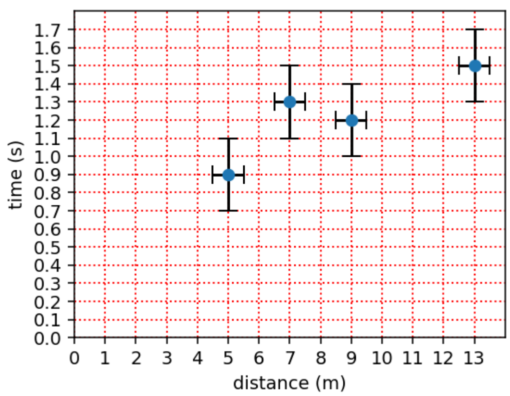
Homework set 02. Physics 141, Fall 2022

Due date: Friday Sept 16 at noon.
Total of 11 points.

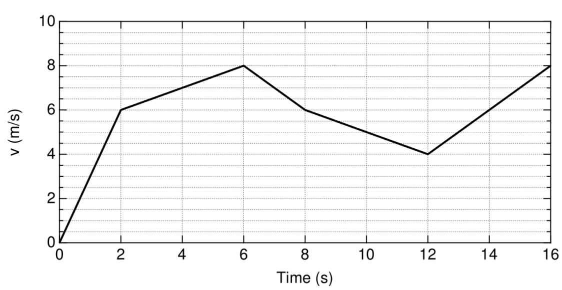
1. (2 points) A technique to measure the gravitational acceleration g is to measure the time t it takes an object to fall a distance d. The results of such a measurement are shown in the Figure (the error bars in this Figure are $\pm 1\sigma$).



What is the most probable measured value for the gravitational acceleration g?

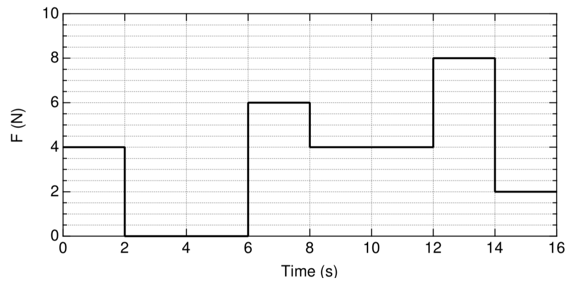
What is the standard deviation of your estimate?

2. (2 points) How far does the runner, whose velocity v versus time t graph is shown in the Figure, travel in the first 13.5 s?



Hint: $x = \int v \, dt$.

3. (2 points) The force exerted on a 9.0 kg block is shown in the figure as a function of time. Assume that the motion is one dimensional and that the velocity of the block at time $t = 0$ s is 0 m/s.



How far does the block travel in the first 7.5 s?

What is the average velocity of the block during the 16 s time interval?

What is the average acceleration of the block during the 16 s time interval?

4. (1 points) Consider a spacecraft that is far away from planets or other massive objects. The mass of the spacecraft is $M = 1.5 \times 10^5$ kg. The rocket engines are shut off and the spacecraft coasts with a velocity vector $\mathbf{v} = (0, 20, 0)$ km/s. The space craft passes the position $\mathbf{x} = (12, 15, 0)$ km at which time the spacecraft fires its thruster rockets giving it a net force of $\mathbf{F} = (6 \times 10^4, 0, 0)$ N which is exerted for 3.4 s. The ejected gases have total mass that is small compared to the mass of the spacecraft.

a) Where is the space craft 1 hour afterwards?

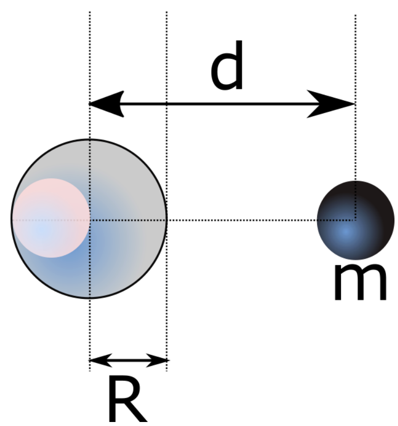
b) What approximations have you made in your analysis?

5. (1 points) M_1 is a spherical mass (46.6 kg) at the origin. M_2 is also a spherical mass (14.5 kg) and is located on the x-axis at $x = 93.4$ m.

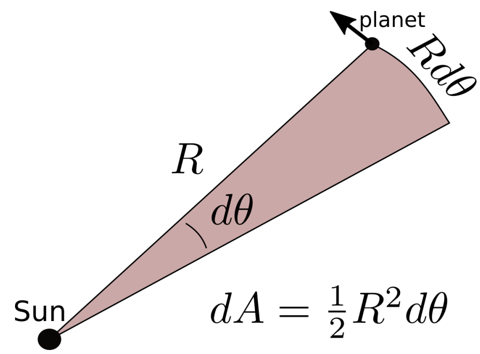
At what value of x would a 17.0-kg mass experience no net gravitational force due to both M_1 and M_2?

6. (2 points) A lead sphere has a radius of $R = 11.3$ cm. Inside this sphere there is a spherical hollow. The hollow touches the surface of the sphere and grazes the center of the sphere as shown in the Figure. The radius of the hollow directly depends on R. The mass of the sphere before hollowing was $M = 57.0$ kg.

What is the magnitude of the gravitational force (in Newtons) between the hollowed-out lead sphere and a small sphere of mass $m = 4.2$ kg, located a distance $d = 0.55$ m from the center of the lead sphere?



7. (1 points) Kepler's second law is this statement: *A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.* We are going to prove this statement.



Consider the wedge in the figure with area

$$dA = \frac{1}{2}R^2 d\theta$$

The rate that area is swept per unit time is

$$\frac{dA}{dt} = \frac{1}{2}R^2\frac{d\theta}{dt} = \frac{1}{2}R^2\dot{\theta}$$

and this is true even if radius R is varying. We take the origin to be the center of the Sun and radius R is the distance between planet and Sun. The angle θ gives the position of the planet in the ecliptic plane.

Kepler's second law is equivalent to

$$\frac{dA}{dt} = \text{constant} \qquad \text{or} \qquad \frac{d^2A}{dt^2} = 0.$$

In class we showed that acceleration in polar coordinates can be written

$$\mathbf{a} = (\ddot{R} - R\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\boldsymbol{\theta}}$$

Because the gravitational force is in the radial direction, the tangential component of acceleration is zero. This means that

$$2\dot{R}\dot{\theta} + R\ddot{\theta} = 0$$

Show that this relation is equivalent to $dA/dt =$ constant and Kepler's second law.