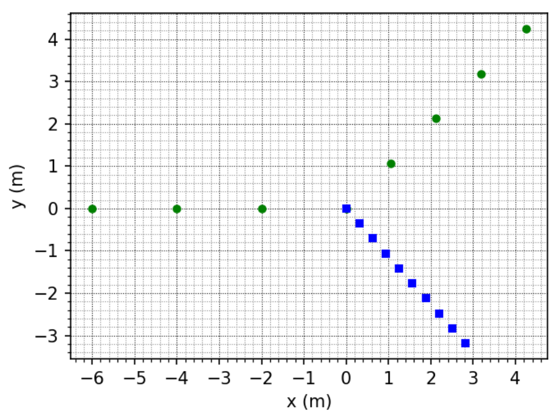
Homework set 07. Physics 141, Fall 2022

Due date: Friday Oct 28, 2022 at noon. Total of 12 points. On collisions.

1. (2 points) The Figure shows the result of a collision of an $m_1 = 1.0$ kg ball with an object located at the origin. The position of the objects are shown at 1.0 s intervals.



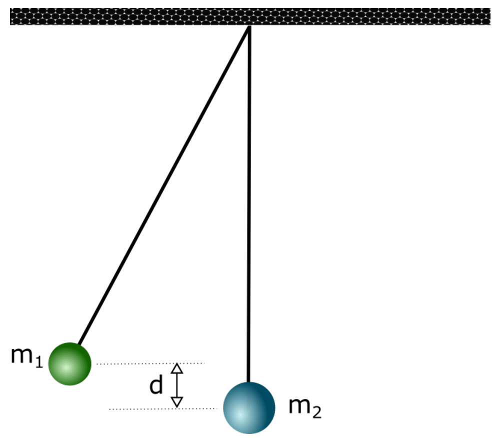
The ball m_1 is initially on the left (at negative x) but leaves the encounter on the top right (with positive y).

a) What is the mass of the object, m_2 that was originally located at the origin?

b) What is the change in the kinetic energy as a result of the collision?

Hint: You need to estimate the sizes of the initial and final velocities and of their velocity components The collision need not be elastic.

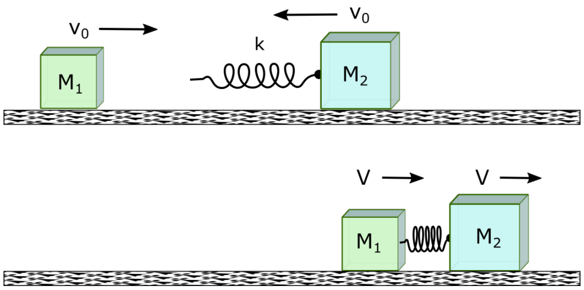
2. (2 points) Two 18-cm long pendulums (each made of a massless string and a ball) are initially situated as shown in the figure. The masses of the left and right balls are $m_1 =$ 90 gram and $m_2 = 330$ gram , respectively. The first pendulum is released from a height $d = 7.2$ cm and strikes the second.



Assuming that the collision is elastic and neglecting the mass of the strings and any frictional effects, how high does the center of mass rises after the collision?

(Note the two pendulums have the same frequency as long as the amplitudes are low. This means if they start at the lowest point at the same time, they will reach their maximum heights at the same time. When they reach their maximum heights, their kinetic energy is zero.).

3. (2 points) A block of mass M_1 and initial velocity v_0 collides with a block of mass M_2 and initial velocity of $-v_0$. Attached to M_2 is a spring with a force constant k. Ignore friction.



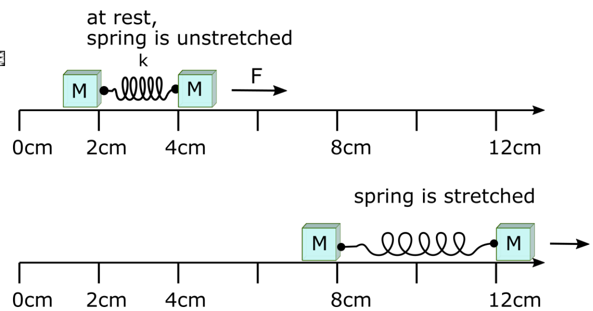
a) At a later moment in time the velocities of block M_1 and block M_2 are equal. What is the speed V of the blocks at that moment?

(Hint: compute the center of mass velocity).

b) What is the compression of the spring when the velocities of the blocks are equal?

(Hint: write the energy as a sum of center of mass kinetic energy, vibrational kinetic energy and vibrational potential energy).

4. (2 points) Two identical blocks of mass $M = 0.1$ kg are at rest on a nearly frictionless surface, connected by an un-stretched massless spring with spring constant $k = 100$ N/m. At time $t = 0$ a constant force $F = 5$ N, directed towards the right, is applied to the rightmost block (see upper diagram). At a later time the blocks are in a new position, as shown in the lower diagram and they are moving. Note that the spring is stretched at the later time. The rest length of the spring is 2 cm.



All of these questions are for quantities at the time shown in the bottom panel in the Figure.

a. At the later time, what is the sum of the kinetic energy and potential energy of the two-block system?

b. What is the total kinetic energy of the two-block system?

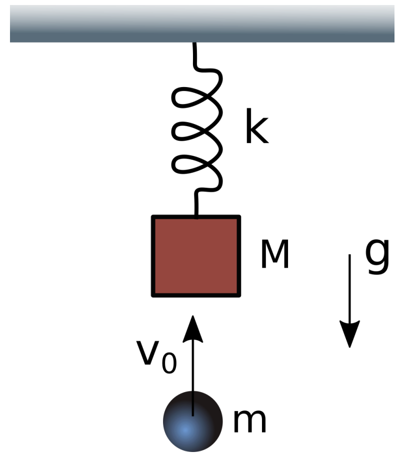
c. What is the translational kinetic energy of the two-block system?

d. What is the speed of the center of mass of the two-block system?

e. What is the vibrational kinetic energy of the two-block system?

By vibrational kinetic energy, we mean the total kinetic energy subtracted by the kinetic energy of the center of mass. Translational kinetic energy is that associated with center of mass motion alone. The total kinetic energy is the sum of the translational kinetic energy and the vibrational kinetic energy.

5. (2 points)



A mass M is hanging from a spring of rest length L and spring constant k that is connected to a fixed point on the ceiling. The mass M is pulled downward by gravity and it is initially at rest.

a) What is the length of the spring in this equilibrium position?

A mass $m < M$ collides elastically with M. Its velocity at the moment of collision is v_0. After the collision, m rebounds and falls to the ground.

b) What is the length of the spring as a function of time after the collision?

Hint: You need to find the position and velocity of M just after the collision. These can be used as initial conditions to find the solution of the mass spring system (k and M) at subsequent times.

6. (2 points)

   The **coefficient of restitution** is the ratio of the velocity after a collision to that before the collision.

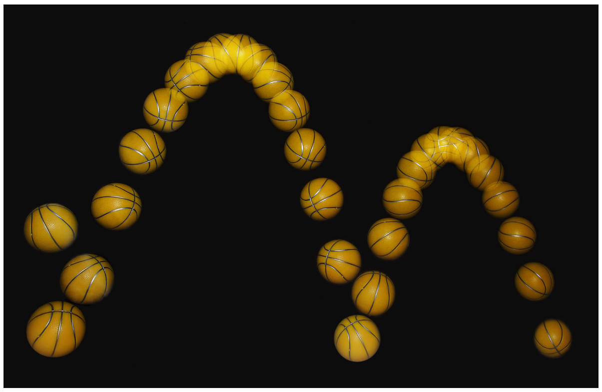


Figure 1: This image is by Michael Maggs with Edits by Richard Bartz and from Bouncing ball strobe edit.jpg under the license Creative Commons Attribution-Share Alike 3.0 Unported license.

   Consider a ball bouncing on a flat table top with coefficient of restitution COR. The ball's initial velocity is $\mathbf{v} = (v_{0x}, 0, v_{0z})$ and the ball is initially touching but leaving the table top and on its way upward.

   When the ball bounces, the z component of velocity is reversed and reduced by the coefficient of restitution

$$v_z \to -v_z \times COR$$

   The coefficient of restitution is positive, unit-less and less than 1. We assume that when the ball bounces the x component of the velocity remains unchanged.

   Between the bounces, the ball is affected by gravity alone. It would be on a parabolic trajectory with vertical acceleration g. The time it takes to go up and come back down to hit the table top again depends only on its vertical velocity component when it leaves the table top. (You can compute the time it takes to do this).

   a. How long does it take the ball to undergo N bounces?

   b. How far does the ball go horizontally as the number of bounces $N \to \infty$?

   It may be helpful to know that the geometric series

$$\sum_{k=0}^{n} z^k = 1+z+z^2+z^3+.....+z^n = \frac{1-z^{n+1}}{1-z}$$

   This problem illustrates a numerical problem that arises by trying to integrate a trajectory from bounce to bounce in an idealized model. Here an infinite number of bounces can occur in a finite time and this would stop an integrator from advancing. In a real system, additional physics would prevent an infinite number of bounces from taking place. We have assumed that bounces happen instantaneously and that is a poor approximation when the time between bounces is short.