Use the Planck function and known facts about Vega (alpha Lyra) to determine it's radius.

Known facts:

Effective Temperature  $T_e := 9900 \cdot K$ 

From its spectral type (A0) and Cecilia Payne-Gaposhkin's calibration of spectral types

Parallax 
$$p := 0.129$$
·arcsec

From Hipparchos via the SIMBAD Astronomical Database, this immediately gives the distance:

$$d := \frac{pc}{\left(\frac{p}{\operatorname{arcsec}}\right)} \qquad \qquad d = 7.752 \, pc$$

Vega is zero magnitudes at all wavelengths, therefore it's flux is that of a star of zero mag. From handout #2, I gave the V-band zero mag flux as

$$F_{V} := \frac{10^{3} \cdot \frac{\text{photon}}{\text{s}}}{\text{cm}^{2} \cdot \text{Angstrom}}$$

3 photon

The central wavelength of the V-band is

So the energy of one photon is

photon :=  $h \cdot \frac{c}{\lambda_V}$ 

 $\lambda_{\mathbf{V}} := 5500 \cdot \text{Angstrom}$ 

and

$$F_{V} := \frac{10 \cdot \frac{10}{\text{sec}}}{\text{cm}^{2} \cdot \text{Angstrom}}$$

$$F_{V} = 3.604 \times 10^{-12} \frac{\text{W}}{\text{cm}^{2} \cdot \mu\text{m}}$$

The Planck function:

$$B_{\lambda}(\lambda, T) := 2 \cdot h \cdot c^{2} \cdot \left[ \frac{1}{\lambda^{5} \cdot \left( exp\left( \frac{h \cdot c}{\lambda \cdot k \cdot T} \right) - 1 \right)} \right]$$

Use this to determine the solid angle of Vega:

$$\Omega_{\text{Vega}} \coloneqq \frac{F_{\text{V}}}{B_{\lambda}(\lambda_{\text{V}}, T_{\text{e}})} \quad \Omega_{\text{Vega}} = 0.199 \, 10^{-15} \cdot \text{ster}$$

The solid angle subtended by a sphere of radius R at distance d is given by:

So the radius of Vega is:

phere of radius R at distance d is given by:  

$$\Omega = \frac{\pi \cdot R^2}{d^2}$$

$$R_{Vega} \coloneqq \sqrt{\frac{\Omega_{Vega} \cdot d^2}{\pi}}$$

$$R_{Vega} = 1.908 \times 10^{11} \text{ cm}$$
or
$$R_{Vega} = 2.725 R_{sun}$$

Which compares favorably with the radius of an A0 main sequence star from Appendix E of Carroll and Ostlie (2.7 solar radii)

## **NEEDED CONSTANTS**

$$\mu m \equiv 10^{-6} \cdot m \quad \text{Angstrom} \equiv 10^{-8} \cdot \text{cm} \quad \text{arcsec} \equiv \frac{\pi}{180 \cdot 60 \cdot 60} \cdot \text{rad} \quad \text{ster} \equiv 1 \quad W \equiv \frac{\text{joule}}{\text{sec}}$$
$$c \equiv 2.99 \cdot 10^{10} \cdot \frac{\text{cm}}{\text{sec}} \qquad k \equiv 1.38 \cdot 10^{-16} \cdot \frac{\text{erg}}{\text{K}} \qquad h \equiv 6.63 \cdot 10^{-27} \cdot \text{erg} \cdot \text{sec}$$
$$AU \equiv 150 \cdot 10^{6} \cdot \text{km} \qquad pc \equiv \frac{AU}{\text{arcsec}} \qquad pc = 3.094 \times 10^{18} \text{ cm} \qquad R_{\text{sun}} \equiv 7 \cdot 10^{10} \cdot \text{cm}$$