Homework set 01. Physics 141, Fall 2022

Due date: Friday Sept 9 at noon. Turn it in at the homework locker on the first floor of Bausch and Lomb Hall. Note there are also 7 webwork problems assigned this week. These should be done on line using the Web link on the home page on blackboard for this course.

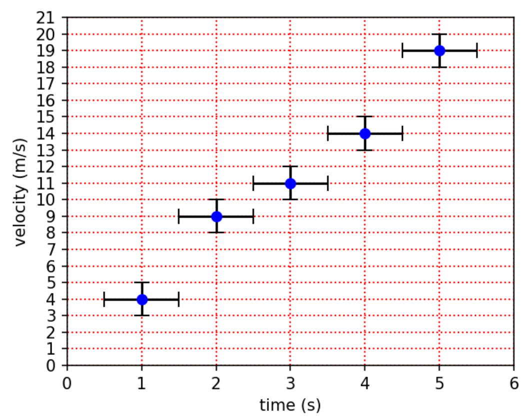
Total of 10 points. On errors.

This problem set is long. I would recommend automating with a calculator or spreed sheet or with python the computation of sums. I would also recommend comparing your results to those of yours colleagues so you can find errors in your computations. We have given sum formulas for computing the means and standard deviations of a list of numbers. These can also be computed automatically by entering the lists of numbers into an array in python (or another software package).

1. (2 points) A pendulum is made from a uniform density rod that is suspended from one end. It can be used to measure the gravitational acceleration g (on the surface of Earth). The value of acceleration g can be computed from measurements of the length of the rod L and the period of the pendulum T with
$$g = \frac{8\pi^2 L}{3T^2}.$$
Experimental measurements gave
$$L = 68.6 \text{ cm} \pm 0.3 \text{ cm}$$
and
$$T = 1.31 \text{ s} \pm 0.06 \text{ s}$$
with errors that are $\pm 1\sigma$.

What is the standard deviation for the measured value of g?

2. (2 points) The measured velocity v of a car, moving with a constant acceleration a, is shown in the Figure. Note: the indicated error bars are $\pm 1\sigma$.



We assume that the measured velocity v is proportional to the measured time t
$$v = at.$$
Our goal is to estimate most probable value of the acceleration a and an uncertainty for this estimate.

a) Use each point labeled with index i to compute an acceleration $a_i = v_i/t_i$.

b) For each point we have an uncertainty in v, which we describe with σ_v and is shown with the vertical error bars. What is σ_v?

c) For each point we have an uncertainty in t, which we describe with σ_t and is shown with the horizontal error bars. What is σ_t?

d) What is the uncertainty σ_i of each acceleration point?

Hint: propagate errors using the relation $a = v/t$.

e) Combine your measurements using weights $w_i = 1/\sigma_i^2$. Your estimate for a should be

$$\mu_a = \frac{\sum_i a_i w_i}{\sum_i w_i}$$

This is the mean value weighted by the errors.

f) Compute an estimate for the standard deviation of your measurement for acceleration.

$$\sigma_\mu = \sqrt{\frac{1}{\sum_i w_i}}$$

3. (2 points) An individual wants to make an estimate for the area of a sheet of paper and makes the following measurements:

   The height: 11.53, 11.14, 10.67, 11.39, and 12.12 cm.

   The width: 13.51, 13.37, 13.20, 13.71, 13.62, and 13.98 cm.

   What is the best estimate for the area of the paper and what is the estimated error for this best estimate?

4. (2 points) Astronomical magnitudes are related to brightness or flux f with

$$m = -2.5 \log_{10} f + \text{constant}$$

   where m is the magnitude. The constant sets the *zero-point*. Astronomical magnitudes, like the Richter scale and decibels, are on a logarithmic scale.

   A star with flux f_* has a measurement error with standard deviation $\sigma_f = 0.1 f_*$. Here the fractional error is 0.1 or 10%. What is the standard deviation of the measurement of the astronomical magnitude?

   It may be helpful to know that

$$\frac{d}{dx} \log_{10} x = \frac{1}{x \ln 10}.$$

5. (2 points) The length of a steel rod is measured repeatedly. The results of the measurements are:

   19.57 cm
   19.94 cm
   20.41 cm
   19.91 cm
   20.26 cm
   19.82 cm
   20.13 cm
   19.97 cm
   19.92 cm
   20.16 cm
   20.40 cm
   19.79 cm
   20.28 cm
   19.62 cm
   19.74 cm

   - What is the best estimate for the length of the steel rod? (Here you combine the measurements to make an accurate measurement).
   - What is the standard deviation of a single measurement? (Here you use the scatter in the data to estimate the uncertainty in your measurements).
   - What is the uncertainty (standard deviation) of your best estimate for the length of the steel rod? (Here you estimate the uncertainty in your combined measurement).