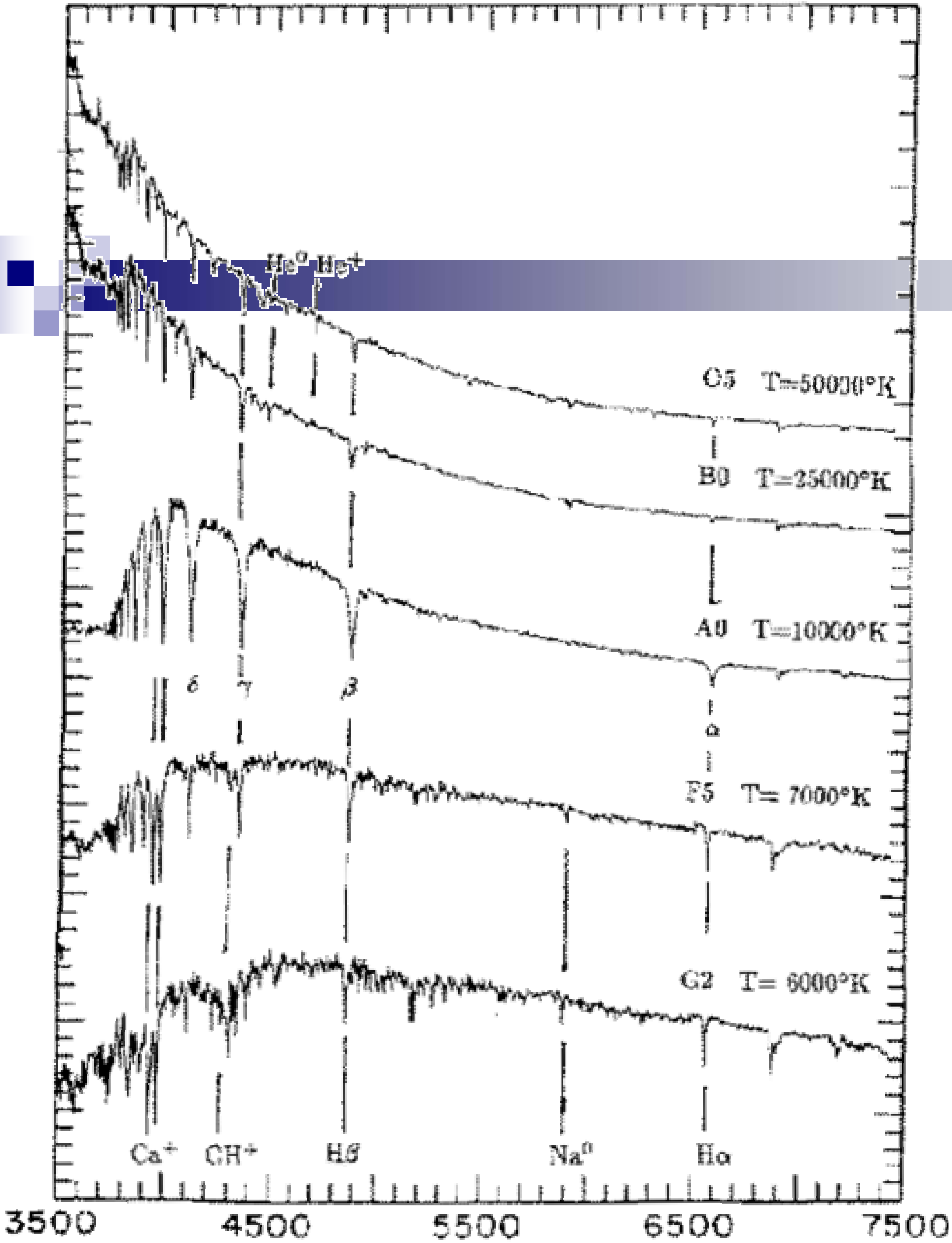
Handout 3: Chap. 5: Light & Matter

If everything radiated like a blackbody, spectra would be quite boring. In fact, deviations from a blackbody tell us a lot about stars (and the rest of the universe). Deviations occur for many reasons. Some objects (i.e. dust grains smaller than λ) don't radiate as blackbodies. Resonances (such as the Si-O vibration mode at 10 μm) imply enhanced absorption. According to Kirchoff (and thermo), this implies enhanced emissivity. Hence the observed flux is given by $F_\lambda = (\Omega B_\lambda)\varepsilon_\lambda$, where the emissivity ε_λ shows peaks at resonances which are characteristic of the chemical composition.

Importance of Stellar Spectra

Stars show dark lines (Fraunhoffer 1814, missed by Newton). The dark lines occur at the same λ's as emission lines of terrestrial substances in Bunsen burners – i.e. the Na D lines near 5900 Angstroms. Also, many lines of Fe. This implies there is Na and Fe in the atmospheres of stars. In fact, an important goal of understanding stellar spectra is to be able to accurately measure the abundances of, eg., Fe, Na, O, C, etc.w.r.t. H.

Rainbow is natural Solar spetrum



Flux (y-axis) versus wavelength (Angstroms) for various stars.

The sun is spectral type G2 with an effective temperature near 6000 K.

It shows dark (absorption) lines from H, neutral Na (D lines), singly ionized Ca (H and K lines) and many others.

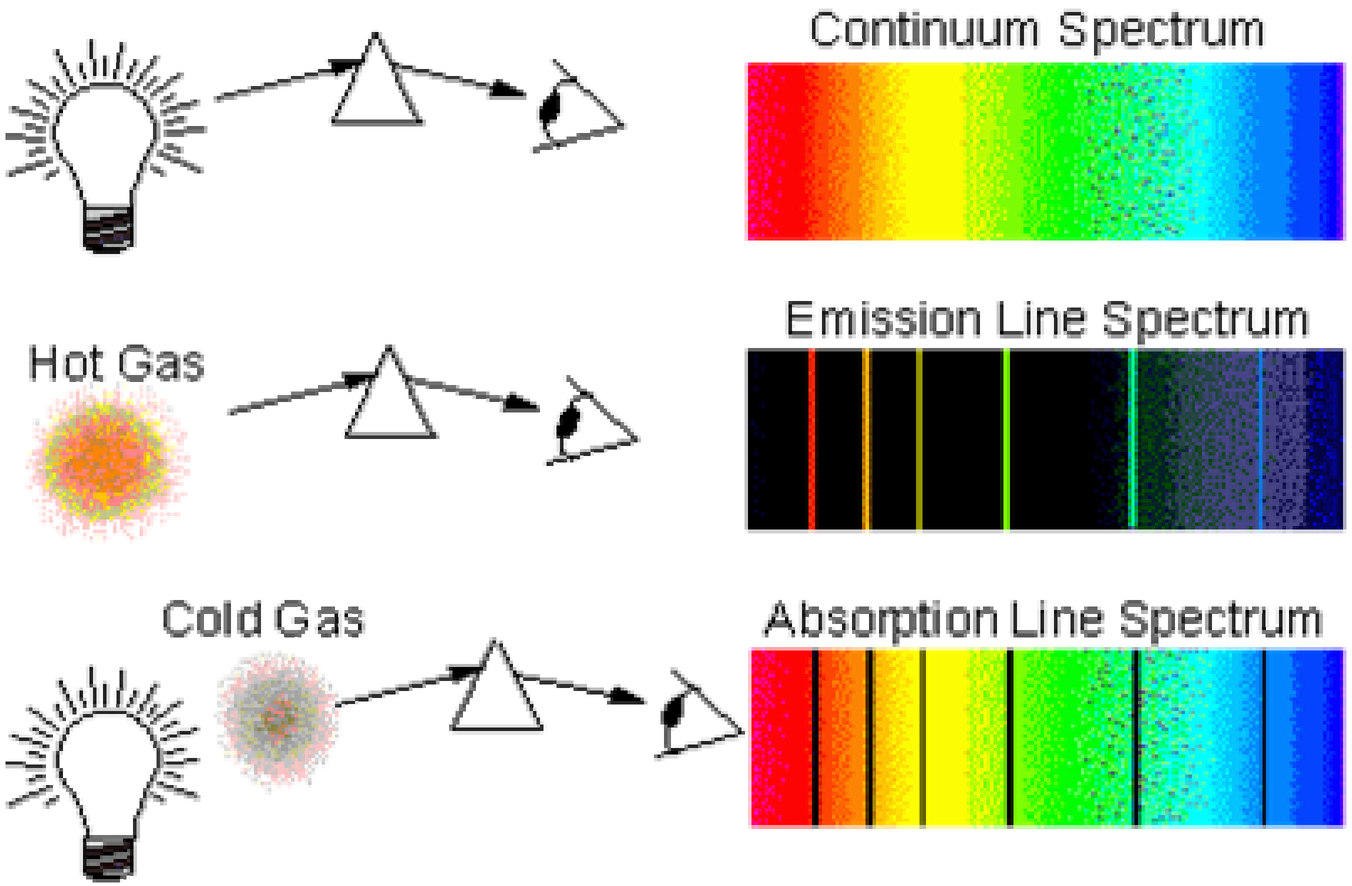
The Na and Ca+ lines are as strong as the H lines, even though the abundances of Na and Ca are about 10^{-5} that of H!

Kirchoff's 3 laws of Spectra

We will prove these laws in our treatment of radiative transfer -- which is fundamental to understanding the meaning of light

- 1) Dense, hot object
  - Emits a continuous spectrum
    - Intimately related to the Planck function
- 2) Heated, diffuse gas
  - Emission line spectrum
- Cooler gas in front of hotter object
  - Absorption lines at same λ's as 2)

Kirchoff’s three laws



Solar Spectrum

- Dark lines such as
  - Hα (‘C’), Hβ (‘F’), Na D, Ca II H&K strongest
    - Ions (i.e. Ca II strong) ➔ Saha eq’n
    - Na D lines comparable to H-lines
      - Excitation conditions
- Lines from discrete E-levels in atoms
  - Simplest case H (i.e. lab)

H spectrum

- Balmer, visible, series (Hα, β, etc.) fit by:

$$\frac{1}{\lambda} = R_H \cdot \left(\frac{1}{4} - \frac{1}{n^2} \right)$$

note: $\frac{1}{\lambda} = \frac{\nu}{c}$, so $E = h \cdot \nu = \frac{h}{c} \cdot \frac{1}{\lambda}$

i.e. energy levels of H atom proportional to $1/n^2$.

Other series (Paschen, Lyman, etc.) given by

$$\frac{1}{\lambda} = R_H \cdot \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

m=2 Balmer, 1 Lyman, 3 Paschen, 4 Brackett, etc.

Bohr model – semi-classical

- Allowed orbits have $L = n(h/2\pi)$
  - Should use 'reduced mass' for electron for more accuracy
    - $m_{proton}/m_{electron} = 1836$

Attractive force, cgs units $$F = \frac{|q_1 \cdot q_2|}{r^2}$$ set equal to centripetal accelaration

$$m \cdot \frac{v^2}{r} = \frac{|q_1 \cdot q_2|}{r^2}$$ but requiring

$$L = m \cdot v \cdot r = n \cdot \left(\frac{h}{2 \cdot \pi}\right)$$ with n = 1, 2, 3, etc gives.

$$E = KE + PE = \frac{PE}{2} = -KE$$ Virial theorem

Since $v^2 \sim 1/r$ and $vr \sim n$, $n^2 \sim r$ and therefore $E \sim -(1/n^2)$

Nature of H atom

- Radius of ground n=1 state 0.5 Angstrom
  - “Size” of H atom
    - Excited states bigger by factor n^2
- Energy gnd state $E_{n=1} = -13.6$ eV
  - Ionization energy of H
    - T for ionization??, kT = E
      - kT = 1/40 eV for room T
      - 12,000 K for 1 eV
      - 120,000 K for 10 eV
    - Only one part of story
      - Saha equation → 10,000 K H starts to ionize in atmosphere

Excitation needed for Balmer series

- For n=2, $1/n^2 = \frac{1}{4}$.
  - n=2 level 10.2 eV above gnd state
  - Probability of n=2 compared to n=1
    - Boltzmann factor $e^{-(E2-E1)/kT}$ most of story
    - kT = 0.5 eV sun's atmosphere
    - Factor = $e^{-20} \sim 10^{-8}$
      - Reason Na D lines as strong as H-Balmer lines
        - Na D lines from **ground** state
        - Na D lines **resonance** lines