Handout 6
Chap. 8 Spectral Classification

- Class based on strength of absorption lines in spectrum (see v.g.)

	← T increasing						
	O	B	A	F	G	K	M
H lines	*weak* ←	*S*	→ *weak*				
Ca II			*weak* ←	*S*	→ *weak*		
TiO molecules				*weak* ←	*S*		

"S", strong

Visible (Balmer) H lines

- Arise from n=2 level
  - i.e. 2➔3 is Balmer α (H α)
- $E_2 - E_1$ = 10.2 eV, need to be excited to n=2
  - Boltzmann analysis of probabliity vs T and E (assuming Thermal Equilibrium)

$$\frac{P(E_2)}{P(E_1)} = \frac{g_2}{g_1} \cdot e^{\frac{-(E_2 - E_1)}{k \cdot T}}$$

g = "statistical weight", i.e. for H

n=1 l=0 s = +/- 1/2 ➔ g = 2

n=2 l=0 s = +/- 1/2 2
l=1 m=+1,0,-1 s= +/- 1/2 6
➔ g = 8

Statistical weights partition function

In general, for H, l = 0, …, n-1

m = -l, -l+1, …, +l

and s = +/- ½

So $g_n = 2n^2$

The generalization of the Boltzmann equation for any number of excited states involves the "partition function" Z:

$$Z = g_1 + \sum_n g_n \cdot e^{\frac{-(E_n - E_1)}{k \cdot T}}$$

so

$$P(n = 2) = \frac{g_2}{Z} \cdot e^{\frac{-(E_2 - E_1)}{k \cdot T}}$$

Temperature dependence, Balmer lines

- From the B'mann equation, expect Balmer lines to increase up to 120,000 K
  - kT = 1/40 eV at T =300 K
  - kT = 1 eV at T = 12,000 K
- But peak strength of H-lines at spectral type A0
  - T_e = 9500 K
- ➔ ionization reduces strength of H-lines above 9500 K

Ionization, Saha equation

Boltzmann considered thermal equilibrium balance

$H_{n=1} \leftrightarrow H_{n=2}$

E_2-E_1 activation energy

Saha considered balance between neutral and ionized H:

$H \leftrightarrow p + e$

χ_i ionization energy

$$\frac{n_p \cdot n_e}{n_H} = \frac{Z_p \cdot Z_e}{Z_H} \cdot e^{\frac{-\chi_i}{k \cdot T}}$$

Z_e = # of places to put an electron in position-momentum phase space considering

1) the velocities are **Maxwell Boltzmann** at temperature T
2) The **Heisenberg** uncertainty $\Delta x \Delta p = h/2\pi$
3) **Pauli**: since s=1/2, only 2 electrons allowed in each vol. of phase space

H ionization in stellar atmospheres

We have

$$Z_H = g_1 + \sum_n g_n \cdot e^{\frac{-(E_n - E_1)}{k \cdot T}}$$

which is very close to $g_1 = 2$

$$Z_p = 1$$

a naked proton, only one possibility

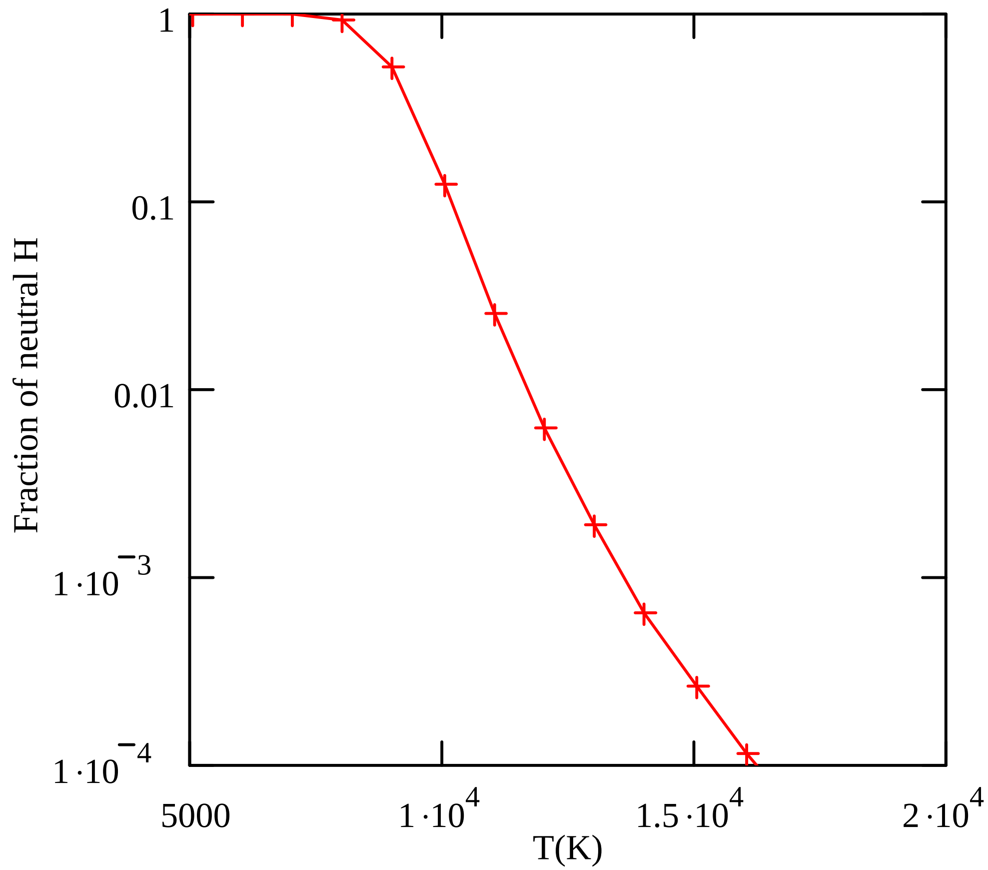
$$Z_e = 2 \cdot \left(\frac{2 \cdot \pi \cdot m_e \cdot k \cdot T}{h^2} \right)^{\frac{3}{2}}$$

the first 2 accounts for the spin of the electron

At typical photospheric densities n_e, ionization occurs when T exceeds about 10,000 K

Ionization of H in a stellar atmosphere

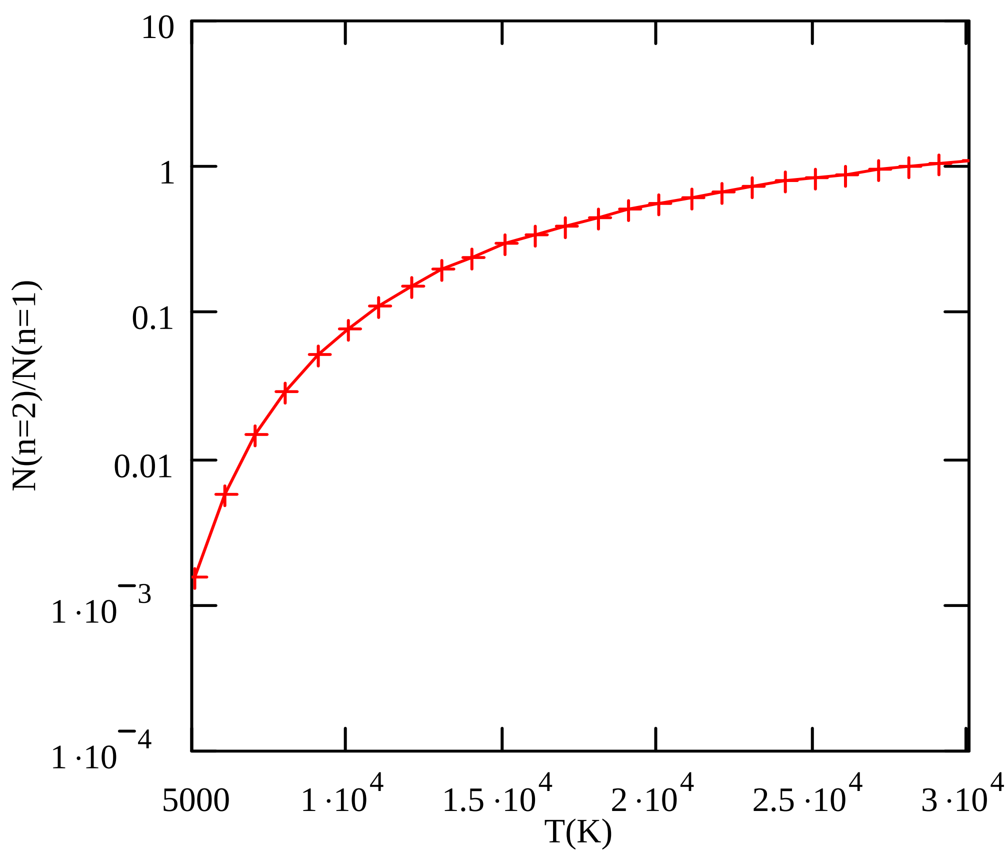
$n_e(10000\,\mathrm{K}) = 4.572\times 10^{13}\ \mathrm{cm}^{-3}$



Ionization of H in a stellar atmosphere

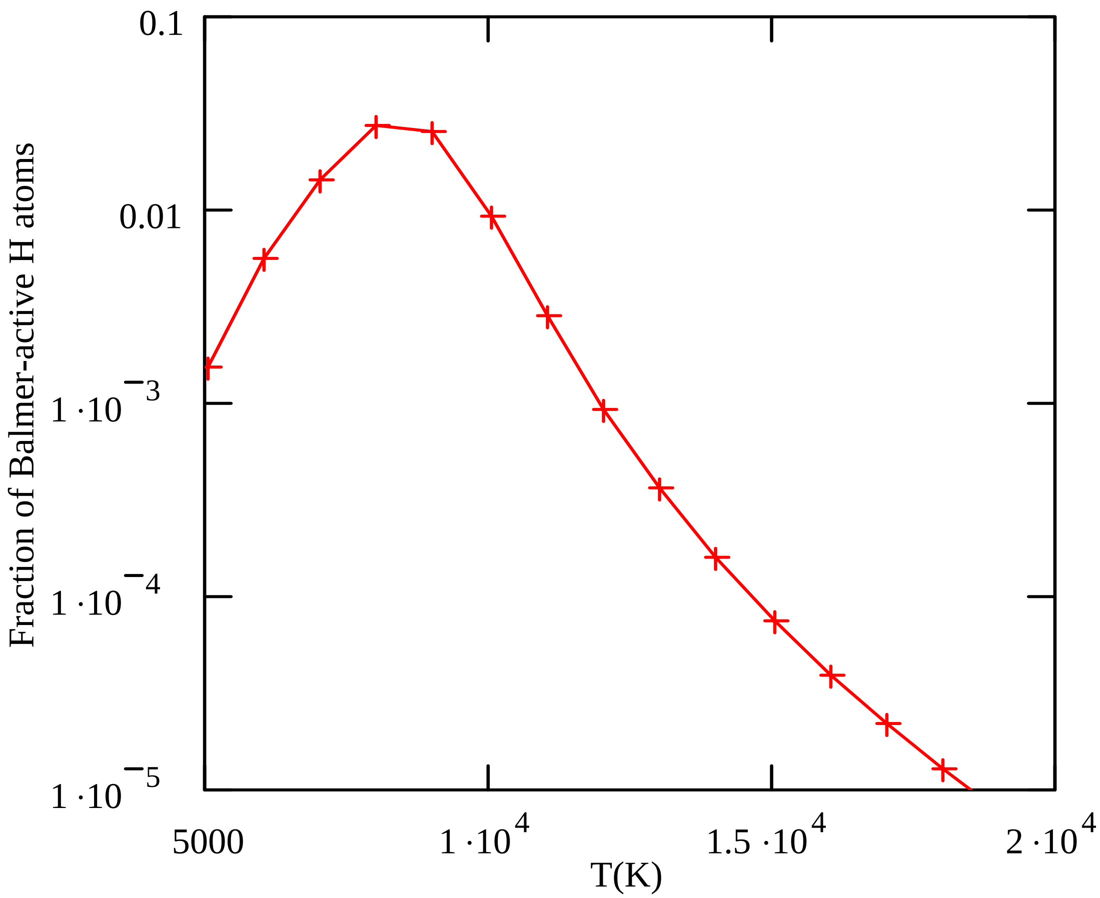
Excitation of n=2 level of H

Ratio of densities of H atoms in the n=2 state to the n=1, ground, state



Strong Balmer lines, product of last two functions

Product of previous 2 functions: fraction of neutral H in the n=2 state



Ca II H&K, Na I D-lines

- From **ground** state (resonance)
  - Most Ca II is in gnd. State (99.9% at stellar T)
  - Strength governed by ionization
    - M stars Ca I mainly
    - G-K stars Ca II mainly
    - A stars Ca III mainly
- Explains why Ca II stronger than H in solar spectrum
  - Even though abundance Ca ~ 10^{-6} H