Homework set 06. Physics 141, Fall 2022

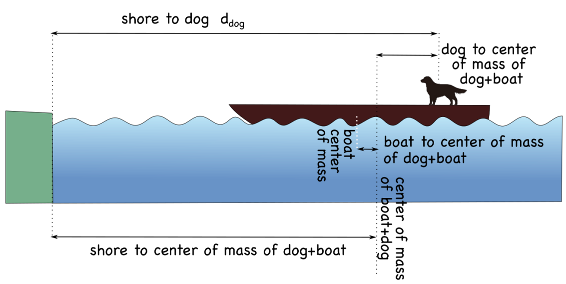
Due date: Friday Oct 21, 2022 at noon.
Total of 9 points. On center of mass.

1. (2 points) A stone is dropped at $t = 0$ s. A second stone, with a mass 3.0 times that of the first, is dropped from the same point at $t = 0.10$ s.

   How far from the release point is the center of mass of the two stones at $t = 0.58$ s? Assume that neither stone has yet reached the ground.

   What is the speed of the center of mass of the two stone system at that time?

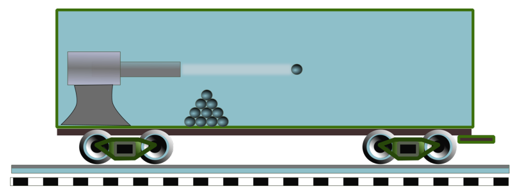
2. (2 points) A dog, with a mass of 10.0 kg, is standing on a flatboat so that he is 25.5 m from the shore. He walks 5.0 m on the boat toward the shore and then stops. The boat has a mass of 44.0 kg.



   Assuming there is no friction between the boat and the water, how far is the dog from the shore now?

   Hint: Relate the distance of the dog from the shore (or the center of mass of dog+boat) to the distance between the dog and boat's centers of mass. Consider subtracting before and after versions of this relation.

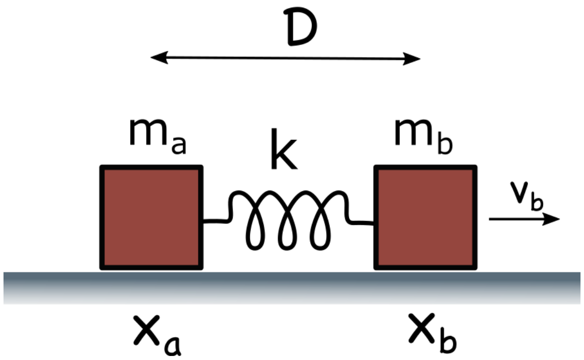
3. (2 points) A 500 kg cannon and a supply of 68 cannon balls, each with a mass of 39.0 kg, are inside a sealed railroad car with a mass of 45000 kg and a length of 73 m. The car is initially at rest. The cannon fires to the right; the car recoils to the left. The cannon balls remain in the car after hitting the wall.



   After all the cannon balls have been fired, what is the greatest distance the car can have moved from its original position?

   What is the speed of the car after all the cannon balls have come to rest on the right side?

4. (3 points)



Two equal mass blocks are connected with a spring. The blocks have mass m and the spring constant is k. The rest length between the centers of the two blocks is L. The blocks are free to move on a line with coordinate x. There is no friction or damping. We denote quantities for the first mass with an a subscript and for the second one with a b subscript. The block positions are $x_a(t), x_b(t)$ and their velocities are $v_a(t), v_b(t)$ with $v_a = \frac{dx_a}{dt}$ and similarly for v_b.

Initially the first block has velocity

$$v_a(t=0) = 0$$

and the second block has velocity

$$v_b(t=0) = A.$$

Initially the spring is at its rest position. Initially the positions of the two blocks are at

$$x_a(t=0) = 0 \qquad \text{and} \qquad x_b(t=0) = L.$$

The distance between the two blocks

$$D(t) = x_b(t) - x_a(t)$$

The equations of motion are

$$m\frac{d^2x_a}{dt^2} = -k(x_a - x_b + L)$$
$$m\frac{d^2x_b}{dt^2} = k(x_a - x_b + L)$$

The spring force is applied equally and opposite. We add these equations together, and we subtract them to find

$$\frac{d^2}{dt^2}(x_a + x_b) = 0$$
$$\frac{d^2}{dt^2}(x_b - x_a) = -\frac{2k}{m}(x_b - x_a - L) \quad (1)$$

Equations 1 can be written

$$\frac{d^2x_{cm}}{dt^2} = 0 \quad (2)$$
$$\frac{d^2D}{dt^2} = -\omega^2(D - L) \quad (3)$$

where x_{cm} is the position of the center of mass and frequency ω depends on k and m.

a) What is the reduced mass, μ? This should only depend on m. The reduced mass is relevant because ω can be written in terms of the reduced mass, $\omega = \sqrt{k/\mu}$.

b) What is the center of mass position $x_{cm}(t)$ at later times? Here I mean the center of mass of the whole block/spring system assuming that the spring's mass is negligible. Your formula should depend on $t, L, m, k,$ or/and A.

Hint: integrate equation 2. Find the initial velocity and position of the center of mass. Use the initial position and center of mass velocity to find the solution at later times.

c) What is the difference between the two masses $D(t)$ at later times?

Hint: Find a general solution to 3. Find $D(t=0)$ and its time derivative at $t=0$. Use the initial value of D and the initial value of its time derivative to find the solution for $D(t)$ at later times.

d) What are the positions of the blocks a and b at later times? Find formulas for $x_a(t)$ and $x_b(t)$ in terms of t, L, m, k, A.

Hint: You have relations for x_{cm} and D in terms of x_a, x_b. Invert the transformation so that you have x_a, x_b in terms of $x_{cm}(t)$ and $D(t)$.