# Homework set 05. Physics 141, Fall 2022

**Due date: Friday Oct 7, 2022 at noon.**

Total of 11 points. On potential energy and energy levels. Special relativity.

1. (1 points) The potential energy of two atoms separated by a distance $r$ may be written as

$$U(r) = 4U_{\text{atomic}}\left[\left(\frac{r_0}{r}\right)^{12} - \left(\frac{r_0}{r}\right)^{6}\right].$$

We take radius $r_0 = 3.3\mathring{A}$ and strength $U_{\text{atomic}} = -0.013045$ eV.

What is the distance at which there is no net force between the atoms? Express your answer in $\mathring{A}$.

What is the potential energy at the position where the net force is 0 N? Express your answer in eV.

2. (3 points) The Great Pyramid of Cheops at El Gizeh in Egypt had a height $H = 150$ m before its topmost stone fell. Its base is a square with edge length $L = 200$ m. It is made of solid rocks that have a uniform density of $\rho = 3000$ kg/m$^3$.

In terms of $\rho, L, H$ and gravitational acceleration $g$, how much work was required to lift the rocks from base level to build this pyramid?

In units of $J$ (Joule) how much work was required to lift the rocks from base level to build this pyramid?

(Hint: you need to compute the height of the center of mass).

3. (2 points)

   A particle of mass $m$ and rest mass $E_0 = mc^2$ is in a potential well described by potential energy that is a function of radius $r$,

   $$U = -U_0 e^{-kr}$$

   with coefficients $U_0, k > 0$.

   At radius $r$ what velocity is needed to escape the potential well?

   Do not assume that $v \ll c$.

   Hint: $E = \gamma(r)mc^2 + U(r)$ where $\gamma$ is the Lorenz factor.

4. (2 points) A particle with mass $M$ and charge $+e$ and its antiparticle (same mass $M$, charge $-e$) are initially at rest, far away from each other. They attract each other and move toward each other.

   a. Make a graph of the dependence of the various energies, the potential energy, the particle energies (including the rest energy), and the total energy, as function of distance.

   b. When the particle and antiparticle collide, they annihilate and produce a different particle with rest mass $m$ (which is much smaller than $M$) and charge $+e$ and its antiparticle (same rest mass $m$, charge $-e$). When these particles have moved far away from each other, how fast are they going? Is this speed large or small compared to the speed of light?

   c. Now take the specific case of a proton and antiproton (with rest mass $1.67262192 \times 10^{-27}$ kg) colliding to form a positive and negative pion. Each pion has a rest mass of $2.488 \times 10^{-28}$ kg. When the pions have moved far away from each other, how fast are they going?

5. (1 points) Suppose we have reason to suspect that a certain quantum object has only three quantum states. When we excited such an object we observe that is emits electromagnetic radiation of three different energies: 2.50 eV (green), 1.90 eV (orange), and 0.60eV (infrared). Propose two possible energy-level schemes for this system. Equivalently: If the energy of the ground state is $E_0$, what are the possible energy values for $E_1$ and $E_2$?

6. (2 points) Consider a microscopic spring-mass system (with a single spring and a single mass) whose spring stiffness is $k = 50$ N/m and mass is $m = 4 \times 10^{-26}$ kg.

   a. What is the smallest amount (a quantum) of vibrational energy that can be added to this system?

   b. In a collection of microscopic oscillators, the temperature is high enough that the ground state and the first three excited states are occupied. What are the possible energies and wavelengths of the photons emitted by the oscillators.