3$
  - T = effective temperature
    - We “see” to $\tau = 1$ at each θ
    - This corresponds to $\tau_v < 1$
    - Average over all θ gives $\langle\tau_v\rangle = 2/3$
- With the source function, we can calculate limb darkening

$$S = \frac{\sigma}{\pi}\cdot\frac{3}{4}\cdot T_e^4\cdot\left(\tau_v + \frac{2}{3}\right) = \frac{\sigma}{\pi}\cdot\frac{3}{4}\cdot T_e^4\cdot\left(\tau\cdot\cos(\theta) + \frac{2}{3}\right)$$

  - Use eqn. of radiative transfer
  - Go to deep enough layers so $I_0 e^{-\tau} \rightarrow 0$

Limb darkening

$$I(\theta, \tau = 0) = \int_0^{\infty} S \cdot e^{-\tau} \, d\tau$$

$$\int_0^{\infty} e^{-\tau} \, d\tau = 1 \qquad \int_0^{\infty} \tau \cdot e^{-\tau} \, d\tau = \int_0^{\infty} e^{-\tau} \, d\tau - \int_0^{\infty} \frac{d}{d\tau} \tau e^{-\tau} \, d\tau = 1 - 0$$

$$I(\theta) = \frac{\sigma}{\pi} \cdot \frac{3}{4} \cdot T_e^{\,4} \cdot \left(\cos(\theta) + \frac{2}{3} \right)$$

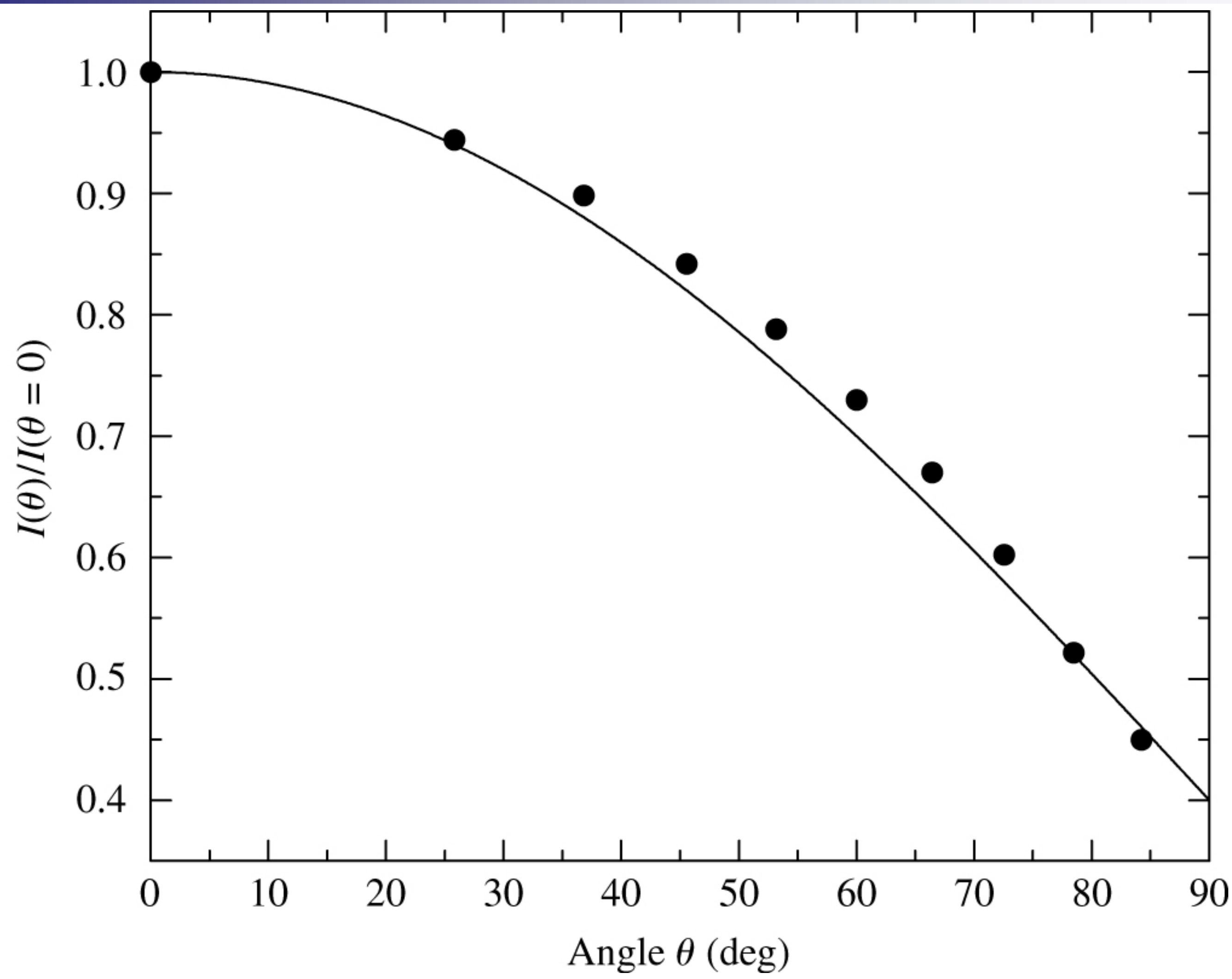
$$\frac{I(\theta)}{I(0)} = \frac{\cos(\theta) + \frac{2}{3}}{\frac{5}{3}} = \frac{2}{5} + \frac{3}{5} \cdot \cos(\theta)$$

Note that $I = S(\tau = 1)$

- We "see" to optical depth 1
  - True when source fct. Linear in τ
    - i.e. $S = a + b\tau$
  - Approximately true for more complex dependence
- At $\theta = 90$ degrees
  - Minimum $I = (1/2)(\sigma/\pi)T_e^4$
- At $\theta = 48$ degrees
  - $\text{Cos}(\theta) = 2/3$, $I = (\sigma/\pi)T_e^4$
    - i.e. we see the effective T at this angle

Fig. 9-17 Carroll and Ostlie

Compare measured limb darkening of sun (points) to the Eddington approximation (line)



Absorption lines, equivalent widths

- The overall strength of a line is judged by its “Equivalent Width” W
  - Width of a perfectly black line that removes the same amount of energy from the spectrum
  - Combination of width times depth
  - Larger W implies higher abundance

defined as $$W \cdot F_{\text{continuum}} = \int_{\text{line}} \left(F_{\text{coninuum.}} - F_{\text{line}}\right) d\lambda$$

Equivalent widths and abundances

- Stronger lines (larger W) ➔ more opacity
  - More atoms/cm^2 above the $\tau_{continuum}$ = 1 level
  - ➔ abundance i.e. of CaII, etc.
  - Strengths of absorption lines used to measure abundances of all the elements
    - In sun log values, H=12, He=11, C=8.5, N=8, O=8.8, Ca=6.3, etc.
      - Sun representative of present solar neighborhood
    - Some stars have much weaker lines *cf*. H
      - Globular clusters metals 10^{-2} or less *cf*. sun
        - ➔ nucleosynthesis after the big bang

Line widths

- An important component of W is the width of a line
  - Defining the width as the Full Width at Half Maximum FWHM
    - W = area ~ (fractional depth) x FWHM
      - Fractional depth is limited to 1 (100% absorption)
      - W increases because FWHM increases
- First learn about optically thin line shapes
  - Natural, thermal doppler, turbulence, pressure

Natural line width

- The narrowest possible line
  - Governed by the uncertainty principle
  - Uncertainty in time and energy related
    - $\Delta E \Delta t$ at least $h/2\pi$
      - Δt = lifetime of energy level
    - $\Delta E = h\Delta\nu$
    - $\Delta\nu = (2\pi\Delta t)^{-1}$
    - $\Delta\lambda/\lambda = \Delta\nu/\nu$
  - Typical Δt very small
    - $H\alpha \sim 10^{-8}$ s, $\Delta\lambda \sim 10^{-3}$ Angstrom
  - Other broadening mechanisms dominate

Thermal doppler line width

- In T.E., velocities are Maxwellian

$$\frac{1}{2}\cdot m\cdot v_z^2 = \frac{1}{2}\cdot k\cdot T$$

on average, z is along line of sight

Maxwellian distribution

$$n(v_z) = \text{const}\cdot e^{\frac{\frac{-1}{2}\cdot m\cdot v_z^2}{k\cdot T}}$$

Gaussian

each velocity gives a different wavelength through

$$\lambda = \lambda_0\cdot\left(1+\frac{v_z}{c}\right)$$

If line optically thin, FWHM at v such that

$$e^{\frac{\frac{-1}{2}\cdot m\cdot v_z^2}{k\cdot T}} = \frac{1}{2}$$

$$|v_z| = \sqrt{\frac{\ln(2)\cdot 2\cdot k\cdot T}{m}}$$

$$\text{FWHM} = 2\cdot\sqrt{\frac{\ln(2)\cdot 2\cdot k\cdot T}{m}}\cdot\frac{\lambda_0}{c}$$

Line widths, Turbulence

- For $m = m_H$, $T = 5800$ K (sun), $H\alpha$ line
  - $\Delta\lambda = 0.4$ Angstrom
    - 1000 times wider than natural width
  - Even for Ca $m = 40\ m_H$
    - Velocities sqrt(40) = 6 times smaller
    - Thermal Doppler width 6 times narrower
    - Still 100 times the natural width
- Turbulence velocities usually gaussian
  - Will add to Thermal Doppler width
    - Velocity widths added in quadrature

Pressure/collisional broadening

- Close encounters disturb atom
  - Make excited state decay faster
  - ➔ Δt reduced ➔ ΔE increase ➔ $\Delta\lambda$ increased

$$\Delta t = \frac{L}{v} = \frac{1}{n \cdot \sigma} \cdot \sqrt{\frac{m}{3 \cdot k \cdot T}}$$

    - ΔE increases ~ $n\sigma$ sqrt(T/m)
  - Why H lines broader in dwarf stars than giants
    - Higher g ➔ higher n ➔ larger width
    - In detail, atomic physics is complex
  - Line profile like natural width
    - "damped oscillator" ➔ Lorentzian profile