Saha equilibrium in early Universe. The universe became transparent to photons when hydrogen recombined. Supposedly this happened at 3000 K. Test this using the Saha equation. Use a pure hydrogen universe.

The fraction of ionization is governed by

$$n = n_{ion} + n_0$$ **total H density = ionized + neutral**

but $$\frac{n_{ion} \cdot n_e}{n_0} = \frac{g_{ion}}{g_0} \cdot \left[\frac{2 \cdot (2 \cdot \pi \cdot m_e)^{\frac{3}{2}}}{h^3} \cdot (k \cdot T)^{\frac{3}{2}} \right] \cdot e^{-\frac{\chi}{k \cdot T}}$$ **Saha equation**

Hydrogen $g_0 := 2$ $g_{ion} := 1$ $\chi := 13.6 \cdot eV$

and $n_e = n_{ion}$ **pure H, #protons = #electrons**

solving for the density of neutral H

$$n_0 = \frac{n_{ion}^2}{f(T)}$$

where $$f(T) := \frac{g_{ion}}{g_0} \cdot \left[\frac{2 \cdot (2 \cdot \pi \cdot m_e)^{\frac{3}{2}}}{h^3} \cdot (k \cdot T)^{\frac{3}{2}} \right] \cdot e^{-\frac{\chi}{k \cdot T}}$$

substituting into the first equation gives a quadratic in n.ion

$$\frac{1}{f(T)} \cdot n_{ion}^2 + n_{ion} - n = 0$$

taking the positive root gives:

$$n_{ion}(n, T) := \left(\sqrt{1 + \frac{4 \cdot n}{f(T)}} - 1 \right) \cdot \frac{f(T)}{2}$$

and finally, the fractional ionization is

$$\%\_ionized(n, T) := \frac{n_{ion}(n, T)}{n} \cdot 100$$

We know that the present T of the microwave background is 2.7 K.

$$T_0 := 2.7 \cdot K$$

From Big Bang nucleosynthesis calculations, and the oserved light element abundances, the density is about 3.6% of the critical density. The critical density follows from consideration of the present velocities in the Hubble flow. i.e. set the velocity equal to the escape velocity from the radius R:

$$\frac{1}{2} \cdot m \cdot v^2 = \frac{G \cdot M \cdot m}{R}$$ **solve for the critical density**

$$\frac{M}{R^3} = \frac{1}{2 \cdot G} \cdot \left(\frac{v}{R} \right)^2$$

$$\rho_c = \frac{3}{8 \cdot \pi \cdot G} \cdot \left(\frac{v}{R} \right)^2$$

The quantity v/R is called the Hubble constant. The current value is

$$H_0 := \frac{70 \cdot km \cdot sec^{-1}}{10^6 \cdot pc}$$

So the critical density is

$$\rho_c := \frac{3}{8 \cdot \pi \cdot G} \cdot H_0^2 \qquad \rho_c = 9.2 \times 10^{-30} \, gm\,cm^{-3}$$

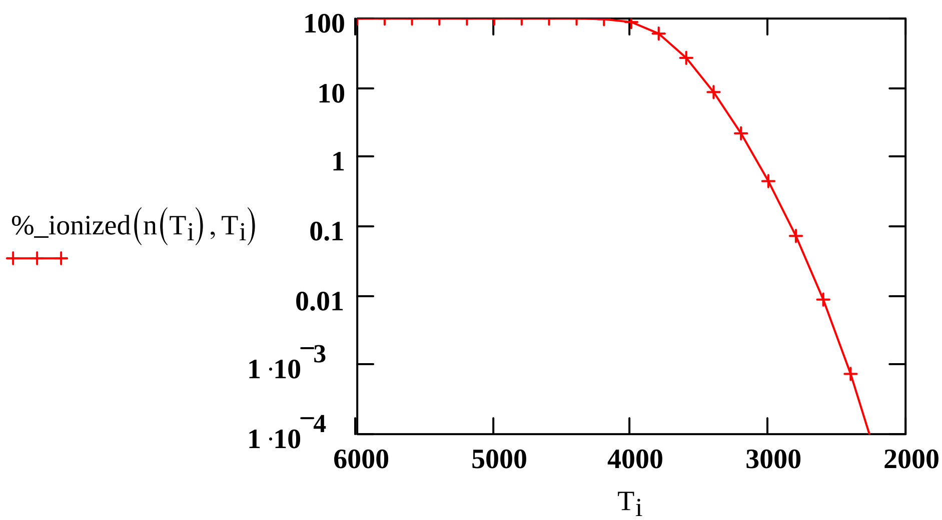
The actual baryon density is $\rho_0 := 3.6 \cdot \% \cdot \rho_c$

Corresponding to a H number density of $n_0 := \dfrac{\rho_0}{m_H}$

As the universe expands, the density scales as 1/R^3 and the temperature scales as 1/R, so the densities in the past were

$$n(T) := n_0 \cdot \left(\frac{T}{T_0}\right)^3$$

$i := 6000, 5800 .. 2000$ $\qquad T_i := i \cdot K$



So at 3000 K, the ionization dropped below 1%, and the universe became transparent

The H number density at that time was $n(3000 \cdot K) = 272.4 \, cm^{-3}$

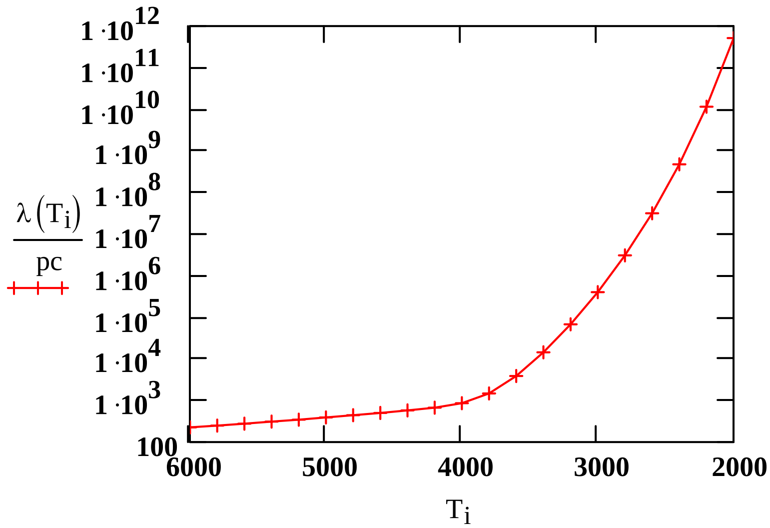
Which is quite high, considering the gas density near the sun is about 1 cm^-3!

The 'visibility' in the early universe will be

$$\sigma_e := 6.6 \cdot 10^{-25} \cdot cm^2$$ electron scattering cross section

$$\frac{\sigma_e}{m_H} = 0.4\, cm^2\, gm^{-1}$$ mass opacity

$$\lambda(T) := \frac{100}{\sigma_e \cdot n(T) \cdot \%\_ionized(n(T), T)}$$ visibility = mean free path of photons



At 3000 K, the visibility reaches 1 Mpc, or 3 M light years. This is greater than photons could travel since the birth of the Universe

$$H(T) := \frac{T}{T_0} \cdot H_0$$ Hubble constant for an empty universe

$$\frac{1}{H_0} = 1.4 \times 10^{10}\, yr$$ age of the universe

$$\frac{1}{H(3000 \cdot K)} = 1.3 \times 10^{7}\, yr$$ age of universe at recombination

$$\frac{1}{H\left(10^9 \cdot K\right)} = 1.2 \times 10^{9}\, sec$$

$$k \cdot \left(10^9 \cdot K\right) = 8.6 \times 10^{4}\, eV$$ era of nucleosynthesis?

$nm \equiv 10^{-9} \cdot m$

$gauss \equiv 1.00 \cdot gm^{0.5} \cdot cm^{-0.5} \cdot sec^{-1}$ **magnetic field strength**

$c \equiv 3.00 \cdot 10^{10} \cdot \frac{cm}{sec}$

$M_{sun} \equiv 2 \cdot 10^{33} \cdot gm$

$AU \equiv 1.5 \cdot 10^{13} \cdot cm$

$k \equiv 1.38 \cdot 10^{-16} \cdot erg \cdot K^{-1}$

$R_{sun} \equiv 7.0 \cdot 10^{10} \cdot cm$

$arcsec \equiv \frac{\pi}{180 \cdot 3600} \cdot rad$

$h \equiv 6.63 \cdot 10^{-27} \cdot erg \cdot sec$

$L_{sun} \equiv 3.9 \cdot 10^{33} \cdot \frac{erg}{sec}$

$pc \equiv AU \cdot \frac{rad}{arcsec}$

$ster \equiv 1$ **dimensionless constant**

$pc = 3.1 \times 10^{18}\, cm$

$\mu m \equiv 10^{-6} \cdot m$ $\quad W \equiv watt$

$sc \equiv 1.36 \cdot 10^{6} \cdot erg \cdot cm^{-2} \cdot sec^{-1}$

$ly \equiv c \cdot yr$

$Angstrom := 10^{-8} \cdot cm$

$pc = 3.3\, ly$

$G \equiv 6.67 \cdot 10^{-8} \cdot erg \cdot \frac{cm}{gm^2}$

$\sigma \equiv 5.67 \cdot 10^{-5} \cdot erg \cdot cm^{-2} \cdot sec^{-1} \cdot K^{-4}$

$a \equiv \frac{4 \cdot \sigma}{c}$

$A \equiv 6.022 \cdot 10^{23}$

$e_{\_} \equiv 1.602 \cdot 10^{-19} \cdot coul$ $\quad e_{\_} = 4.8 \times 10^{-10}\, esu$ $\quad eV \equiv e_{\_} \cdot volt$

$m_H \equiv \frac{1 \cdot gm}{A}$ $\quad m_H = 1.7 \times 10^{-24}\, gm$ $\quad eV = 1.6 \times 10^{-12}\, erg$

$m_e \equiv 9.109 \cdot 10^{-31} \cdot kg$

$\frac{e_{\_}^{2}}{10^{-8} \cdot cm} = 14.4\, eV$

$Hz \equiv sec^{-1}$ $\quad Jy \equiv 10^{-26} \cdot \frac{W}{m^2 \cdot Hz}$

$B_\nu(\nu, T) := \frac{2 \cdot h \cdot \nu^3}{c^2} \cdot \frac{1}{\left(\frac{h \cdot \nu}{\dots}\right)}$

$B_\lambda(\lambda, T) := 2 \cdot h \cdot \frac{c^2}{\lambda^5 \cdot \left[\exp\left[h \cdot \frac{c}{[\lambda \cdot (k \cdot T)]}\right] - 1\right]}$