Handout 10: Opacity Continued

- Free-free continuum absorption
  - A free electron interacts with an ion
  - The accelerating electron emits E-M radiation
    - Photon energy taken from electron KE
    - Gas cools
  - A good emitter is also a good absorber
    - Free electrons and ions provide opacity
    - Photon energy given to electrons KE
      - Heating the gas
  - Most effective at longer wavelengths

Electron scattering

- Thompson cross section
  - $\sigma_T = 6.7\ 10^{-25}\ \mathrm{cm}^2$
    - Independent of wavelength
    - Small
      - *c.f.* free-bound ~ $10^{-17}\ \mathrm{cm}^2$
    - Important when everything is ionized
      - Hot star atmospheres and interiors
    - Ionized H, $\kappa = \sigma/m = 0.4\ \mathrm{cm}^2/\mathrm{gm}$
      - Lower limit to opacity
      - Leads to Eddington limit to Luminosity

Total opacity

Besides keeping those photons inside the star, the continuum opacity is crucial For understanding absorption lines. The strength of a line is really given by the Ratio $\kappa_{line}/\kappa_{continuum}$. For many years, the source of continuum opacity in The solar atmosphere was unknown, making the interpretation of the absorption Lines difficult.

To determine the total opacity, the various sources (b-f, b-b, f-f, e-scattering) from Each of the various atoms, ions, molecules, and electrons must be summed together to give the total opacity κ_λ.

- Example: e-scattering
  - From P_e, calculate $n_e = P_e /kT$
  - Then $\kappa_e \rho = n_e \sigma_T$ (ρ = total gas density) gives e-scattering contribution to κ_{total}.
- Etc. for b-f HI, CaI, CaII, etc.

Total opacity – Rosseland mean

- To track the overall flow of photon energy, we need an average opacity
  - Average over wavelength
  - Weighted for most important wavelengths
    - Use the Planck, blackbody function naturally
- Called the Rosseland mean opacity
  - About to 1 cm^2/gm in the solar atmosphere
    - Also the interior
    - Distance we "see" to given by $\int \rho ds = 1\ gm/cm^2$

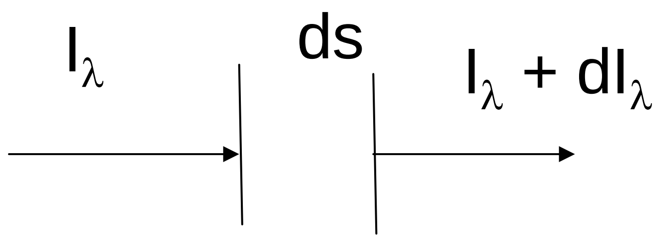
Poor man's radiative transfer

- Many of the most important results in radiative transfer follow from the dictum we "see" to optical depth 1 (i.e. visibility)
  - Absorption lines
    - In the continuum, see to $\tau_{cont} = 1$, $I = B(T_1)$
    - In Hα line, reach $\tau = 1$ sooner, higher, cooler layer in the atmosphere $T_2 < T_1$ and we see $I = B(T_2) < B(T_1)$
    - In stronger CaII K line, we reach $\tau = 1$ even sooner, in a higher, cooler layer $T_3 < T_2$ and $B(T_3) < B(T_2)$. CaII line is deeper and wider than Hα line

Limb darkening

- Same consideration gives limb darkening
  - The optical depth to a given radius (and hence T) in the star depends on the viewing angle as $\tau(\theta) = \tau_0/\cos(\theta)$
    - Plane-parallel atmosphere
  - At disk center, $\theta = 0$
    - $I = B(T_1)$, $T = T_1$ at $\tau_0 = 1$
  - Near the limb
    - $\tau(\theta) = 1$, $\tau_0 < 1$, $T_2 < T_1$, $I = B(T_2) < B(T_1)$
  - Limb darkening!

Emission – leads to **increase** in I



- Emission proportional to
  - Density of atoms n
  - Emissivity of each atom
  - Path length ds
- We write $dI_\lambda = j_\lambda \rho ds$

$$j_\lambda = \frac{\left(\frac{\text{erg}}{\text{sec}}\right)}{\text{gm}\cdot d\lambda \cdot d\Omega}$$

atoms usually radiate equally over 4π ster unit sphere

Fundamental Eq'n of Radiative Transfer

  - $j_\lambda \rho ds$ has units of intensity
- Including both gain and loss

$$dI_\lambda = -\kappa_\lambda \cdot \rho \cdot I_\lambda \cdot ds + j_\lambda \cdot \rho \cdot ds \qquad \text{or} \qquad \frac{dI_\lambda}{ds} = -\kappa_\lambda \cdot \rho \cdot I_\lambda + j_\lambda \cdot \rho$$

This is the fundamental equation we seek. If we know the state of matter, we should be able to calculate j and k and solve this. Problem: to know state of matter (i.e. ionization, excitation), we need to know the intensity and to solve the intensity we need the state of matter -- a circular conundrum.

Approach solution iteratively. Guess one, solve for the other, refine the guess, etc.

Problem greatly simplifies in Thermodynamic Equilibrium.

$$\frac{-1}{\kappa_\lambda \cdot \rho} \cdot \frac{dI_\lambda}{ds} = \frac{-dI_\lambda}{d\tau_\lambda} = I_\lambda - \frac{j_\lambda}{\kappa_\lambda} = I_\lambda - S_\lambda$$

The "source fct." S = B(T) in T.E (Kirchoff), emissivity = absorptivity in T.E..

Solution of equation

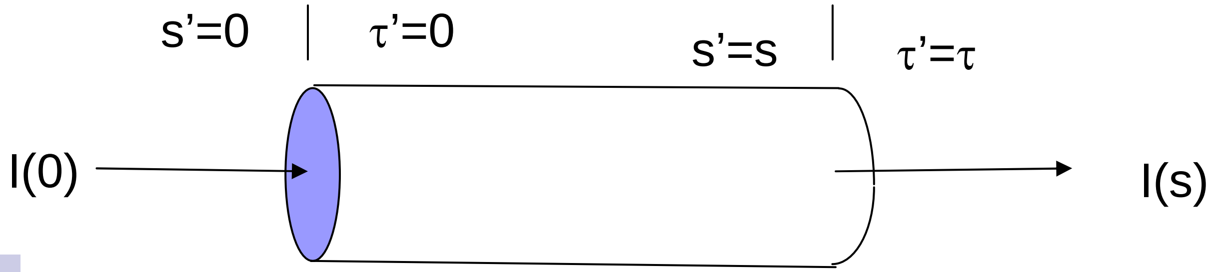
- First note:
  - If S > I, then I is increasing
    - Or S < I, I decreases
    - Until I = S
- Solution

$$\frac{dI_\lambda}{d\tau_\lambda} + I_\lambda = S_\lambda$$

$$e^{\tau_\lambda} \cdot \left(\frac{dI_\lambda}{d\tau_\lambda} + I_\lambda \right) = e^{\tau_\lambda} \cdot S_\lambda$$

$$\frac{d}{d\tau_\lambda}\left(I_\lambda \cdot e^{\tau_\lambda} \right) = e^{\tau_\lambda} \cdot S_\lambda$$

integrate over optical depth



$$I_\lambda(\tau_\lambda)\cdot e^{\tau_\lambda} - I_\lambda(0) = \int_0^{\tau_\lambda} e^{\tau'_\lambda}\cdot S_\lambda \, d\tau'_\lambda$$

$$I_\lambda(\tau_\lambda) = I_\lambda(0)\cdot e^{-\tau_\lambda} + \int_0^{\tau_\lambda} e^{-(\tau_\lambda-\tau'_\lambda)}\cdot S_\lambda \, d\tau'_\lambda$$

Incident flux,
attenuated

Attenuation
by intervening
material

dI added to I