Handout 7
Chap. 9 Stellar Atmospheres

- Goals
  - Understand why stellar spectra closely resemble black bodies
    - i.e. shape of F_λ vs $\lambda \sim B_\lambda(T_e)$
    - $L = 4\pi r^2 \sigma T_e^4$
  - Deviations from a blackbody
    - i.e. absorption lines
      - Give Temperature
      - Give abundances of n_{Ca}/n_H, etc.
      - Give Luminosities

Radiative transfer

- Need to understand transfer of rad'n through a medium which
    - 1) Absorbs some
    - 2) Scatters some at angle θ
    - 3) Emits some
  - 1 and 2 reduce the intensity, while 3 increases it
- First need a formal description of the radiation field

Classical view of radiation field

- Picture photons streaming in one direction until they hit something
  - A stream of energy carried by photons at speed c
    - Ignore polarization for the time being
  - Fundamental unit of this radiation is the **intensity**
    - Figures 3.9 and 9.1 show light described by I_λ
      - Travelling in direction θ,ϕ
      - Emanating from dA

Definition of intensity

- Into solid angle $d\Omega = dA'/r^2 = \sin\theta d\theta d\phi$
- In wavelength interval $\lambda \rightarrow \lambda + d\lambda$
- In time interval dt
- Which carries energy dE_λ (ergs or joules)

- Taking the limit dA, $d\lambda$, dt, $d\Omega \rightarrow 0$, the intensity is

$$I_\lambda = \frac{dE_\lambda}{d\lambda \cdot dt \cdot (dA \cdot \cos(\theta)) \cdot d\Omega}$$

note: Flux F_λ would have the same **units**, except for the $d\Omega$

Intensity conserved in free space

- In the limit $d\Omega \rightarrow 0$
  - I_λ is a **ray**
  - Energy doesn't diverge
  - I_λ is conserved in free space
    - Also conserved in a perfect optical system of lenses and mirrors
- Knowledge I_λ of as a function of position **r** and direction θ,ϕ is a complete description of the radiation field
  - Example, inside b.b. cavity $I_\lambda = B_\lambda(T)$

Moments of radiation field

Maybe it's obvious, but the solution of the equations of radiative transfer is very difficult. Only with extensive computer calculations can solutions be found. This method leaves one without physical insight – therefore we seek simplifying assumptions.

- Take $\cos^n(\theta)$ weighted averages of I_λ over directions in space = "moment"
  - Zeroth moment = average, $\langle I_\lambda \rangle$
  - First moment = Flux F_λ
  - 2nd moment = Radiation pressure

Average I_λ

$$\langle I_\lambda \rangle = \frac{1}{4\cdot\pi}\cdot\int_{4\cdot\pi} I_\lambda(\theta,\phi)\,d\Omega$$

- Important for heating of a dust grain, or excitation of an atom
  - If the radiation field is isotropic $< I_\lambda > = I_\lambda$
    - Inside a blackbody cavity, $< I_\lambda > = B_\lambda$ (T)
    - What is $< I_\lambda >$ at the orifice of a b.b. cavity?
      - $I_\lambda = 0$ in upper hemisphere
      - $I_\lambda = B_\lambda$ (T) in lower hemisphere
      - $< I_\lambda > = B_\lambda$ (T)/2
    - If a black grain of dust were placed at the orifice, theheating power would be half that of one inside the cavity
      - Equilibrium temperature given by

$$\sigma\cdot T_{dust}{}^4 = \frac{1}{2}\cdot\sigma\cdot T_{bb}{}^4 \qquad T_{dust} = \left(\frac{1}{2}\right)^{\frac{1}{4}}\cdot T_{bb}$$

incidentally, this is the minimum temperature in a stellar atmosphere, i.e. **not** zero

Average I_λ and energy density of rad'n field

- The energy density is given by
  - $u_\lambda = (4\pi/c) < I_\lambda >$
  - Integrating over wavelengths

$$u = \int_0^\infty u_\lambda \, d\lambda = \frac{\text{erg}}{\text{cm}^3}$$

  - Inside a b.b. cavity

$$u = \frac{4\pi}{c} \cdot \int_0^\infty B_\lambda(T) \, d\lambda = \frac{4 \cdot \sigma}{c} \cdot T^4 = a \cdot T^4$$

    - Which makes sense since flux F_λ from a black body is given by π B_λ and flux integrated over wavelength is σT^4 .