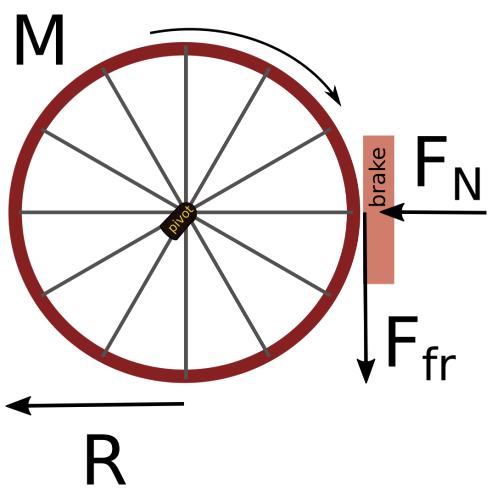
Homework set 09. Physics 141, Fall 2022

Due date: Friday Nov 11, 2022 at noon
Total of 8 points. On torque.

1. (2 points) **Friction brake on a wheel**



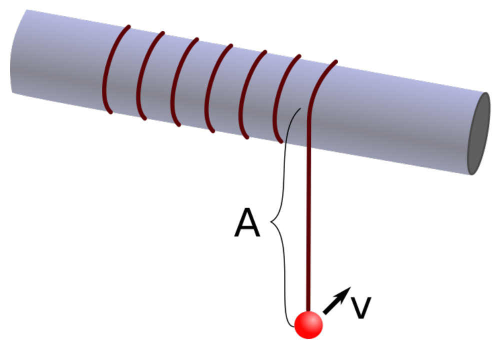
A wheel of mass M and radius R is held by a pivot through its center. It is spinning with initial angular rotation rate ω_0. A brake applies a normal force F_N to the rim of the wheel and this gives a friction force between wheel rim and brake. The coefficient of dynamic friction is μ. Assume that most of the mass of the wheel is its rim.

a. How long do you need to apply the brake to stop the wheel from rotating?

b. What is the work done on the wheel by the brake?

c. How many rotations does the wheel undergo while slowing down?

2. (2 points) **A string winding up on a cylinder**



A mass m is on the end of a string and is winding up around a fixed cylinder. The radius of the cylinder is R. The velocity of the mass is initially v_0. The length of the string is L_{string}. The distance between m and where the string touches the cylinder is A. This is the length of the free part of the string. The rate that the string winds up is $\dot{A}$.

For this problem ignore gravity and assume that the distance between each wind on the cylinder is small.

a) Is there work done on m by the tension in the string? Find the mass's velocity $v(t)$ as a function of time.

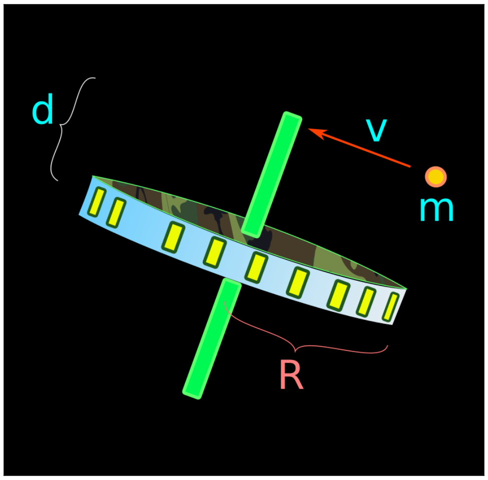
b) Is there a torque on m? Use the tension on the string to compute the torque on m. Show that $\dot{A}A \approx -Rv_0$.

c) Use the rate that the string winds up and the angular rotation rate of m about the center of the cylinder to show that $\dot{A}A \approx -Rv_0$.

d) Solve for $A(t)$.

Hint: You can assume $A > R$ when estimating the angular rotation rate ω, computing $\dot{\omega}$ and computing the rate that the string winds up, $\dot{A}$.

3. (2 points) **Tilting a space station**



A space station of mass M is a rigid body but it is spinning, to simulate gravity. The radius of the space station wheel is R and its centripetal acceleration is a which is at 1g so that people are comfortable inside. Assume that most of the mass of the station is in the wheel and not in the narrow central axis. The angular momentum of the station is aligned with its central axis. Assume that the space station maintains rotation about its axis of symmetry.

a) What is the angular rotation rate ω of the space station?

b) What is the angular momentum of the space station?

A planetesimal of mass $m \ll M$ hits the axis of the space station with a relative velocity v. The planetesimal bounces elastically off the space station axis, leaving in the same direction as it came. The distance of the impact from the center of the space station is d. The velocity v is perpendicular to the spin axis of the space station.

The impact causes a change in the angular momentum of the space station.

c) Does the size of the angular rotation rate of the space station change after the impact?

d) What is the difference in angle between the space station's spin axis before and after the collision?

Hint: The angle $\theta = \frac{\Delta L}{L}$.

4. (2 points) **On an elliptical orbit**

A space craft is in an elliptical orbit with semi-major axis a and eccentricity e is orbiting a planet of mass M.

At pericenter, (where the radius is the smallest), the space craft has a short burn giving it an impulse or change in velocity. The space craft enters a circular orbit.

a) Prior to the impulse, what was the radius and velocity at the orbit's pericenter?

b) After the impulse what is the velocity?

c) What is the velocity of the impulse, Δv, and which direction was it applied?

If it is more convenient, you can assume that e is small and work to first order in e.

Hints: The orbit has shape $r = \frac{a(1-e^2)}{1+e\cos f}$ where f is an angle known as the true anomaly. Orbital angular momentum per unit mass is

$$l = \sqrt{GMa(1-e^2)} = rv_\theta.$$

Orbital energy per unit mass is

$$E/m = -\frac{GM}{2a} = \frac{1}{2}(v_\theta^2 + v_r^2) - \frac{GM}{r}$$

where v_θ is the tangential velocity component and v_r is the radial velocity component. The radial component $v_r = 0$ at pericenter and apocenter.