Handout 4: Chap. 7: Binary Stars

The key information we gain from binary stars are their masses. When Cavendish first measured Newton's gravitational constant G, he asserted that he was "weighing the Earth." In like manner, we can weigh stars by measuring the gravitational forces they exert.

This information (masses) is crucial to any theoretical understanding of the nature of stars. In like manner, gravitational theory is used to measure the masses of galaxies.

Secondarily, eclipsing binaries can give accurate stellar radii, which is also needed for stellar theories. In this case, there are alternative methods, i.e. interferometry, lunar occultation, black body theory, which can give reasonably accurate stellar radii.

Mass derived from acceleration

- Recall Newton's
  - Laws of gravity and motion
  - Measurement of acceleration
    - Implies Force
      - Which implies mass of attractor

$$-\overrightarrow{F_{21}} = \overrightarrow{F_{12}} = G\cdot\frac{m_1\cdot m_2}{{r_{12}}^3}\cdot\overrightarrow{r_{12}}$$

$$\overrightarrow{a_1} = \frac{\overrightarrow{F_{12}}}{m_1} \qquad \overrightarrow{a_2} = \frac{\overrightarrow{F_{21}}}{m_2}$$

Radii derived from timing

- If Binary orbit is edge on
  - Eclipses
  - Timing of eclipses gives stellar radius
- Note sun's angular diameter 0.5 deg
  - At a distance of 1 pc diameter 0.01 arcsec
  - Hard to measure size of dwarf, main sequence stars

Binaries tell us about star formation

Majority of stars form in multiple systems. Understanding the nature of binaries can inform our theories of star formation.

- What sorts of masses m form?
  - Mass spectrum called the “Initial Mass Function”
- What size are the orbits?
  - From 0.1 to 10,000 AU
- What are the eccentricities?
  - All the way from zero to 1!

Binaries and star/planet formation

- Star formation/binaries seems distinct from planet formation
  - Planet mass << sun
    - Binaries often have nearly equal masses
  - Orbits all in ecliptic
    - triple stars have random planes
  - Planets eccentricity near zero (nearly circular)
- But "extra solar" planets different
  - Jupiters found near star << 1 AU
  - Eccentricities definitely not near zero
  - Is it a planet or a brown dwarf?

Kepler/Newton Laws of Planetary Orbits

- Kepler's three laws
  - I Orbits are ellipses, with sun at one focus
  - II Radius vector sweeps out equal areas in equal times
  - III $P^2 = a^3$ in solar system units
- Newton (laws of motion and gravity)
  - I Ellipses, but center of mass at one focus
    - The 2 bodies orbit about their common c.m.
      - Each body follows an elliptical orbit
      - Jupiter-sun c.m. at surface of the sun
      - The c.m. moves at constant velocity

Motion about center of mass

Center of mass defined by

$$\left(m_1 + m_2\right)\cdot\overrightarrow{r_{cm}} = m_1\cdot\vec{r_1} + m_2\cdot\vec{r_2}$$

teeter totter

assuming $\overrightarrow{r_{cm}} = 0$ i.e. inertial frame of reference

$$-\vec{r_2}\cdot m_2 = \vec{r_1}\cdot m_1 \qquad \text{or} \qquad \frac{m_2}{m_1} = \frac{r_1}{r_2}$$

taking the time derivative, we also have

$$-\vec{v_2}\cdot m_2 = \vec{v_1}\cdot m_1 \qquad \text{or} \qquad \frac{m_2}{m_1} = \frac{v_1}{v_2}$$

For the velocity projected along the line of sight (i.e. Doppler Shift)

$$\frac{m_2}{m_1} = \frac{-v_{1.z}}{v_{2.z}}$$

Newtons view of Kepler's Laws

- For KII, Newton asserted
  - Force of gravity is <u>radial</u>
    - Torque (**r** x **F**) is zero
    - Angular momentum **L** is constant
      - dA/dt = **r** x **v** = **L**/m
- For KIII, first consider m1>>m2, and circular orbits

$$r_2 = a \qquad \frac{2\cdot\pi\cdot r_2}{v_2} = P \qquad \frac{m_2\cdot {v_2}^2}{r_2} = G\cdot\frac{m_1\cdot m_2}{{r_2}^2}$$

Newtons version of KIII

substitute $v_2 = \frac{2\cdot\pi\cdot r_2}{P}$ gives

$$\frac{G}{(2\cdot\pi)^2}\cdot m_1\cdot P^2 = a^3$$

Converting the 2-body to the equivalent 1-body problem

reduced mass $\mu = \frac{m_1\cdot m_2}{m_1 + m_2}$ orbiting around a fixed mass M = m1 + m2

at a distance $\vec{r}_{..} = \vec{r_{12}}$ gives

$$\frac{G}{(2\cdot\pi)^2}\cdot M\cdot P^2 = a^3$$ or, in solar system units $M^1\cdot P^2 = a^3$

Application to binary orbits

- Two basic types
  - Visual
    - Orbit can be resolved on the sky
      - Larger orbits & closer stars
      - 1 AU at 1 pc is 1”, a small angle
  - Spectroscopic
    - Doppler shifts measured
      - For v << c, only radial velocity gives shift

$$\frac{\Delta\lambda}{\lambda_0} = \frac{-\Delta\nu}{\nu_0} = \frac{v}{c}$$

$\Delta\lambda > 0$ red-shift, v>0, receeding

$\Delta\lambda < 0$ blue-shift, v<0, approaching