Handout 9: Radiative transfer in a star

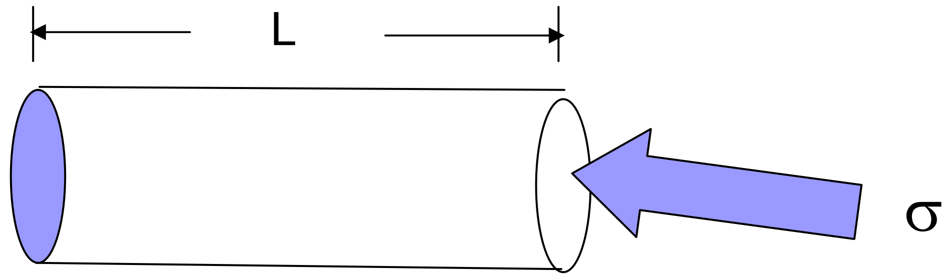
- Since Intensity is conserved in free space, we need only deal with interaction of light with matter
  - To properly account for these interactions
    - We need the density of matter
    - Also, the composition
      - i.e. H:He:C:N:O:etc. (20-30 important elements)
    - The state of excitation and ionization
      - i.e. CaI:CaII:CaIII etc.
      - Fraction of H in a given excitation state n

Assumption of Thermal Equilibrium

- Actually, **TE** can't be blithely assumed
  - There is a definite T gradient
    - Energy flows from hotter to cooler
    - T increases inward into a star
- To first approx., we can often assume **Local Thermodynamic Equilibrium**
  - Locally, atoms in TE at a temperature T
    - Very good assumption in the interior
    - Not so good for the atmosphere

Applicability of LTE

- Check how rapidly T changes with height in a model atmosphere
  - T scale height H = T/(dT/dr) ~ 600 km
    - Near the T = T_e level in the solar atmosphere
  - Compare H to the mean free path L of atoms
    - Distance, on average, travelled before collision
    - If σ is cross-section, n density, L = 1/(σn)

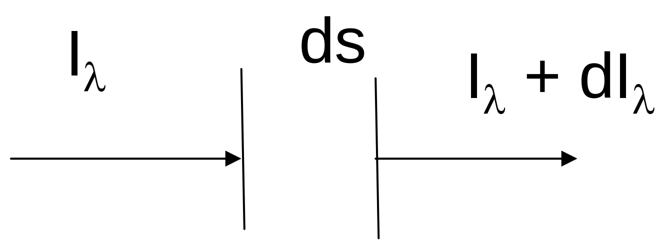


Mean free path

- $n = \rho/m \sim 10^{17}\ cm^{-3}$ at $T = T_e$ level
- If collision radii are r_1 and r_2
  - $\sigma = \pi(r_1 + r_2)^2$
- H – H collision, r = 0.5 Angstrom
  - $\sigma = \pi\ 10^{-16}\ cm^2$
- L = 0.02 cm << H
  - i.e. gas collisions tend to establish TE, but photons can travel long distances
    - Especially high in the atmosphere

How far will photons fly?

- This is the subject of radiative xfer
  - Eventually, they fly freely into outer space!
- Consider a pencil beam travelling in s direction, over a distance ds
  - Intensity will be lost due to absorption or scattering
    - Amount lost proportional to σ, n, and ds



Optical depth over path s'=0 to s'=s gives diminution of intensity

Defining $\rho = n \cdot m$ and $\kappa_\lambda = \dfrac{\sigma_\lambda}{m}$ the mass-opacity

$n \cdot \sigma_\lambda = \kappa_\lambda \cdot \rho$

The probability of collision is $ds' \cdot \sigma_\lambda \cdot n = ds' \cdot \kappa_\lambda \cdot \rho$ << 1

The change in intensity is $dI_\lambda = -I_\lambda \cdot \kappa_\lambda \cdot \rho \cdot ds'$

or $\dfrac{dI_\lambda}{I_\lambda} = -\kappa_\lambda \cdot \rho \cdot ds'$

integrating both sides from s' = 0, [$I_\lambda = I_\lambda(0)$] to s' = s [$I_\lambda = I_\lambda(s)$]

$$\ln\left(I_\lambda(s)\right) - \ln\left(I_\lambda(0)\right) = -\int_0^s \kappa_\lambda \cdot \rho \, ds'$$

or $I_\lambda(s) = I_\lambda(0) \cdot e^{-\tau_\lambda}$

where $\tau_\lambda = \int_0^s \kappa_\lambda \cdot \rho \, ds'$ is the "optical depth"

Optical depth = 1, mean free path

- Over distance such that $\tau_\lambda = 1$
  - Intensity reduced by factor e^{-1}
- This is the mean free path for photons
  - $1 = \kappa_\lambda \rho L = n\, \kappa_\lambda L$
  - In the solar atmosphere, near 5000 Angstrom
    - L ~ 150 km
- Since L <u>not</u> << H, **LTE** is only an approximation
  - caution

"Visibility" = mean free path

- The weather report gives the "visibility", important for flying airplanes safely
    - See runway for landing, visibility > 100 m needed
  - Example, light rain falling
    - 1 mm dia drops, separation d =10 cm
      - $n = d^{-3} = 10^{-3}$ cm^{-3}
      - $\sigma = (1\text{ mm})^2$
    - $L = 1/(n\sigma) = 10^5$ cm = 1 km, adequate visibility
  - True whether scattering (snow, rain), or absorbing (soot)
  - At radio wavelengths, L >> 1 km, because the cross section is much smaller

Plane parallel atmosphere

- The sun is a sphere, but we can treat its atmosphere as an infinite plane
  - Thickness (600 km) is much less than the radius (700,000 km)
    - True also for earth's atmosphere
      - 9 km thick versus 6000 km radius
- The "plane parallel" approximation is very good
  - Consider rays traveling at angle θ to vertical

Extinction law in earth's atmosphere and stellar atmosphere

$$\tau_\lambda = \int_0^h \kappa_\lambda \cdot \rho \, dz$$

vertical optical depth z = 0 to h

$$ds = \frac{dz}{\cos(\theta)} \qquad \tau'_\lambda = \int_0^{z=h} \kappa_\lambda \cdot \rho \, ds$$

optical depth along slanted path

Therefore extinction increases with θ

$$I_\lambda(z = h) = I_\lambda(0) \cdot e^{-\tau'_\lambda} \qquad \ln\left(\frac{I_\lambda(z = h)}{I_\lambda(z = 0)}\right) = -\tau_\lambda \cdot \sec(\theta)$$

Same extinction law in stellar atmosphere as earth's atmosphere

Sources of opacity

- Both absorption and scattering count
  - Note that in TE, a good absorber is a good emitter
    - So losses can also lead to gain
- Bound-bound
  - i.e. H goes from n=1 to n=3 state
    - Absorbing photon of energy E3 – E1
  - Later on, atom could emit a E3-E1 photon
    - But in a random direction
    - Or it could emit E3-E2 followed by E2-E1 photons

Bound-bound, Bound-free

- Bound-bound only blocks light at discrete frequencies
  - Not so effective in blocking the whole Planck function
    - Important because otherwise the 10 million K photons in the interior would escape in 2 sec rather than 10,000 years, and the solar luminosity would be $10^4 \times 10^7\ L_{sun}$ – Yikes !
- Bound-free (*a.k.a* photoionization) blocks a wide range of frequencies

H bound-free (photoionization)

- Photons with energy > $\chi = -E_n$
  - > 13.6 eV, Lyman continuum
    - Excess energy goes to KE of electron, heating the gas
  - > 3.4 eV, Balmer continuum
    - Beyond energy of Balmer-limit as n → infinity
    - Shortward of 3647 Angstrom, the Balmer-jump
      - Sketch cross-section vs. wavelength
      - Peak ~ 10^{-17} cm^2, calculate L
    - Excess opacity blocks emergent intensity (solar spectrum)
    - Depresses U-band (v.graph)
      - Strongest in A0 stars