LECTURE 15

Synchrotron Radiation Polarization

Notice that for an electron, the emission is elliptically polarized as the electron moves around the field line. Looking within the cone angle of emission, the electron moves around counterclockwise and the emission is left circularly polarized. Seen from outside the cone of emission the electron appears to move clockwise, and the emission is right circularly polarized.

For a distribution of particles that varies relatively smoothly with $Sin\alpha$, the elliptical polarization should cancel and we would get a linear polarization, largely in plane perpendicular to the magnetic field. fig 6.5

The power perpendicular and parallel to the magnetic field as projected onto the plane of the sky can then be used to calculate the polarization. For particles of a single energy

$$\Pi(\omega) = \frac{P_\perp - P_{||}}{P_\perp + P_{||}}. \tag{448}$$

The frequency integrated value is 0.75. For a power law distribution $N(\gamma) \propto \gamma^{-p}$, the polarization is

$$\Pi(\omega) = \frac{P_\perp - P_{||}}{P_\perp + P_{||}} = \frac{p+1}{p+7/3}. \tag{449}$$

Note that the larger p the higher the polarization. Why?

Transformation from Cyclotron to Synchrotron Radiation

For low energies the electric field oscillates with the same frequency of gyration of the electron around the magnetic field. This produces a single line in the emission spectrum. fig 6.8 ab

For slightly higher energies, the beaming kicks in and so the E-field is stronger when the particles move toward the observer. As this effect increases for larger and larger velocities, the spectrum picks up multiple harmonics as shown in the expansion discussed earlier in the course. fig 6.10 ab

Eventually mapping out the $F(\omega/\omega_c)$ function.

The spectrum becomes continuous for a system in which there are a distribution of particle energies, or for which the field is not exactly uniform. The electric field received by observer from a distribution of particles is the random superposition of many pulses. The spectrum is the sum of the spectra from the individual pulses as long as the average distance between particles is larger than a gyro-radius.

Distinction Between Emitted and Received Power

Emitted power is not equal to received power. We need to consider the Doppler effect as the particle moves toward us. fig 6.11

Thus for period of the projected motion of $T = 2\pi/\omega_B$, we have the period measured by the received emission.

$$T_A = T(1 - v_{||} cos\alpha/c) = T(1 - \frac{v}{c} cos^2\alpha) = \frac{2\pi}{\omega_B} sin^2\alpha \tag{450}$$

for $v = c$.

The fundamental observed frequency is $\omega_B/sin^2\alpha$. This implies that the spacing between harmonics is larger, being $\omega_B/sin^2\alpha$.

Recall that we got the emitted power by dividing by ω_B, so now we have instead

$$P_r = \frac{P_e}{sin^2\alpha}. \tag{451}$$

Should we include this in astrophysical situations?

In fact not usually. This is because in general the gammas of the particles are large compared to the gamma of the bulk flow, and are often more or less randomly moving. When the particle moves toward the observer, the power is increased by $sin^2\alpha$ but the particle emission only arrives to the observer per period for a time $T/sin^2\alpha$. fig 6.13 fig S.8

We can see this as follows. If the time to travel the bottle half way is $t/2$ as measured by the observer with proper clocks, then the correction for the Doppler effect is

$$\frac{t}{2}(1 - v_{||}cos\alpha/c) = \frac{t}{2}(1 - vcos^2\alpha/c) = (t/2)sin^2\alpha. \tag{452}$$

Suppose we arrange the bottle such that $t/2 = \pi/\omega_B$. Then the cancellation with the factor for the observed power follows: That is

$$P_{r,ave} \sim \frac{(P_e/sin^2\alpha)(2\pi sin^2\alpha/\omega_B)}{2\pi/\omega_B} = P_e \tag{453}$$

Synchrotron self-absorption

Photons emitted by synchrotron can be re-absorbed. Or such photons can stimulate more emission in some component of phase space where photons already exist. (Stimulated emission).

There is one state for each element of phase space h^3. We break up the continuous volume in to discrete elements of size h^3 and consider transitions between states.

We must sum over all upper and lower states. We can use the Einstein coefficient formalism. We then have for the absorption coefficient, assuming a tangled magnetic field and isotropic particle distribution (thus assuming isotropy)

$$\alpha_\nu = h\nu/4\pi \sum_{E_1}\sum_{E_2}[n(E_1)B_{12} - n(E_2)B_{21}]\phi_{21}(\nu). \tag{454}$$

The $\phi(\nu)$ is approximately a delta function restricting sum to states differing by $h\nu = E_2 - E_1$.

We want to relate the Einstein coefficients to (441), the microscopic components of emission. We write

$$P(\nu, E_2) = 2\pi P(\omega) \tag{455}$$

where ν is the frequency and E_2 is the energy of the radiating electron.

In terms of the Einstein coefficients and using Einstein relations:

$$P(\nu, E_2) = h\nu \sum_{E_1} A_{21}\phi_{21}(\nu) = (2h\nu^3/c^2)h\nu \sum_{E_1} B_{21}\phi_{21}(\nu), \tag{456}$$

where Einstein relations have been used. Note there are no statistical weights.

Thus for stimulated emission we have

$$-\frac{h\nu}{4\pi}\sum_{E_1}\sum_{E_2} n(E_2)B_{21}\phi_{21} = -\frac{c^2}{8\pi h\nu^3}\sum_{E_2} n(E_2)P(\nu, E_2) \tag{457}$$

and for true absorption

$$\frac{h\nu}{4\pi}\sum_{E_1}\sum_{E_2} n(E_1)B_{12}\phi_{21} = \frac{c^2}{8\pi h\nu^3}\sum_{E_2} n(E_2 - h\nu)P(\nu, E_2), \tag{458}$$

where we use $B_{21} = B_{12}$. Note that the argument of n corresponds to those particles which have radiated. Thus, plug into (454) to obtain

$$\alpha_\nu = \frac{c^2}{8\pi h\nu^3}\sum_{E_2}[n(E_2 - h\nu) - n(E_2)]P(\nu, E_2). \tag{459}$$

Now consider an isotropic electron distribution $f(p)$ such that $f(p)d^3p$ is the number density of electrons within momentum range d^3p.

The number of quantum states per volume range d^3p is $h^{-3}\xi d^3p$, where $\xi = 2$ is the statistical weight for electrons. Electron density per quantum state is then $f(p)d^3p/h^{-3}\xi d^3p = (h^3/\xi)f(p)$. Thus we can make the following exchanges: number density is then

$$n(E_2) \rightarrow (h^3/\xi)f(p_2), \tag{460}$$

and sum over quantum states is then

$$\sum_2 \rightarrow (\xi/h^3)\int d^3p_2. \tag{461}$$

We then have

$$\alpha_\nu = \frac{c^2}{8\pi h\nu^3}\int d^3p_2[f(p_2^*) - f(p_2)]P(\nu, E_2), \tag{462}$$

where the $*$ labels the momentum for the $E_2 - h\nu$ energy. The reason for keeping the 2 is to remind us that it is the 2 which labeled the initial energy state of particles that radiate.

Note that for a thermal distribution of particles

$$f(p) = Ce^{-E(p)/kT}. \tag{463}$$

We note that

$$f(p_2^*) - f(p_2) = Ce^{-(E_2-h\nu)/kT} - Ce^{E_2/kT} = f(p_2)(e^{h\nu/kT} - 1) \tag{464}$$

so

$$\alpha_\nu = \frac{c^2}{8\pi h\nu^3}(e^{h\nu/kT} - 1)\int d^3p_2 f(p_2)P(\nu, E_2). \tag{465}$$

The integral is just the total power per frequency for isotropic thermal emission here, which is $4\pi j_\nu$. Thus

$$\alpha_{\nu,ther} = \frac{j_\nu}{B_\nu(T)} \tag{466}$$

which is Kirchoff's law as expected.

Using energy rather than momentum for the distribution function, we have, assuming $E = pc$ for extremely relativistic particles,

$$N(E)dE = 4\pi p^2 f(p)p^2 dp \tag{467}$$

so

$$\alpha_\nu = \frac{c^2}{8\pi h\nu^3}\int\left(\frac{N(E-h\nu)}{(E-h\nu)^2} - \frac{N(E)}{E^2}\right)P(\nu, E)E^2 dE. \tag{468}$$

Now assume $h\nu << E$ as required for classical e&m. Then from the definition of the derivative (e.g. expanding for small $h\nu$)

$$\alpha_\nu = -\frac{c^2}{8\pi\nu^2}\int P(\nu, E)E^2\frac{\partial}{\partial E}\left[\frac{N(E)}{E^2}\right]dE. \tag{469}$$

Then for thermal distribution of ultra-relativistic particles

$$N(E) = CE^2 e^{-E/kT}. \tag{470}$$

This leads to

$$\alpha_{\nu,ther} = \frac{c^2}{8\pi\nu^2 kT}\int N(E)P(\nu, E)dE = \frac{j_\nu c^2}{2\nu^2 kT}, \tag{471}$$

which is Kirchoff's law for the Rayleigh-Jeans regime.

For a power-law distribution, we have

$$-E^2\frac{d}{dE}\left(\frac{N(E)}{E^2}\right) = (p+2)CE^{-(p+1)} = \frac{(p+2)N(E)}{E} \tag{472}$$

so

$$\alpha_\nu = \frac{(p+2)c^2}{8\pi\nu^2}\int dEP(\nu,E)\frac{N(E)}{E} \tag{473}$$

Recall that in (445) we ignored ignored the hidden E dependence in the bounds of of the integral. We do the same here. This allows us to avoid the frequency dependence in $P(\nu, E)$ (the emission for a single particle, peaked near ω/ω_c) since we will just leave it in the integrand as in (444). After changing variables from E to γ to ω_c/ω, the frequency dependence in (473) then becomes

$$\alpha_\nu \propto \nu^{-(p+1)/2+1/2-4/2} = \nu^{-(p+4)/2}, \tag{474}$$

where we get the 4/2 from the ν^{-2}, the $\nu^{1/2}$ from the measure, and the $\nu^{-(p+1)/2}$ from changing $N(E)/Eto\gamma^{-(p+1)}$ from (472).

The source function is then

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{P(\nu)}{4\pi\alpha_\nu} \propto \nu^{-(p-1)/2+(p+4)/2)} = \nu^{5/2}, \tag{475}$$

where here $P(\nu)$ is now the emission spectrum for a power-law distribution calculated earlier.

Can derive this result also like this:

$$S_\nu = j_\nu/\alpha_\nu \propto \nu^2\bar{E} \tag{476}$$

where $\bar{E}$ is some mean particle energy. This approximate relation comes from noting that $P(\nu) = \int dEN(E)P(\nu,E)$ and then noting that $\alpha_\nu \propto \nu^{-2}\int dEN(E)P(\nu,E)/E$ and then taking ratio to get S_ν. Then note/recall $\nu_c \propto \gamma^2 \propto \bar{E}^2$. Thus each frequency in the spectrum corresponds to a ν_c for a particle of a specific energy. Thus $\nu_c = \nu$ when applied to the source function which is integrated over all particles. Thus $S_\nu \propto \nu^{5/2}$.

For optically thin synchrotron emission

$$I_\nu \propto j_\nu \propto P_\nu \propto \nu^{-(p-1)/2}, \tag{477}$$

since for optically thin plasmas, intensity is proportional to the emission function. For optically thick plasmas, intensity is proportional to the source function

$$I_\nu \propto S_\nu \propto \nu^{5/2}. \tag{478}$$

fig6.12

This optically thin regime occurs at high frequencies, and the optically thick regime occurs at low frequencies.

The absorption produces a cutoff in the frequency. Note that the electrons are non-thermally distributed so even though the system is optically thick to synchrotron emission in the $\nu^{5/2}$ regime, the system is not in thermal equilibrium, and thus shape is different from Rayleigh jeans B-Body which is proportional to ν^2.

No synchrotron masers in a vacuum

The absorption coefficient is positive for an arbitrary distribution of particle energies $N(E)$. This means that if there were stimulated emission resulting from making $N(E)$ at a certain energy E_0 larger so that emission from E_0 to $E_0 - h\nu$ was a maser transition, then someplace else in the distribution, there would be more positive absorption that would more than compensate. This means that α_ν is positive since it is integrated over energy. Can show this is also true for separate polarization states of synchrotron emission.