Nature of Light

- Two aspects – both important
  - Wave
    - Young's double slit – destructive and constructive interference of monochromatic light. i.e.
      - If Δ(path length) = nλ, constructive
      - If Δ(path length) = (n+1/2)λ, destructive
        - Where $\lambda\nu$ = c, fundamental constant
    - Maxwell: wave is E and B fields transverse to propogation (draw on board)

Light = Wave

- Pointing vector **S**
  - Strength of E & B fields gives ***energy flux***
    - ***(erg/s)/cm2 =*** $\langle \mathbf{S} \rangle_{\text{one cycle}} = (c/8\pi)E_0B_0$
      - ***E_0 and B_0*** amplitude of electric, magnetic sine waves
        - Monochromatic here
      - For $\langle S \rangle$ = solar constant, E_0 = 10 volt/cm

Particle nature of light

- Einstein's analysis of PhotoElectric effect clearly showed this
  - Minimum unit of E-M energy is the <u>photon</u>
    - $E = h\nu$
      - note: for $E = 1$ eV, $\lambda \sim 1\ \mu m$
        - ?? λ for a keV X-ray?
  - Photons also carry momentum, $P = E/c$
    - Rad'n force = dP/dt = photons/s (absorbed) x $h\nu/c$

Radiation force (pressure)

- Radiation pressure typically weak, but clearly seen in comet tails.
  - The curved, dust tail is particles whose orbit was changed by solar radiation force
  - Red Giant stars have $L \sim 10^4 L_{sun}$, but $M \sim M_{sun}$, so radiation force 10^4 times larger than for sun
- Rad'n pressure can be significant in stars
  - Interiors and atmospheres

Comet West 1975

The blue tail is the gas tail
It points directly away form the sun pushed by the solar wind

The curved tail to the left is the dust tail
- Microscopic dust grains
- In orbit around the sun
- Orbit is modified by rad’n pressure
- Spectrum = reflected sunlight

The coma is the bright glow at the bottom

The 10 km nucleus would appear as small as the stars in this image

Radiation pressure, Eddington limit

- Note that for ideal gas
  - P = nkT, and E-density = (3/2)nkT
- For Black Body rad'n, E-density ~ T^4
  - Rad'n pressure most important in hottest stars
  - Rad'n pressure sets max. mass of star ~ 100 M_{sun}
    - If M > 100 M_{sun}, L is so high that radiation pressure exceeds gravity
      - Liftoff!
      - Called Eddington limit

Blackbody radiation

- Spectrum depends on T, not composition
  - Blackbody = perfect absorber of rad'n
      - By thermodynamics, also a perfect emitter
    - Black paint ~ 90% absorber, i.e. <u>not</u> perfect
    - Put small hole in opaque cavity held at T
      - Each time photons hits wall, 90% absorbed
      - $(10\%)^N$ approaches zero as N increases
  - All such bodies emit a universal ***blackbody*** or ***Planck*** spectrum

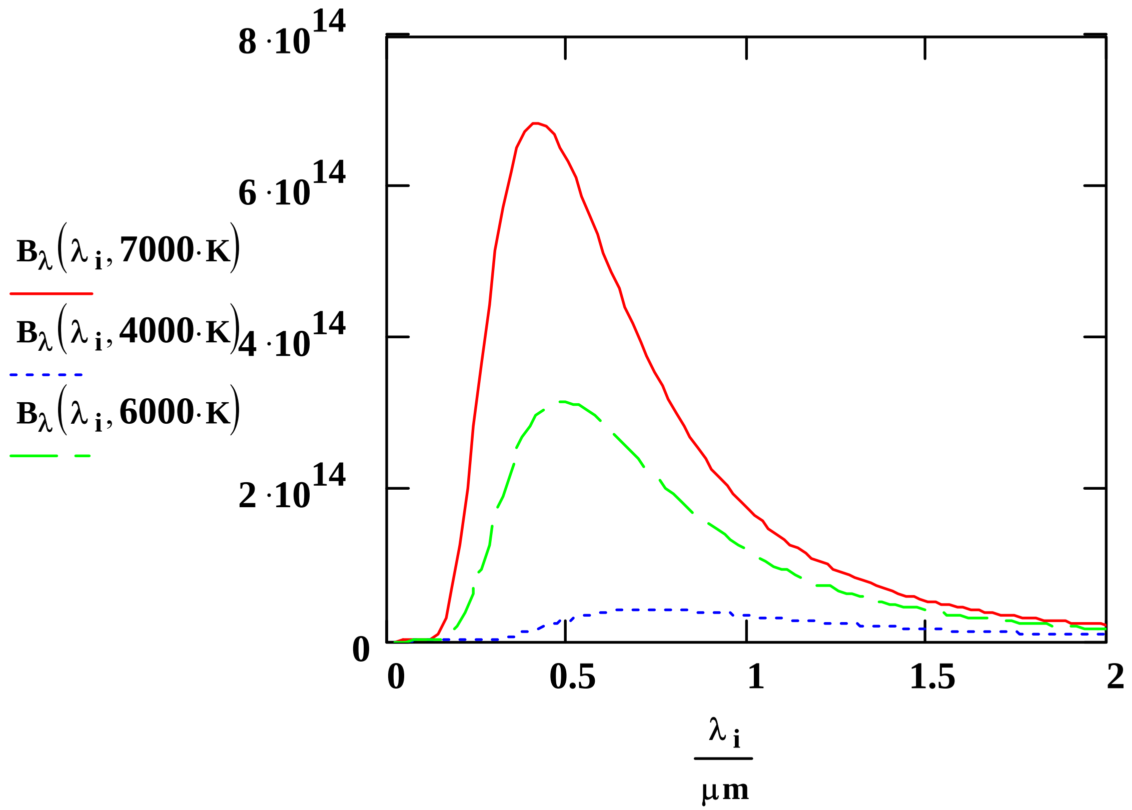
Blackbody or 'Planck' spectrum

Flux F_λ is related to surface brightness B_λ through the solid angle Ω of the blackbody

$$F_\lambda = \Omega \cdot B_\lambda$$

$$F_\lambda = \frac{\text{power}}{\text{area} \cdot \Delta\lambda} = \frac{\text{"erg/s"}}{\text{cm}^2 \cdot \text{Angstrom}}$$

Spectral plots (i.e. F_λ versus λ) are illuminating:



Blackbody rules

- There is radiation at ***all*** wavelengths
- Hotter bodies radiate more at ***all*** wavelengths
- The peak wavelength of flux is related to T through
  - $\lambda_{max}T = 0.29$ cmK (Wien displacement law)
    - Short of the peak is a very steep falloff
      - Slower falloff longward of peak
- Total power emitted = (area)*σ*T^4
  - Integrate over λ's, and 2π ster hemisphere
  - $\sigma = 5.7\ 10^{-5}$ cgs (Stefan Boltzmann constant)
    - Use this to determine effective T of sun

Effective T of the sun, from solar constant, angular size, and distance of sun:

$\theta_{sun} := 0.54 \cdot deg$ — angular diameter of sun

$d_{sun} := 1 \cdot AU$ — distance of sun

$sc = 1.36 \dfrac{10^3 \cdot W}{m^2}$ — bolometric radiant flux from sun at earth

---------------------------- calculations ----------------------------------

$R_{sun} := \dfrac{\theta_{sun}}{2} \cdot d_{sun}$ — physical size of sun

$L_{sun} := sc \cdot 4 \cdot \pi \cdot d_{sun}^{\;2}$ — Luminosity sun — $L_{sun} = 3.845 \times 10^{33} \dfrac{erg}{sec}$

using $L := 4 \cdot \pi \cdot R^2 \cdot \sigma \cdot T^4$

we find $T_{sun} := \left(\dfrac{L_{sun}}{4 \cdot \pi \cdot R_{sun}^{\;2} \cdot \sigma} \right)^{0.25}$ — $T_{sun} = 5.733 \times 10^3 K$

Temperatures, λ's of stars, planets

- Applying the Wien law to the sun

$$\lambda_{\mathbf{max}} = 5 \cdot 10^{-5} \cdot \mathbf{cm} = 5000 \cdot \mathbf{Angstrom} = 500 \cdot \mathbf{nm} = 0.5 \cdot \mu\mathbf{m}$$

    - Eyeballs are sensitive to these radiations
  - ?? λ_{max} for human, room at 300 K?
    - 20 times colder, 20 times longer λ, λ_{max} = 10 μm
      - **Infrared!**
        - Likewise for all the planets, 50-600 K
- Stars 2000 – 50,000 K
  - Near infrared, visible, UV

Planck's Function

Planck was able to derive the observed blackbody spectrum using Thermodynamics and Statistical Mechanics -- but only if the walls of the cavity emitted energy in multiples of the minimum energy = h ν. He showed:

$$B_\lambda = 2 \cdot h \cdot c^2 \cdot \frac{1}{\lambda^5 \cdot \left(e^{\frac{hc}{\lambda k T}} - 1 \right)}$$

Note: $\frac{hc}{\lambda kT} = \frac{h\nu}{kT} = \frac{E}{kT}$ **and the exponential e $^{-E/kT}$ dominates at short λ's**

The units of the Planck function are "surface brightness" or "intensity"

$$dE = B_\lambda \cdot d\lambda \cdot \left(dA \cdot \cos(\theta) \right) \cdot dt \cdot d\Omega$$

energy

λ

area

time

Solid angle

Solid angle, area

- See fig. 3.9 for definition of
  - Solid angle $d\Omega = dA'/r^2 = \sin\theta d\theta d\phi$
  - $dA\cos\theta$ = projected area perpendicular to line of sight as seen from $d\Omega$
    - Where our detector of area dA' (perpendicular to this line of sight) is placed, a distance r from dA
- Solid angle is to area as
  - Angle is to length
    - Units ster
      - 4π ster solid angle surrounds a point in 3-d space
      - Hemisphere is 2π ster

Integrals over Planck function

Integrating the Planck function over wavelength gives:

$$\mathbf{B} = \int_0^{\infty} \mathbf{B}_\lambda \, d\lambda = \frac{\sigma}{\pi} \cdot \mathbf{T}^4$$

integrating over hemisphere (angles: ϕ 0 ==> 2π, θ 0 ==> π/2) gives

$$\frac{dE}{dA \cdot dt} = B \cdot \int_{\text{hemisphere}} 1 \, d\Omega = B \cdot \int_0^{2\cdot\pi} \int_0^{\frac{\pi}{2}} \sin(\theta) \cdot \cos(\theta) \, d\theta \, d\phi = \sigma \cdot T^4$$

i.e. **blackbodies radiate (effectively) into π ster.**

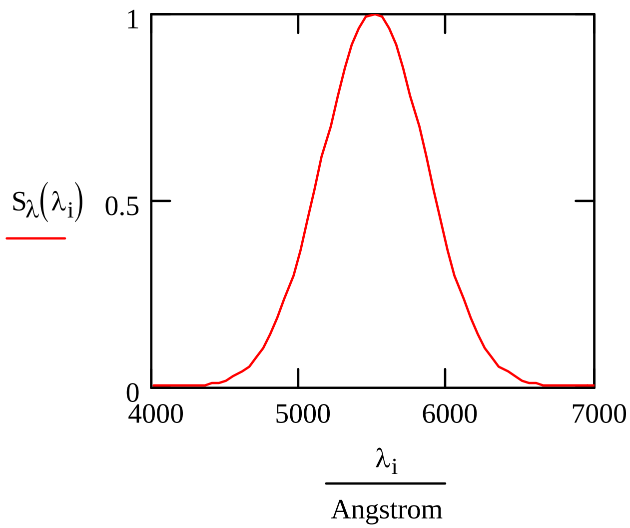
Fluxes, magnitudes at various λ's

recalling F_λ = "monochromatic flux"

defining T_λ = "filter transmission" and ε_λ = "detector efficiency"

$$S_\lambda = T_\lambda \cdot \varepsilon_\lambda = \text{"system sensitivity"}$$

a typical system sensitivity will look like



Filter	λ_0(Angstrom)	$\Delta\lambda$(Angstrom)
U	3650	680
B	4400	980
V	5500	890

Now, define the in-band flux, i.e. $F_B = \int_0^\infty S_\lambda \cdot F_\lambda \, d\lambda$ approximately $F_B := F_{\lambda 0} \cdot \Delta\lambda$

Then a blue magnitude, m_B, which is denoted B, is defined w.r.t. a standard star of known mag.

The star α Lyr (Vega), Sp type A0, T ~ 9500 K is very nearly m = 0 at all λ's. Thus from the definition of magnitude, with 1 = unknown star and 2 = Vega, we have

$$m_B = -2.5 \cdot \log\left(\frac{F_B}{F_{B,\,Vega}}\right)$$

Likewise for U, V, and other filters.

Colors, which are related to temperatures, are defined as differences between magnitudes:

$"color" = m_{\lambda 1} - m_{\lambda 2}$ where, by convention $\lambda_1 < \lambda_2$

i.e. $m_B - m_V = B - V$ color related to temperature

The color of **Vega** is zero, for all wavelengths. For a star like the **sun, cooler** than Vega, the flux ratio F_B/F_V is less than Vega, i.e. redder. Hence the magnitude **B** is **bigger** than **V**. Therefore the color **B - V** is **positive** (red). For the sun, B - V ~ 0.6 mag.

For reference, zero mag at V is about $10^3 \cdot \frac{\left(\frac{photons}{sec}\right)}{cm^2 \cdot Angstrom}$