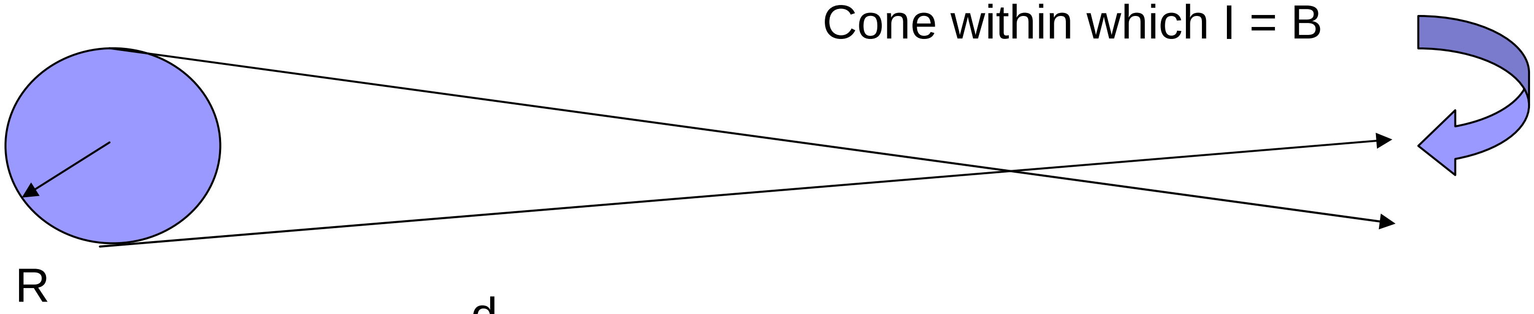
Handout 8
Chap. 9 Radiative Transfer

- First moment – a vector quantity – Flux
  - Usually, there is a special direction of interest
    - For telescopes, in the direction of the optic axis
    - In a star, in the outward direction
  - Flux in the z direction

$$F_\lambda = \frac{dE_\lambda}{d\lambda \cdot dt \cdot dA} = \int_{4\cdot\pi} I_\lambda(\theta,\phi)\cdot\cos(\theta)\, d\Omega = 4\cdot\pi \langle\cos(\theta)\, I_\lambda\rangle$$

    - Intensity is independent of distance, but flux depends on distance
    - Consider a black body at T, radius R, distance d >> R

Flux ~ $1/d^2$



- Within the cone of half-angle R/d, I = B
- Outside this cone, I = 0
- The flux is

$$F_\lambda = B_\lambda(T) \cdot \int_{\theta=0}^{\frac{R}{d}} \cos(\theta)\, d\Omega$$

and the cos(q) ~ 1 here so

$$\int_{\theta=0}^{\frac{R}{d}} \cos(\theta)\, d\Omega = \int_{\theta=0}^{\frac{R}{d}} 1\, d\Omega = \Omega_{star} = \frac{\pi \cdot R^2}{d^2}$$

and

$$F_\lambda = \Omega_{star} \cdot B_\lambda(T) = \frac{\pi \cdot R^2}{d^2} \cdot B_\lambda(T)$$

i.e. ~ $1/d^2$

Flux and intensity

- If we can **resolve** the source on the sky
  - We can measure the intensity
    - Which gives the temperature through identification with the Planck, black body function
      - This is called **brightness temperature**
      - **Nothing radiates better than a blackbody,** so it's a lower limit to the actual temperature
  - Integrating over the source gives the Flux
- For unresolved sources (i.e. stars)
  - We only measure the flux

2^{nd} moment – Radiation pressure

- Same derivation as gas pressure
  - Given a Maxwell-Boltzmann velocity distribution
- Pressure = Force/area = (momentum/time)/area
  - Pressure is like flux, but there's an extra cos(θ) term because we need momentum in the **z** direction
  - Photons act like gas, except the relationship between momentum and energy is slightly different
    - Gas: momentum = m v = 2 (E/v)
    - Photons: momentum = E/c = E/v, factor of 2 difference

$$P_{rad\_\lambda} = \frac{1}{c} \cdot \int_{4\cdot\pi} I_\lambda(\theta, \phi) \cdot \cos(\theta)^2 \, d\Omega = \frac{4\cdot\pi}{c} \langle \cos^2(\theta) \, I_\lambda \rangle$$

Radiation pressure P_{rad}

- If rad'n field **isotropic**
  - **cos²(θ)** integral gives (4/3)π
- Rad'n field **not** isotropic
  - Pressure depends on direction of d**A**
- Integral over wavelength gives total P_{rad}

$$P_{rad} = \int_0^{\infty} P_{rad\_\lambda}\, d\lambda$$

- For a blackbody rad'n field

$$\int_0^{\infty} B_\lambda(T)\, d\lambda = \frac{\sigma}{\pi}\cdot T^4 \qquad P_{rad} = \frac{4\cdot\sigma}{3\cdot c}\cdot T^4 = \frac{1}{3}\cdot a\cdot T^4 = \frac{1}{3}\cdot u$$

Radiation pressure

- Note: for ideal gas
  - $P = nkT = (2/3)\ u$
    - Twice as much pressure *cf*. radiation
- Importance of rad'n pressure
  - In interiors, helps support star against gravity
    - $P_{rad} \sim T^4$, but $P_{gas} \sim T$
    - Important in **hottest stars**, very small for sun
  - In atmosphere and interstellar space
    - Rad'n pressure can accelerate things

Force of radiation on dust

- Dust tails of comets
  - $F_{rad} \sim$ area $\sim r^2$
  - $F_{gravity} \sim$ volume $\sim r^3$
  - $F_{rad} / F_{gravity} \sim 1/r$
- Most important for small dust grains
  - i.e. not the 10 km nucleus
- Orbits of dust grains differ from nucleus
  - Curved dust tail (Lecture 2 slide 5)
- Smallest grains ejected into interstellar space

Eddington limit

- Atoms in stellar atmosphere
    - Energy is flowing outward
    - Therefore more pressure on bottom than top
    - Therefore **outward** force
  - If $F_{rad} / F_{gravity} > 1$, liftoff!
    - Gas accelerated away, atmosphere lost
  - Since $P_{rad} \sim T^4$, important in hottest stars
- This determines the max possible mass of a star – The *Eddington Limit* $\sim 100\ M_{sun}$