Column density for 0 km/s cloud AF01n1 from Dickey et al. 21 cm paper

$$\begin{split} T_{B} &\coloneqq 15 \cdot K & \text{Brightness T of cloud} \\ B_{\lambda}(\lambda, T) &\coloneqq 2 \cdot h \cdot \frac{c^{2}}{\lambda^{5} \cdot \left[exp \left[h \cdot \frac{c}{\left[\lambda \cdot (k \cdot T) \right]} \right] - 1 \right]} \end{split}$$

Planck function. At low frequencies (1420 MHz, and high T (> 3 K) this reduces to

i.e. linearly proportional to T. This is the Rayleigh-Jeans tail $B_{\lambda}(\lambda, T) := 2 \cdot \frac{c}{\lambda^4} \cdot k \cdot T$ of the Planck function, correct far beyond the peak wavelength λ_{max}

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Setting observed intensity I equal to the Planck function B defines the brightness Temperature, so the intensity of cloud AF01n1 is:

$$I_{21cm} \coloneqq B_{\lambda}(21 \cdot cm, T_B) \qquad \qquad I_{21cm} = 6.386 \times 10^{-13} \frac{W}{m^2 \cdot cm \cdot ster}$$

To find the H column density, we need the integrated line strength:

ed line strength:
$$I = I_{\lambda} \cdot \Delta \lambda = I_{\nu} \cdot \Delta \nu$$

$$\Delta \lambda := 21 \cdot \text{cm} \cdot \frac{5 \cdot \frac{\text{km}}{\text{sec}}}{\text{c}} \qquad I := I_{21\text{cm}} \cdot \Delta \lambda$$

 $I = \frac{A \cdot h \cdot v}{4 \cdot \pi} \cdot N_u$

The width of the line is given by the Doppler shift:

From Lecture 21, the integrated line strength is given by:

where
$$A := (3 \cdot 10^6 \cdot yr)^{-1}$$
 is the spontaneous decay rate (Einstein A) of the upper level

and
$$hv := h \cdot \frac{c}{21 \cdot cm}$$
 $hv = 5.912 \times 10^{-6} eV$ so

$$N_{u} \coloneqq \frac{I \cdot 4 \cdot \pi}{A \cdot h \nu} \qquad \qquad N_{u} = 2.808 \times 10^{19} \text{ cm}^{-2}$$

We showed in class that 3/4 of the H atoms will be in the excited state, so the atomic H column density is merely:

$$N_{\rm H} := \frac{4}{3} \cdot N_{\rm u}$$
 $N_{\rm H} = 3.743 \times 10^{19} \,{\rm cm}^{-2}$

Our result on dust extinction in the Milky Way, i.e. the "visibility" is about 500 pc at visible wavelengths implies:

$$A_{V_N.H} := \frac{1.1}{500 \cdot \text{pc} \cdot 1 \cdot \text{cm}^{-3}}$$
 $A_{V_N.H} = 0.711 \frac{1}{10^{21} \cdot \text{cm}^{-2}}$

So this cloud has a visual extinction of

 $A_{V_N.H} \cdot N_H = 0.027$ magnitudes

The cloud is translucent. This is typical of HI "cirrus" clouds detected by IRAS from their infrared emission. Since A.V is less than one magnitude, molecular H will not form here!