LECTURE 19

Inverse Compton Power for repeated scatterings: relativistic electrons small optical depth

Interesting result: we will find that a power law spectra can result even when electrons are not a power law, if there are repeated scatterings.

Define A as the amplification per scattering

$$A = \epsilon_f/\epsilon = \frac{4}{3}\langle\gamma^2\rangle = 16(kT/mc^2)^2, \tag{561}$$

for a thermal distribution of electrons. Consider initial photon energy distribution with mean photon energy ϵ_i satisfying

$$\epsilon_i << mc^2\langle\gamma^2\rangle^{-1/2}, \tag{562}$$

and number intensity $I_n(\epsilon_i)$. Then after k scatterings, the energy of a mean initial photon is

$$\epsilon_k = \epsilon_i A^k. \tag{563}$$

If the medium has small scattering optical depth then the probability of a photon making k scatterings is $p_k(\tau_{es}) \sim \tau_{es}^k = (1 - e^{-\tau_{es}})^k$. This is because the probability going τ_{es} before single scattering is $e^{-\tau_{es}}$.

The intensity is also proportional to this quantity since the width of the probability distribution (histogram) is of order the peak frequency. Thus

$$I(\epsilon_k) \sim I(\epsilon_i)\tau_{es}^k \sim I(\epsilon_i)\left(\frac{\epsilon_k}{\epsilon_i}\right)^{-s} \tag{564}$$

where $s = -ln(\tau_{es})/ln(A)$. To prove, take ln_τ of (564) and get back (563).

This holds only when

$$\epsilon_k \leq \langle\gamma^2\rangle^{1/2}mc^2, \tag{565}$$

to ensure last scattering is still in Thomson type limit in rest frame. The final photons are those which emerge therefore with energy of order kT for thermal distribution of relativistic electrons. The total Compton power is

$$P \propto \int^{A^{1/2}mc^2} I(\epsilon_k)d\epsilon_k = I(\epsilon_i)\epsilon_i\left[\int_1^{A^{1/2}mc^2/\epsilon_i} x^{-s}dx\right], \tag{566}$$

where the upper bound is $\sim kT$ as required for the upper limit on the allowed energy range.

You can see that the initial energy is amplified by the factor in brackets, which is important for $s < 1$. When does this happen? When

$$-ln\tau_{es} < lnA \tag{567}$$

that is, when $A\tau_{es} \geq 1$.

This is equivalent to the condition that $y_R > 1$ for small τ_{es}, since A is given by (561). Compare to (557). Thus energy amplifciation important in this regime.

Repeated Scatterings by non-relativistic electrons

In general we need to solve an explicit equation for evolution of the photon distribution function due to repeated non-relativistic inverse Compton scatterings (Kompaneets equation). For frequencies with $y << 1$ and $y >> 1$ we can get away with approximations. We we consider on the approximate regimes here.

We are interested in thermal media with both emission and absorption take place by bremsstrahlung, and we want to see the effect of Compton scattering in such a medium.

Note that bremsstrahlung absorption takes place more strongly at lower frequencies.

There are three key frequencies of interest. First define the frequency ν_0 such that the scattering and the absorption opacities are equal. We recall that $\alpha_\nu = \rho\kappa_{\nu,a}$, $\sigma(\nu) = \rho\kappa_{es}(\nu)$, and $\kappa_{es} = \sigma_T/m_p = 0.4\text{cm}^2\text{g}$, we have

$$\kappa_{es} = \kappa_{ff}(\nu) \tag{568}$$

$$\alpha_\nu^{ff} = 3.7 \times 10^8 T^{-1/2} Z^2 n_i n_e \nu^{-3}(1 - e^{-h\nu/kT})\bar{g}_{ff}. \tag{569}$$

That is then

$$\frac{x_0^3}{1 - e^{-x_0}} \sim 4 \times 10^{25} T^{-7/2} \rho \bar{g}_{ff}(x_0), \tag{570}$$

or

$$x_0 = 6 \times 10^{12} T^{-7/4} \rho^{1/2} [\bar{g}_{ff}(x_0)]^{1/2} \tag{571}$$

, for $x_0 << 1$. where $\bar{g}_{ff}(x) = 3\pi^{-1/2} ln(2.25/x)$. For $x = h\nu/kT < x_0$ scattering is not important. We assume $x << x_0$ below.

The second frequency ν_t is that for which the medium becomes "effectively thin." Using

$$(L/l_*)^2 = 1 = \rho^2 \kappa_a(\nu)(\kappa_a(\nu) + \kappa_{es})L^2. \tag{572}$$

Then dividing by $\tau_{es}^2 = \rho^2 L^2 \kappa_{es}^2$, and ignoring the term $(\kappa_a/\kappa_{es})^2$ we have

$$\kappa_{es} = \kappa_a(\nu)\tau_{es}^2. \tag{573}$$

This gives the condition

$$\frac{x_t^3}{1-e^{-x_t}} \sim 4\times 10^{25} T^{-7/2} \rho \bar{g}_{ff}(x_t) \tau_{es}^2. \tag{574}$$

The τ_{es} is the total optical depth to electron scattering. For values $x > x_t$ the absorption is unimportant. For $x_0 < x < x_t$ both scattering and absorption are important.

Third, define the frequency ν_{coh} where inverse Compton is important. This is defined such that $y(\nu_{coh}) = 1$. That is for $\nu > \nu_{coh}$, inverse Compton is important. This frequency is only of interest if the y parameter for the full medium is greater than unity, implying that scattering is important for some photons at least.

In this case we then have

$$1 = \frac{4kT}{mc^2}\tau_{es}^2(\nu) \tag{575}$$

and using

$$\tau_{es} = \left(\frac{\kappa_{es}/\kappa_a(\nu)}{1+\kappa_a(\nu)/\kappa_{es}}\right)^{1/2} \tag{576}$$

gives

$$\kappa_{es} = \frac{mc^2}{4kT}\kappa_f f(\nu_{coh}). \tag{577}$$

Then again using $\kappa_{ff} = \alpha_{ff}/\rho$ we have

$$x_{coh} = 2.4\times 10^{17} \rho^{1/2} T^{-9/4} [\bar{g}_{ff}(x_{coh})]^{1/2}. \tag{578}$$

Note that x_{coh} must be less than x_t because for $x > x_t$ the medium is effectively thin. Inverse Compton cannot be significantly important when the medium is effectively thin.

Now let us use these frequencies to get info about various regimes.

Modified blackbody spectral regime $y << 1$

For $y << 1$ there is only coherent (elastic) scattering. You derived in problem 1.10 that the formula for the intensity in a scattering and absorbing medium is

$$I_\nu = \frac{2B_\nu}{1+\sqrt{(1+\kappa_{es}/\kappa_{ff})}}. \tag{579}$$

For $x << x_0$, this reduces to blackbody spectrum.

For $x >> x_0$ this reduces to modified blackbody spectrum, that is

$$I_\nu^{mb} = 2B_\nu \frac{\kappa_{ff}}{\kappa_{es}} = 8.4\times 10^{-4} T^{5/4} \rho^{1/2} \bar{g}_{ff}^{1/2} x^{3/2} e^{-x/2} (e^x - 1)^{-1/2}. \tag{580}$$

Note that for $x_0 << x << 1$, $I_\nu^{mb} \propto \nu$ not $I_\nu^{rj} \propto \nu^2$ as in Rayleigh Jeans. (Refer to (571)). Less steep than RJ regime.

The total flux is

$$F^{mb} = \sigma T^4 (\kappa_R/\kappa_{es})^{1/2} \sim 2.3 \times 10^7 T^{9/4} \rho^{1/2} \mathrm{erg/s\ cm}^{-2}, \tag{581}$$

here using the Rosseland mean absorption coefficient, (defined as inverse of expectation value of $\frac{1}{\alpha_{ff}}$, weighted by $\partial B_\nu/\partial T$.) where $\alpha_R = 1.7 \times 10^{-25} T^{-7/2} Z^2 n_e n_i \bar{g}_R$ The flux is reduced from the bbody flux. This is because if you think of absorption and emission in equilibrium as "trapping" radiation density, then the radiation density is higher where there is more trapping.

** For finite media, we have to consider x_t. When $x_t < x_0$, emission is blackbody for $x < x_t$ and optically thin bremsstrahlung for $x > x_t$ where scattering is not important.

For $x_0 < x_t < 1$, the emission is (579) for $x < x_t$, and is optically thin brems. for $x > x_t$.

(all this is because scattering is more important than abs for $x > x_0$, but system is effectively thin for $x > x_t$.

For $x_t > 1$ (579) is good for all x.