(AST 461) LECTURE 1: Basics of Radiation Transfer

Almost all we know about the astronomical universe comes from from radiation emanating from faint sources. We need to know how to interpret this radiation.

Electromagnetic spectrum, wavelength and frequency:

$$\nu\lambda = c, \tag{1}$$

ν is frequency, λ is wavelength. The speed of light depends on the index of refraction.

Justification for Macroscopic Treatment of Radiation

Scale of system $>>$ λ of the radiation, the radiation can been considered to travel in "straight lines" =rays. $dA >> \lambda^2$

Amount of energy passing through source dA is in time dt is $FdAdt$ where F is the flux. Units are (erg/cm.s^2) in CGS.

Flux from an isotropic source

Isotropic means energy emitted in all directions. Consider 2 different spherical surfaces S_1 and S_2. (fig 1.)

$$F_1 dA_1 dt_1 = F_2 dA_2 dt_2 \tag{2}$$

but $dt_1 = dt_2$ so that

$$F_1/F_2 = dA_2/dA_1 = r_2^2/r_1^2. \tag{3}$$

This is the inverse square law.

Intensity

Flux measures energy from all rays in given area. A more detailed approach is to consider energy carried from individual rays, or sets of rays differing infinitesimally from the initial ray. (fig 2.)

$$dE = (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}) I_\nu dA dt d\Omega d\nu, \tag{4}$$

where I_ν is the Specific Intensity and represents Energy /(time $\times$ area $\times$ sold angle $\times$ frequency).

Now suppose we have an isotropic radiation field. The differential flux at a given frequency is

$$dF_\nu = (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}) I_\nu d\Omega = I_\nu cos\theta d\Omega. \tag{5}$$

But if I_ν is isotropic, then $F_\nu = 0$.

Constancy of Intensity in Free Space:

Specific intensity is constant along a ray in free space.

Here's why: Consider all rays passing through both dA_1 and dA_2 and use conservation of energy: (fig 3)

$$dE_1 = I_{\nu,1} dA_1 dt_1 d\Omega_1 d\nu_1 = dE_2 = I_{\nu,2} dA_2 dt_2 d\Omega_2 d\nu_2 \tag{6}$$

Note that $dt_1 = dt_2$, $d\nu_1 = d\nu_2$. Note also that $d\Omega_1$ is the solid angle subtended by dA_2 at dA_1 and $d\Omega_2$ is the solid angle subtended by dA_1 at dA_2. Thus we have

$$d\Omega_1 = dA_2/r^2 \tag{7}$$

and

$$d\Omega_2 = dA_1/r^2. \tag{8}$$

Thus

$$I_{\nu,1} = I_{\nu,2}. \tag{9}$$

The intensity is constant along free space.

Application to a Telescope (fig 3.)

Assume large distance between object and lens $r >> f$, where f is the focal length. Determine image intensity I_i given object intensity I_0. Infinitesimal amount of surface area of object dA_0 has intensity I_0. We have

$$I_0 dA_0 d\Omega_{T,0} = I_0 A_T dA_0/r^2. \tag{10}$$

where $d\Omega_{T,0}$ is the solid angle of the telescope as measured from the dA_0 of the object. All photons from dA_0 must strike dA_1 on focal plane.

Thus

$$I_0 dA_0 d\Omega_{T,0} = I_0 dA_0 A_T/r^2 = I_i dA_i d\Omega_{T,i} = I_i dA_i A_T/f^2, \tag{11}$$

but solid angle sub-tending image from telescope equals solid angle sub-tending object from telescope so

$$dA_0/r^2 = dA_i/f^2, \tag{12}$$

and as expected, $I_i = I_o$.

Do telescopes measure flux or intensity?

If resolution of telescope is crude, and we cannot resolve object, then we measure flux. If telescope can resolve object, then we measure intensity.

Why? Consider the case when the source is unresolved. Now imagine pushing the source to farther distance. As the distance increases, the number of photons falls as r^2. The flux is measured.

If instead the the source is resolved, then as the source is pushed farther away, more area of the source would be included with the solid angle, which compensates for the the increased distance and the collected number of photons remain the same.

Flux from a uniformly bright sphere: read in text.

Relationship of Intensity to Stat Mech Quantities:

Consider photon distribution function f such that $f_\alpha(\mathbf{x}, \mathbf{p}, t)d^3x d^3p$ is the number of photons in $\mathbf{x}, \mathbf{p}$ space, with spin index α.

$$\mathbf{p} = h\mathbf{k} = (h\nu/c)\hat{\mathbf{k}} \tag{13}$$

so

$$dE = \sum_{\alpha=1}^{2} h\nu f_\alpha(\mathbf{x}, \mathbf{p}, t) d^3x d^3p. \tag{14}$$

But photons traveling in direction $\hat{\mathbf{k}}$ for time dt through an element of area dA whose normal $= \hat{\mathbf{n}}$ occupies

$$d^3x = cdt(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}})dA \tag{15}$$

and

$$d^3p = p^2 d\Omega dp = (E^2/c^2) d\Omega dE/c = (h^3\nu^2/c^3) d\Omega d\nu. \tag{16}$$

Thus,

$$dE = \sum_{\alpha=1}^{2} (h^4\nu^3/c^3) f_\alpha(\mathbf{x}, \mathbf{p}, t) dA dt d\Omega d\nu \tag{17}$$

so

$$I_\nu = \sum_{\alpha=1}^{2} (h^4\nu^3/c^3) f_\alpha(\mathbf{x}, \mathbf{p}, t). \tag{18}$$

In stat mech, the occupation number for each photon is

$$N_\alpha = h^3 F_\alpha \tag{19}$$

so

$$I_\nu = \sum_{\alpha=1}^{2} (h\nu^3/c^2) N_\alpha(\mathbf{x}, \mathbf{p}, t). \tag{20}$$

Later we will see for example that I_ν corresponds to B_ν, the Planck function for b-body radiation. The occupation number corresponds to $N_\alpha = (e^{h\nu_\alpha/kT - \mu_\alpha} - 1)^{-1}$ for Bose-Einstein stats.

Radiation Pressure

photon carries momentum $p_\nu = E_\nu/c$. (fig. 4)

Can compute pressure by considering incident photons reflecting off of a wall: The angle of incidence equals the angle of reflection. The change in the z component of momentum of photon between frequency ν and $\nu + d\nu$ reflected in time dt from area dA is

$$dp_\nu d\nu = [(p_\nu)_f - (p_\nu)_i]d\nu = (1/c)(E_\nu cos\theta - (-E_\nu cos\theta))d\nu = (2/c)E_\nu cos\theta d\nu = \left(\frac{2}{c}\right) I_\nu d\nu dt dA cos^2\theta d\Omega, \tag{21}$$

where we used (4). Now the change in momentum per unit area per unit time is a differential force/area= differential pressure. Integrating over solid angle gives the total pressure:

$$P_{rad,\nu} = (dp_\nu/dt)dA = (2/c)\int I_\nu cos^2\theta d\Omega_{hem} = (2/c)\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I_\nu cos^2\theta sin\theta d\phi d\theta. \tag{22}$$

Now imagine removing the reflecting surface. Thus instead of the factor of 2 in momentum, we would have photons coming in from the other side. Thus we can remove the factor of 2, and integrate over all of the sphere.

$$P_{rad,\nu} = dp_\nu/dtdA = (1/c)\int I_\nu cos^2\theta d\Omega = (1/c)\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\nu cos^2\theta sin\theta d\phi. \tag{23}$$

Energy Density in a Cylinder

Consider a cylinder of cross section dA. (fig 5)

$$dE = u_\nu(\Omega)dVd\nu d\Omega = u_\nu(\Omega)cdtd\nu dA = I_\nu dAd\Omega dtd\nu \tag{24}$$

so the energy density is

$$u_\nu(\Omega) = (I_\nu/c). \tag{25}$$

Then

$$u_\nu = \int u_\nu(\Omega)d\Omega = (1/c)\int I_\nu d\Omega = 4\pi\overline{I}_\nu/c \tag{26}$$

For isotropic radiation field

$$\overline{I}_\nu = I_\nu \tag{27}$$

Note also that the pressure for an isotropic radiation field is

$$P = (1/c)\int I_\nu cos^2\theta d\Omega = (\overline{I}_\nu/c)\int cos^2\theta d\Omega = \frac{1}{3}\int u_\nu d\nu. \tag{28}$$