LECTURE 22

Razin effect

For the index of refraction $n_r < 1$ as in a cold plasma, there is no Cherenkov radiation.

In this case however, beaming can be reduced and thus synchrotron radiation made less potent. Since the beaming is the result of the factor $\tilde{\kappa} = 1 - \beta n_r cos\theta$, for $n_r < 1$ beaming is suppressed since no matter how large β becomes, $\tilde{\kappa}$ is still significantly larger than zero.

The angle for the beaming in a vacuum was shown to be $\theta \sim 1/\gamma \sim (1-\beta^2)^{1/2}$. But for a plasma we have instead

$$\theta = (1 - n_r^2\beta^2)^{1/2}. \tag{637}$$

If $n_r \sim 1$ then this reduces to the approximate vacuum case. If $n_r << 1$ but $\beta \sim 1$, then

$$\theta = (1 - n_r^2)^{1/2} = \omega_p/\omega, \tag{638}$$

for a cold plasma. Thus at low frequencies, the medium will quench the wave propagation and the plasma effects dominate the beaming effects. At high frequencies, θ decreases until it gets down to $1/\gamma$ and then the vacuum results hold. Thus the plasma medium effects are unimportant when

$$\omega >> \gamma\omega_p. \tag{639}$$

The suppression of the beaming effect has a large effect on synchrotron emission. Below the frequency $\gamma\omega_p$ the spectrum will be largely cut off. This effect dominates the usual plasma cutoff since it enters for $\omega < \gamma\omega_p$, so large γ means the effect is strong.

Basically, the decrease in beaming occurs because the electron can no longer "catch up" to the wave that it had just emitted to reinforce it with another emission wave. The relativistic beaming enhancement due to this effect is what would boost synchrotron emission effect over other emission processes at a given frequency. (note that the enhancement is the same effect as the Cherenkov only in the latter the particle actually catches up completely and the particle is moving at a constant speed). Thus the Razin effect is a relative effect. Since the particle speed v cannot approach ω/k for sufficiently small n_r (since $\omega/k >> c$) the synchrotron is choked.

Selected Topics in Atomic and Molecular Physics

Note that macroscopic theory of radiative transfer requires that the scale of the system be much larger than the wavelength of the emission. This is not true on the atomic scale. Macroscopic theory OK on larger scales but we have to compute the absoprtion and emission coefficients from the proper electromagnetic or quantum processes.

Quantum calculations are needed when the De Broglie wavelengths of particles are larger than the dimension L which determines the conditions of a given problem, in this case the atomic scale. Note that if classical theory applied, the electron orbiting the atom would radiate away its energy and spiral in indefinitely.

Some special topics to address.

Distribution of energy levels and Ionization

Boltzmann Population of levels:

Relative populations of levels is not easy since it depends on detailed processes of how a level becomes populated or depopulated. But in thermal equilibrium things are simple and determined by temperature T. In any atom collection, the number in a given level is proportional to $ge^{-E/kT}$, with g as a statistical weight describing the level degeneracy.

Let E_i be the energy of the "ith" level with respect to the ground state. Let N_i be the number density of the ith level and $N = \sum_i N_i$ is the total number density, we have Boltzmann's law:

$$N_i = \frac{N}{U} g_i e^{-\beta E_i}, \tag{640}$$

where $\beta = 1/kT$, U is the partition function, and is simply

$$U = \sum_i g_i e^{-\beta E_i}. \tag{641}$$

At small temperatures the only the lowest term matters, the g_0 term. This gives

$$U = g_0. \tag{642}$$

Note that for finite temperature the sum diverges. Thus we have to think about how to cutoff the sum. In fact for a gas, the distance between particles is finite and outer electrons can be stripped by interactions between atoms. The ionization potential is thus reduced. A characteristic cutoff results from assuming the Bohr orbit corresponding to $n = n_{max}$ is that of the inter-atomic spacing.

$$n_{max}^2 a_0 Z^{-1} \sim N^{-1/3}. \tag{643}$$

or

$$n_{max} \sim \left(\frac{Z}{a_0}\right)^{1/2} N^{-1/6}. \tag{644}$$

The Bohr radius $a_0 \sim .5 \times 10^{-8}$cm marks a characteristic scale such that $e^2/a_0 \sim 27$ ev is a characteristic atomic binding energy.

For hydrogen at $10^{12}\mathrm{cm}^{-3}$, $n_{max} = 100$. However note that there is also Debeye screening and such which will complicate this calculation.

In general, the cutoff is not well understood, and people spend lots of time writing thesis on this subject.

For many cases the value of the cutoff is not critical. Up to 10^4K the U is usually equal to g_0. Ok for Solar interior, ISM, AGN accretion disks.

Saha Equation

Above was for distribution of levels in a density of a single atom type in thermal equilibrium. Now we investigate distribution of among various stages of ionization of single atomic type. The Saha equation describes this. Consider the tow states being the neutral atom and the first stage of ionization.

Consider more general Boltzmann law:

$$\frac{dN_0^+(v)}{N_0} = \frac{g}{g_0} exp^{\frac{-(\chi_I + 1/2 m_e v^2)}{kT}}. \tag{645}$$

Here $dN_0^+(v)$ is the differential number of ions in the ground state with free electron in velocity range $v + dv$, χ_I is the ionization potential, N_0 is the number of atoms in the ground state and the statistical weight g. Here $g = g_0^+ g_e$, the product of statistical weights of the electron and the ground state ion. The statistical weight g_e is given by

$$g_e = \frac{2dx_1 dx_2 dx_3 dp_1 dp_2 dp_3}{h^3}, \tag{646}$$

where the 2 represents the two spin states and the rest of the factors define the phase space. In particular $N_e dx_1 dx_2 dx_3 = 1$, when we apply to volume containing 1 electron. For the momentum space

$$dp_1 dp_2 dp_3 = 4\pi m_e^3 v^2 dv, \tag{647}$$

for non-relativistic electrons. Then

$$\frac{dN_0^+(v)}{N_0} = \frac{8\pi m_e^3}{h^3} \frac{g_0^+}{N_e g_0} exp\left[\frac{(\chi_I + \frac{1}{2} m_e v^2)}{kT}\right] v^2 dv. \tag{648}$$

To find the total number density of the ions we integrate over v to get

$$\frac{N_0^+ N_e}{N_0} = \frac{8\pi m_e^3}{h^3} \frac{g_0^+}{g_0} e^{-\chi_I/kT} \left(\frac{2kT}{m_e}\right)^{3/2} \int_0^\infty e^{-x^2} x^2 dx, \tag{649}$$

where $x \equiv (m_e/2kT)^{1/2} v$. Using the Gaussian integral result that $\int_0^\infty e^{-x^2} x^2 dx = \pi/4$, we have

$$\frac{N_0^+ N_e}{N_0} = \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \frac{2g_0^+}{g_0} e^{-\chi_I/kT}. \tag{650}$$

Now if we want the number of atoms or ions in any state not only the ground state, we use

$$\frac{N_0}{N} = \frac{g_0}{U(T)} \tag{651}$$

and

$$\frac{N_0^+}{N^+} = \frac{g_0^+}{U^+(T)}. \tag{652}$$

so that $N^+/N = U^+(T)/U(T)$. Thus

$$\frac{N^+N_e}{N} = 2\frac{U^+(T)}{U(T)}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT}, \tag{653}$$

where N is number density of all neutral atoms and N^+ is that for ions. This is the Saha equation.

A similar relationship holds for successive stages of ionization, that is

$$\frac{N_{j+1}N_e}{N_j} = 2\frac{U_{j+1}(T)}{U_j(T)}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_{j,j+1}/kT}. \tag{654}$$

We can rewrite all this in terms of pressure rather than number density. The ideal gas law is

$$P = nkT, \tag{655}$$

so that

$$\frac{P_{j+1}P_e}{P_j} = \frac{2U_{j+1}(T)}{U_j(T)}\left(\frac{2\pi m_e}{h^2}\right)^{3/2} (kT)^{5/2} e^{-\chi_{j,j+1}/kT}. \tag{656}$$

When you have a mixture of various different species, one must have an equation that gives the conservation of nuclei

$$\sum N_j^{(i)} = N^{(i)} \tag{657}$$

where $N_j^{(i)}$ is the jth ionization state of the ith species, so that $N^{(i)}$ is the total number density of the ith species. In addition we would need charge conservation

$$N_e = \sum_i \sum_j Z_j N_j^{(i)}. \tag{658}$$

In general, the problem must be done numerically.

Also note that trace elements e.g. of alkali metals with low ionization potentials can make important difference in the equilibrium state because they may strongly affect the electron number density. Particularly the alkali metals, like Na, K, Rb, Cs. (Left most column on peroidic table.).