## Homework set 08. Physics 141, Fall 2022

Due date: Friday Nov 4, 2022 at noon
Total of 10 points. On angular momentum.

1. (2 points) The surface density of a thin rectangle varies as:
$\sigma(x, y)=\left(12 \mathrm{~kg} / \mathrm{m}^{2}\right)+\left(6 \mathrm{~kg} / \mathrm{m}^{4}\right)\left(x^{2}+y^{2}\right)$
The rectangle has a length $L=0.55 \mathrm{~m}$ and a width $W=0.80 \mathrm{~m}$.


What is moment of inertia about the z axis?
2. (2 points) On a frictionless table, a $m=0.70$ kg glob of clay strikes a uniform $M=0.82$ kg bar perpendicularly at a point $b=0.49$ m from the center of the bar and sticks to it. The bar is $L=1.22 \mathrm{~m}$ long and the clay is moving at $v=5.50 \mathrm{~m} / \mathrm{s}$ before striking the bar.


What is the final speed of the center of mass?
At what angular speed does the bar/clay system rotate about its center of mass after the impact?
3. (2 points) Next year make sure that $I<M R^{2}$. A spherically symmetric object, with radius $R=0.50 \mathrm{~m}$ and mass $M=3.0$ kg , rolls without slipping across a horizontal floor, with velocity $v=2.2 \mathrm{~m} / \mathrm{s}$. It then rolls up an incline with an angle of inclination $\theta=27^{\circ}$ and comes to rest a distance $d=3.2 \mathrm{~m}$ up the incline, before reversing direction and rolling back down.


Find the moment of inertia of this object about an axis through its center of mass.
4. (2 points) A bullet of mass $m$ is fired with a velocity of $v_{0}$ into a solid cylinder of mass $M$ and radius $R$. The cylinder is initially at rest and is mounted on a fixed vertical axis that runs through its center of mass. The line of motion of the bullet is perpendicular to the axis and at a distance $d$ from the center.


Find the angular speed $\omega$ of the system after the bullet strikes and adheres to the surface of the cylinder.
5. (2 points) A string is wrapped around a cylinder of mass $M$, radius $R$, and moment of inertia $I$ (about its center of mass and axis of symmetry). A constant force $F$ is applied to one end of the string, as indicated in the Figure. We observe that the cylinder does not move up or down, but it increases its angular velocity.

a. What is the angular acceleration of the object?
b. When you have moved the end of the string to a height $y_{0}$ above the floor, the object is rotating with an angular velocity $\omega_{0}$. What is the angular velocity $\omega$ of the object when you have moved the end of the string to a height $y$ above the floor?

