PHY141 Lectures 6,7 notes

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Contents

1 Harmonic Motion

We focus on a point mass upon which we exert a force. Previously we looked at a constant force, like gravitational acceleration on the surface of a planet, and inverse square law forces (gravity or electric). A very **simple** force law that depends linearly on position in 1 dimension

$$F(y) = -k(y - L)$$

where positive k is a spring constant in units of N/m and the spring rest length is L. Spring forces for actual springs are not exactly linear and the force might also depend on velocity as well as position.

If we shift the coordinate system so that $x = y - L$ is a displacement from rest, then the force law is even simpler

$$F(x) = -kx$$

This is known as **Hooke's law**. The force is applied in the direction opposite to the spring displacement.

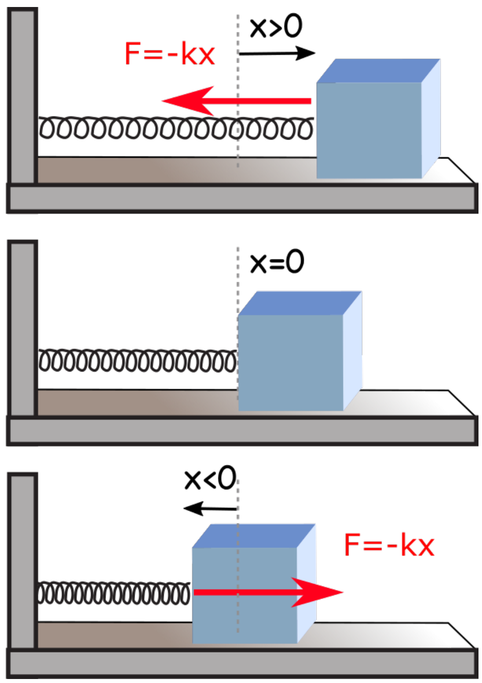

Figure 1: A spring force is linearly dependent on position. Here the block is assumed to be resting on a frictionless surface.

Many materials respond approximately linearly, with force proportional to displacement. It is challenging to design a mechanism with a constant force that is independent of displacement. For a clever mechanism that can be 3D printed see https://www.thingiverse.com/thing:4624094.

1.1 The harmonic oscillator

Consider a point mass of mass m that is connected to a massless spring with fixed endpoint. The value of x gives the position of the mass. We ignore gravity. The equation of motion using $F = ma$ is

$$\begin{aligned} m\frac{d^2x}{dt^2} &= -kx \\ \frac{d^2x}{dt^2} &= -\frac{k}{m}x \end{aligned}$$

The general solution can be written in different ways

$$\begin{aligned} x(t) &= A\cos(\omega t) + B\sin(\omega t) \\ &= Ce^{i\omega t} + De^{-i\omega t} \\ &= a\cos(\omega t + \phi_0) \end{aligned}$$

with angular frequency

$$\omega = \sqrt{\frac{k}{m}}.$$

Depending upon which form you chose, the solutions depend on coefficients A, B or amplitudes C, D or amplitude a and phase ϕ_0.

1.2 Sine and Cosine coefficients

Using the form

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

we differentiate $x(t)$ to find the velocity

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$$

Using initial conditions $x(t = 0) = x_0$ and $v(t = 0) = v_0$ we find that

$$x_0 = A \qquad v_0 = B\omega.$$

We solve for A, B in terms of x_0, v_0.

$$A = x_0 \qquad B = \frac{v_0}{\omega}$$

The solution at later times

$$x(t) = x_0\cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t)$$

1.3 Amplitude and phase coefficients

Using the form

$$x(t) = a\cos(\omega t + \phi_0)$$

we differentiate $x(t)$ to find the velocity

$$v(t) = -a\omega\sin(\omega t + \phi_0).$$

At $t = 0$, the displacement and velocity

$$x_0 = a\cos\phi_0 \qquad v_0 = -a\omega\sin\phi_0$$

We solve for ϕ_0, a

$$a = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$
$$\phi_0 = \mathrm{atan2}\left(-\frac{v_0}{\omega}, x_0\right)$$

Note the atan2 function gives an angle within $[-\pi, \pi]$ or $[0, 2\pi]$. The arctan function alone would only give an angle within $[-\pi/2, \pi/2]$ or $[0, \pi]$.

The solution at later times

$$x(t) = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}\cos\left(\omega t + \mathrm{atan2}\left(-\frac{v_0}{\omega}, x_0\right)\right)$$

1.4 With complex exponentials

Using the form

$$x(t) = Ce^{i\omega t} + De^{-i\omega t}$$

we differentiate $x(t)$ to find the velocity

$$v(t) = Ci\omega e^{i\omega t} - Di\omega e^{-i\omega t}$$

At $t = 0$ the coefficients

$$x_0 = C + D \qquad v_0 = i\omega(C - D)$$

We solve for the coefficients

$$C = \frac{1}{2}\left(x_0 + \frac{v_0}{i\omega}\right)$$
$$D = \frac{1}{2}\left(x_0 - \frac{v_0}{i\omega}\right)$$

The solution at later times

$$x(t) = \frac{1}{2}\left(x_0 + \frac{v_0}{i\omega}\right)e^{i\omega t} + \frac{1}{2}\left(x_0 - \frac{v_0}{i\omega}\right)e^{-i\omega t}$$

It may be disturbing to describe a real system with complex numbers for displacement and velocity. However it can be convenient to solve the problem in a complex form and then assert that the actual solution is the real part of the complex one. Alternatively, by requiring the initial conditions to be real, a real solution at later times is ensured.

1.5 Trajectories of the Harmonic oscillator

Trajectories are shown in different ways in Figure 2. We draw $x(t), v(t)$ vs t and we draw $x(t)$ vs $v(t)$ (which is known as phase space).

What is the period?

$$P = \frac{2\pi}{\omega}$$

The period is independent of oscillation amplitude.

Note that

$$x^2 + v^2/\omega^2 = \text{constant}$$

and this is equivalent to conservation of energy. The trajectory is an ellipse in a v vs x plot.

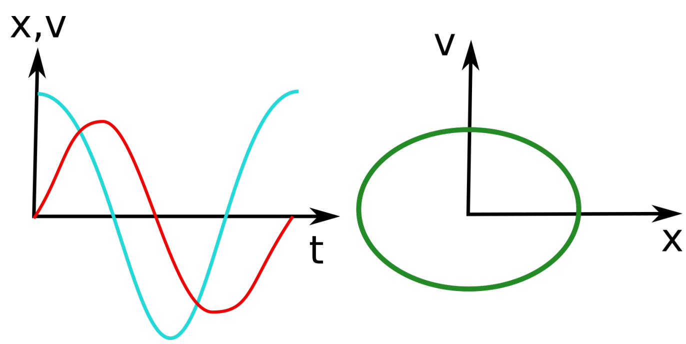

Figure 2: Trajectories for the harmonic oscillator. Displacement and velocity are sinusoidal (on the left) whereas the trajectory is an ellipse in phase space (on the right).

This physical model is known as the **harmonic oscillator** and is a ubiquitous model. If you can make a complicated problem look like a harmonic oscillator, then you can (approximately) solve it.

1.6 The pendulum

We consider a point mass of mass m that is attached to a massless string of length L. The end of the string is held fixed. The point mass feels gravitational acceleration. The state of the system is described with an angle θ.

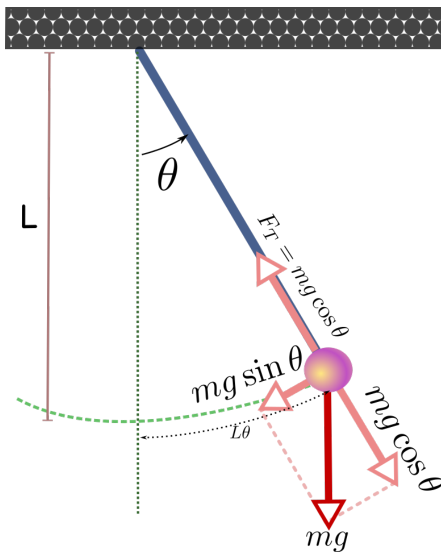

Figure 3: The radial force component is balanced by tension on the string. The tangential force component is $F = mg\sin\theta$.

We decompose the force from gravity (onto m) into radial and tangential components

$$\mathbf{F} = -mg\hat{\mathbf{z}} = mg\cos\theta\hat{\mathbf{r}} - mg\sin\theta\hat{\boldsymbol{\theta}}$$

The radial component is balanced by the tension in the string leaving the tangential component to accelerate the mass m.

Recall that in polar coordinates

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}}$$

As $\dot{r} = 0$ and the radial component is balanced by tension we are left with

$$\mathbf{a} = L\ddot{\theta}\hat{\boldsymbol{\theta}}$$

(here radius is L). Using our tangential force component

$$mL\ddot{\theta} = -mg\sin\theta$$

With the small angle approximation

$$\sin\theta \approx \theta$$

and we find

$$\ddot{\theta} = -\frac{g}{L}\theta.$$

We can solve this equation of motion with

$$\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{L}}t + \phi_0\right)$$

with phase ϕ_0. The frequency of oscillation is

$$\omega = \sqrt{\frac{g}{L}}$$

and the oscillation period is

$$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}.$$

With small amplitudes of oscillation, the period is independent of amplitude and we have equations the same as for a harmonic oscillator. However when the amplitude is large the period depends on amplitude.

For a pendulum the period depends on the amplitude with higher amplitudes having longer periods. Is it possible to design a pendulum that has period independent of amplitude? Then answer is yes. See the cycloidal pendulum animation and The tautochrone curve.

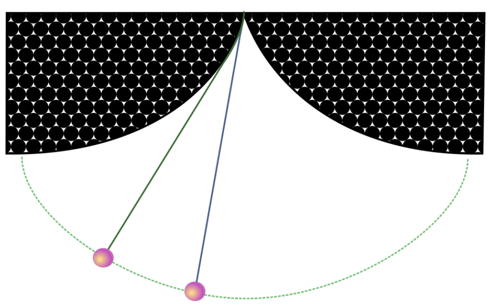

Figure 4: The cycloid pendulum.

2 Some Force diagrams

Some force diagram examples to discuss in class.

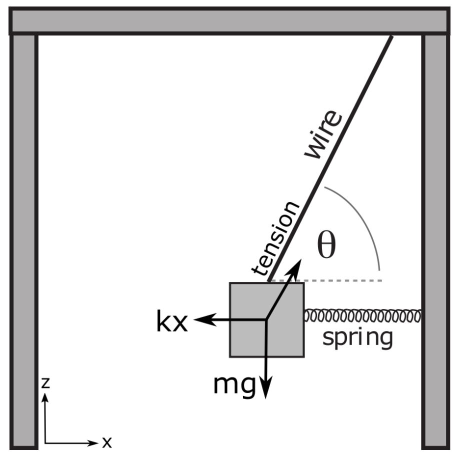

Figure 5: We assume the system is in an equilibrium state. The tension force in the wire $\mathbf{T} = T\cos\theta\hat{\mathbf{x}} + T\sin\theta\hat{\mathbf{z}}$. The z component of tension is balanced by gravity and the x component is balanced by the spring force. Here $+z$ is upward and $+x$ is to the right.

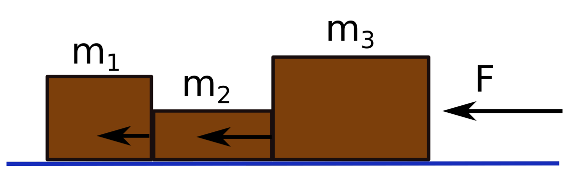

Figure 6: The three blocks are on a frictionless surface. The force $F = (m_1 + m_2 + m_3)a$. The acceleration of all the blocks is the same. We call the contact force $f_{1,2}$ that between blocks 1 and 2, and $f_{2,3}$ the contact force between blocks 2 and 3. The contact forces are exerted equally and oppositely between pairs of blocks.

All three blocks in Figure 6 have acceleration a. This means that

$$F = (m_1 + m_2 + m_3)a.$$

The leftmost block (m_1) must have contact force

$$f_{1,2} = m_1 a$$

on it because there are no other horizontal forces on it. The rightmost block (m_3) has

$$m_3 a = F - f_{2,3} = (m_1 + m_2 + m_3)a - f_{2,3}.$$

This means that the contact force

$$f_{2,3} = (m_1 + m_2)a,$$

which makes sense as $f_{2,3}$ must account for acceleration of $m_1 + m_2$. The middle block has

$$m_2 a = f_{2,3} - f_{1,2} = (m_1 + m_2)a - m_1 a,$$

as expected.

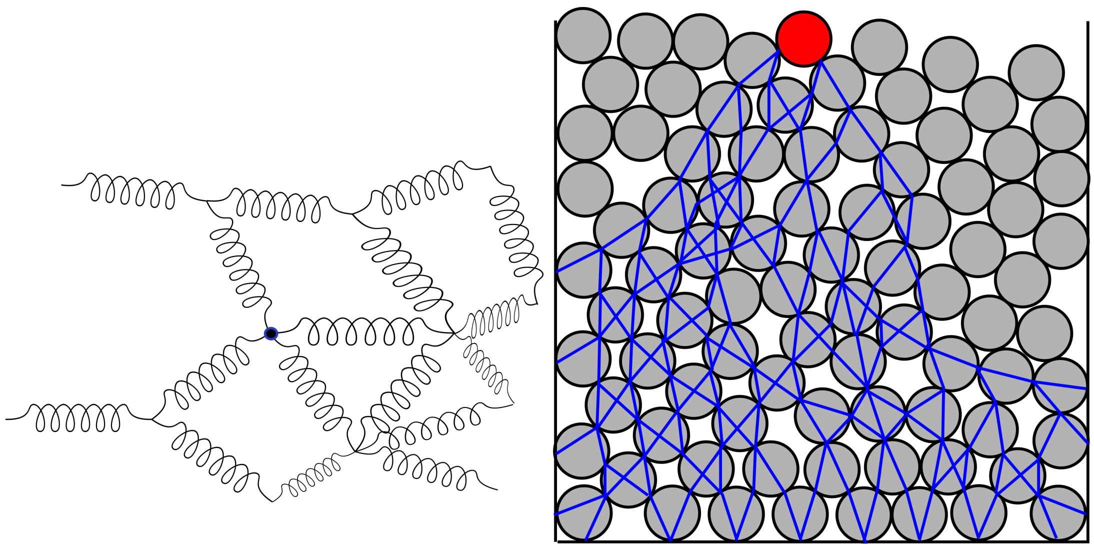

Figure 7: Left: A spring network. If the network is in equilibrium, the sum of the force vectors at each node is zero. Right: An illustration of a chain of transmission of stress forces through contacts in a granular media. The force chain figure is by Gsrdzl - Own work, CC BY-SA 3.0. and licensed under Creative Commons Attribution-Share Alike 3.0 Unported.

3 Damped and driven harmonic motion

3.1 Damped harmonic motion

Often there are velocity dependent forces as well position dependent forces. We modify our spring model so the force is

$$F = -kx - bv. \tag{1}$$

This first term is a spring force. The second term is a velocity dependent force that depends on positive coefficient b that would be caused by a dashpot. The equation of motion

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0. \tag{2}$$

It is useful to define

$$\begin{aligned} \omega_0 &\equiv \sqrt{\frac{k}{m}} \\ \gamma &\equiv \frac{b}{m} \end{aligned} \tag{3}$$

with γ in units of inverse time. The equation of motion (equation 2 becomes)

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0. \tag{4}$$

We assume a general solution

$$x(t) = Ae^{i\omega t}$$

Here ω can be positive or negative and A can be complex.

What does it mean to assume a complex solution when x is a real distance? If the equations are linear then we can solve for complex x and then take the real part.

$$\mathbb{R}\mathrm{e}\, e^{i\omega t} = \cos\omega t.$$

If A is complex then you need to take that into account when you take the real part of $Ae^{i\omega t}$.

Insert our general solution into the equation of motion (eqn 4)

$$\begin{aligned} -\omega^2 Ae^{i\omega t} + \gamma A i\omega e^{i\omega t} + \omega_0^2 Ae^{i\omega t} &= 0 \\ \omega^2 - \gamma i\omega - \omega_0^2 &= 0. \end{aligned}$$

We solve for the frequency ω using the quadratic equation

$$\omega = \frac{i\gamma}{2} \pm \frac{1}{2}\sqrt{4\omega_0^2 - \gamma^2}. \tag{5}$$

We have three cases for solutions

- $\gamma < 2\omega_0$. This case is known as *weakly damped*. The argument inside the square root is positive so the general solution is

  $$x(t) = \mathbb{R}\mathrm{e}\left[Ae^{-\frac{\gamma t}{2}}e^{i\tilde{\omega}t} + Be^{-\frac{\gamma t}{2}}e^{-i\tilde{\omega}t}\right]$$

  with

  $$\tilde{\omega} = \frac{1}{2}\sqrt{4\omega_0^2 - \gamma^2}.$$

- $\gamma > 2\omega_0$. This case is known as *over-damped.* The argument inside the square root is negative so the general solution is

$$x(t) = \left[Ae^{-\frac{(\gamma+\nu)t}{2}} + Be^{-\frac{(\gamma-\nu)t}{2}} \right]$$

with

$$\nu = \frac{1}{2}\sqrt{\gamma^2 - 4\omega_0^2}$$

.

- $\gamma = 2\omega_0$. This case is known as critically damped. The general solution is

$$x(t) = Ae^{-\frac{\gamma t}{2}} + Bte^{-\frac{\gamma t}{2}}.$$

We can write

$$e^{-\frac{\gamma t}{2}} = e^{-\frac{t}{t_{\rm damp}}}$$

in terms of a damping timescale

$$t_{\rm damp} = \frac{2}{\gamma} = \frac{2m}{b}.$$

The amplitude of the weakly damped solution decays on this exponential timescale.

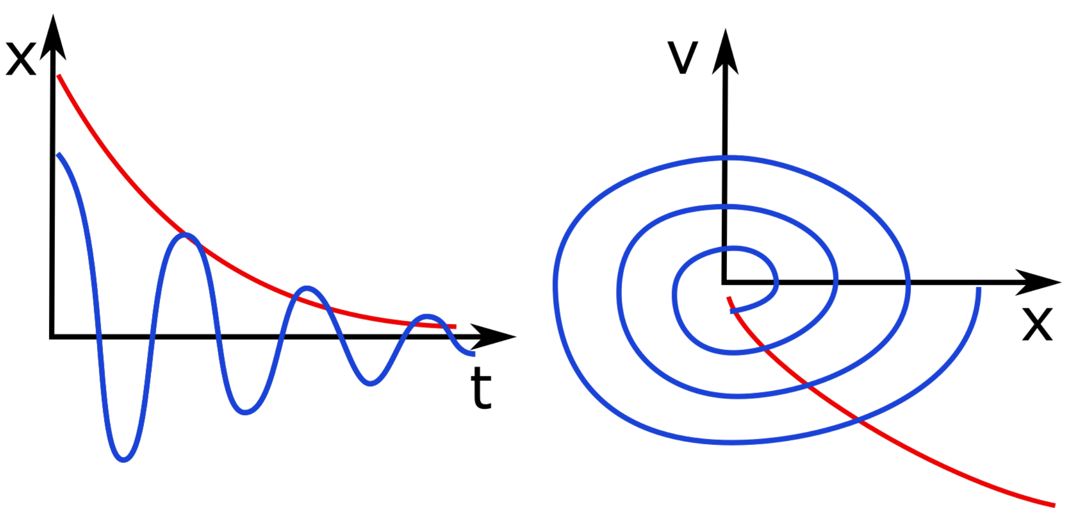

Figure 8: The different types of solutions for the damped harmonic oscillator. The red lines show a highly damped harmonic oscillator. The blue lines show a weakly damped harmonic oscillator. On the left we show $x(t)$ vs t. On the right we show phase space; dx/dt vs x.

3.2 Driven harmonic motion

Consider a damped harmonic oscillator that driven by a sinusoidal force

$$F_{driving} = ma_0 \sin \omega t$$

that is pushing on the mass. Here a_0 is in units of acceleration. We modify equation 2 to include this driving force. The damped driven harmonic oscillator has equation of motion

$$\frac{d^2x}{dt^2} = -\omega_0^2 x - \gamma \frac{dx}{dt} + a_0 \sin(\omega t). \tag{6}$$

We start with the oscillator with some initial position and velocity. The solution will have a **transient** response which decays and then will approach a steady state, which likely will be sinusoidal and described with a particular amplitude, frequency and phase.

To make things simpler, we will ignore the transient response and we look at driven harmonic motion without any damping. With $b = \gamma = 0$, the equation of motion is

$$\frac{d^2x}{dt^2} = -\omega_0^2 x + a_0 \sin(\omega t). \tag{7}$$

The steady state solution (ignoring transient response) must depend on the driving frequency ω

$$x(t) = A\cos(\omega t) + B\sin(\omega t). \tag{8}$$

We insert this into the equation of motion

$$-A\omega^2 \cos(\omega t) - B\omega^2 \sin(\omega t) = -A\omega_0^2 \cos(\omega t) - B\omega_0^2 \sin(\omega t) + a_0 \sin \omega t$$

We get two equations

$$-A\omega^2 = -A\omega_0^2$$
$$-B\omega^2 = -B\omega_0^2 + a_0$$

The first equation gives $A = 0$ (unless $\omega = \omega_0$) and the second one gives

$$B = \frac{a_0}{\omega_0^2 - \omega^2}$$

Units are ok! Inserting our solution for B into equation 8 we find a solution for the undamped but driven harmonic oscillator

$$x(t) = \frac{a_0}{\omega_0^2 - \omega^2} \sin \omega t. \tag{9}$$

This is a solution to the inhomogeneous ordinary differential equation 7. The general solution would be a sum of homogeneous and inhomogeneous terms. (That means we can add any solution of the non-driven harmonic oscillator to get another solution).

If $\omega = \omega_0$ we have a problem as the amplitude is infinite! Had we used a damped spring, the response would have been limited by the damping rate.

This is an example of **resonant** response, as there is a strong response near a resonant frequency.

Notice the response is either in phase or with the opposite phase as the driving force. The sign of the response depends on whether the driven frequency is larger or smaller than the resonant one.

In the adiabatic limit (slow frequency driving) the solution is $B = a_0/\omega_0^2$. In this limit ω_0 can be considered large and the spring constant is large. We balance kx against the driving force (and neglected the inertial force that depends on mass) and this gives the solution.

In the opposite limit (fast frequency driving) the amplitude is small – the forcing averages to zero.

We have assumed an infinite time period for steady driving. There would be transients associated with the onset of the driving force in a real system.

With both damping and forcing, the near resonant response is not infinite and the phase varies continuously from $-\pi$ to π as the driving frequency passes across the resonant frequency.

3.3 Driven and damped harmonic motion

The driven and damped harmonic oscillator

$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0^2 x = \frac{F}{m}\sin(\omega t) \tag{10}$$

(with $\zeta = \frac{\gamma}{2\omega_0} = \frac{b}{2m\omega_0}$ of our damped oscillator; equations 1 and 2). The steady state solution can be written in the form

$$x(t) = A\cos(\omega t + \phi)$$

with amplitude A and phase ϕ being functions of driving force F, frequency ratio ω/ω_0 and damping parameter ζ. What do we mean by steady state? We mean one with constant amplitude, not one that does not oscillate. Independent of its initial conditions, a driven, damped harmonic oscillator will approach this solution. The amplitude of the steady state solution satisfies

$$A = \frac{F_0}{m\omega}\frac{1}{\sqrt{(2\omega_0\zeta)^2 + \frac{(\omega_0^2-\omega^2)^2}{\omega^2}}}. \tag{11}$$

The amplitude of the steady state solution of equation 10 is shown in Figure 9 for different values of damping parameter ζ. I wrote the damping term in terms of ζ so as to be consistent with Figure 9.

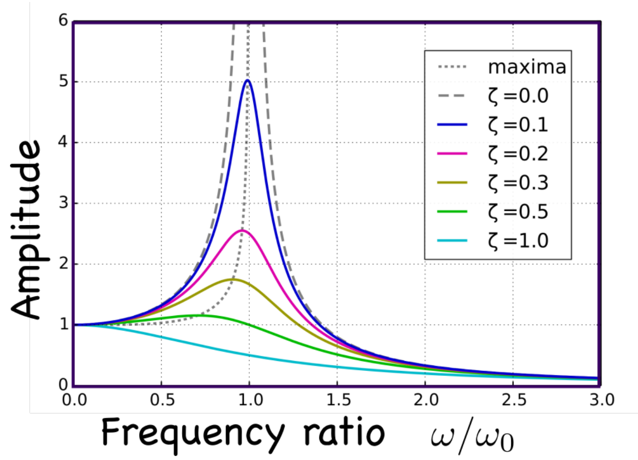

Figure 9: The damped driven harmonic oscillator. The x axis is in units of ω/ω_0. The y axis shows the amplitude of the steady state sinusoidal solution. Here ω is the driving frequency, ω_0 is the resonant frequency and ζ is a damping parameter. The dotted grey line goes through the maxima of each curve. The dashed grey lines show an envelope of possible solutions. This figure is a modified version of Mplwp resonance zeta envelope.svg. Axes have been more clearly labelled. The original file is licensed under Creative Commons Attribution 3.0 Unported license.

The phase difference between driving frequency and harmonic oscillator satisfies

$$\tan\phi = -\frac{\gamma\omega}{\omega_0^2 - \omega^2} = -\frac{2\omega_0\zeta\omega}{\omega_0^2 - \omega^2}. \tag{12}$$

4 Springs in parallel and in series

4.1 Two springs in series

Consider two springs k_1 and k_2 in series (see Figure 10) with the seconnd spring connected to a mass m. The first spring is stretched by x_1 and the second spring by x_2. The total displacement is $x_1 + x_2$. What is the effective spring constant k_s of both springs? Meaning if we think of them as a single spring what would its spring constant be?

If the springs are massless and the bottom one is under a load or weight, then both springs have the same force on them.

$$F = k_1 x_1 = k_2 x_2 = k_s(x_1 + x_2).$$

Equivalently look at the point between the springs. The forces $k_1 x_1 = k_2 x_2$ because equal and opposite forces apply from both springs at this point. The force exerted by the

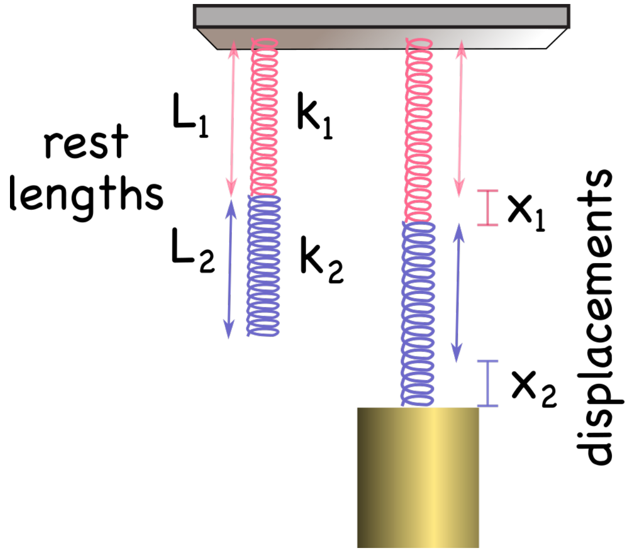

Figure 10: Massless springs in series. The effective spring constant $k_s = \frac{1}{k_1^{-1}+k_2^{-1}}$.

first spring on the second spring must be equal and opposite to that exerted by the second spring on the first spring.

Look at the point touching the mass m. We know that k_2x_2 is the force on m and this must be equal to $k_s(x_1 + x_2)$ as this too must be the force on m.

Using $k_1x_1 = k_2x_2$, we solve for x_2

$$x_2 = \frac{k_1}{k_2}x_1.$$

Using $k_s(x_1 + x_2) = k_1x_1$, we solve for the effective spring constant k_s

$$\begin{aligned} k_s &= \frac{k_1x_1}{x_1+x_2} \\ &= \frac{k_1x_1}{x_1+(k_1/k_2)x_1} \\ &= \frac{k_1k_2}{k_1+k_2} \\ &= \frac{1}{k_1^{-1}+k_2^{-1}}. \end{aligned} \tag{13}$$

There is an analogy for resistors in parallel in electric circuits.

4.2 Two springs in parallel

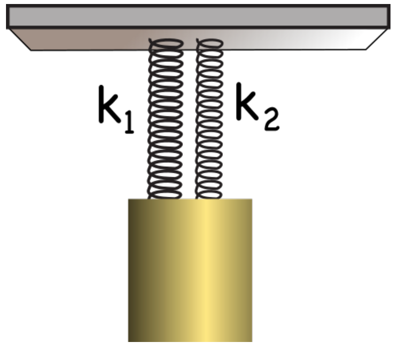

Figure 11: Massless springs in parallel. The effective spring constant $k_s = k_1 + k_2$.

Springs in parallel are more straight forward. The displacements from both springs are the same. The forces add. $F = (k_1 + k_2)x = k_s x$. The effective spring constant for two springs in parallel

$$k_s = k_1 + k_2. \tag{14}$$

The spring constants add. There is an analogy for resistors in series.

4.3 Example

We start with a single spring with spring constant k. We consider a chain of these springs, in series, that is 100 springs long. What is the spring constant of the chain?

$$k_{chain} = \frac{1}{\sum_{i=1}^{100} \frac{1}{k}} = \frac{k}{100}.$$

We have a block that is comprised of 2000 such chains in parallel. What is the spring constant of the 2000 chains?

$$k_{block} = k_{chain} \times 2000 = \frac{k}{100} \times 2000 = 20k.$$

5 Atomic forces in solids, stress and strain

Consider a metal with density ρ and a molar mass of M_{molar}. The density is mass per unit volume. The molar mass is the mass of a mole or Avogadro's number of atoms. Avogadro's number is

$$N_A = 6 \times 10^{23} \frac{\text{atoms}}{\text{mol}}.$$

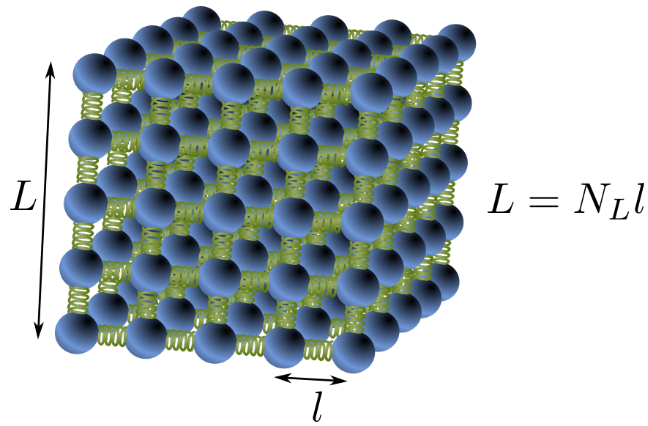

Figure 12: Bond strengths are modeled with springs. A mass spring model approximates the elastic properties of a solid. However, spring forces connect between point masses so sensitivity to bond angle is neglected. This model is classical and neglects thermal fluctuations. Here the inter atom spacing is l and the length of the cube is L. The number of atoms in length L is $N_L = L/l$. The number density $n = N_L^3/L^3 = l^{-3}$.

A mole of this metal has how much volume?

$$\rho \times \frac{1}{M_{\rm molar}} = \frac{\rm mass}{\rm volume} \times \frac{\rm mol}{\rm mass} = \frac{\rm mol}{\rm volume}$$

The volume of a mole

$$V_{\rm mole} = \frac{M_{\rm molar}}{\rho} = \frac{\rm volume}{\rm mol}.$$

The number density is the number of atoms per unit volume; $n = N/V$ where N is a number and V is a volume. For this metal, what is the number density of atoms, n?

$$\begin{aligned} n &= \frac{N_A}{V_{\rm mole}} = \rho \times \frac{1}{M_{\rm molar}} \times N_A \\ &= \frac{\rm mass}{\rm volume} \times \frac{\rm mol}{\rm mass} \times \frac{\rm atoms}{\rm mol} \\ &= \frac{\rm atoms}{\rm volume} \end{aligned}$$

What is the typical length, l, between atoms in our metal? Consider a cubic volume that is $V = L^3$ with L a side of the cube. Then the number of atoms in the cube is $N = nL^3$. We arrange the N atoms in a cubic lattice. Then number of atoms on a side of

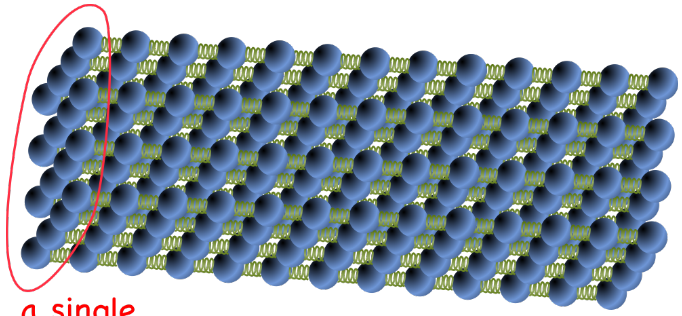

Figure 13: Estimating interparticle force strength from the stretching of a wire. We count the number of atoms in a single layer in the base. Each linear segment can be considered a single long spring.

the cube is N_L with $N = N_L^3$ and $L = N_L l$. This means that

$$\begin{aligned} N &= nV \\ N_L^3 &= nL^3 \\ \left(\frac{N_L}{L}\right)^3 &= n \\ \left(\frac{L}{N_L}\right)^3 &= n^{-1} \\ \left(\frac{L}{N_L}\right) &= n^{-1/3} \end{aligned}$$

We compute $l = L/N_L$ which is the length per atom on a side and this we find is equal to

$$l = n^{-\frac{1}{3}}. \tag{15}$$

What is the typical length between atoms in our metal?

$$l = n^{-\frac{1}{3}} = \left(\frac{M_{\text{molar}}/\rho}{N_A}\right)^{\frac{1}{3}}. \tag{16}$$

To estimate the force strength between atoms we stretch a wire and measure the force required to strength it by a particular length. The wire is modeled as N_a single long springs. Each long spring has N_L atoms in it. Using our formulas for how spring constants add in parallel and in series we can relate the total distance stretched to the amount the distance between atoms is stretched. We can estimate the spring constant between two atoms.

N_a, is the number of single long springs. We estimate this by dividing the area of the wire by the inter atomic spacing l^2. N_L is the number of atoms in a single long chain that is the length of the wire. We estimate this by taking the length of the wire and dividing it by l.

5.1 Stress and strain

Stress is force per unit area. However force is a vector and the direction that it is applied (or the normal vector to the area) is also a vector. So stress is a **tensor**. Compressive stress is exerted when the entire volume shrinks. Tensile stress is exerted when the volume expands, for example when a wire is pulled. Sheer stress is when a cube of jello is pushed on the top so that it tilts.

Strain is a deformation per unit length. It is related to a gradient of displacement. A particle can be displaced w.r.t. to another particle in 3 directions, compared to the vector between the two particles. So strain is also a tensor.

In 1-dimension strain is

$$\epsilon = \Delta L/L \tag{17}$$

where ΔL is the amount of length change and L is the length. The strain ϵ is dimensionless. An elastic solid has stress σ proportional to strain

$$\sigma = E\epsilon \tag{18}$$

and the coefficient E is Young's modulus. However, a more realistic model takes into account additional degrees of freedom (compression vs shear stress and strain). There are additional moduli such as the bulk modulus. Stress and the associated moduli are in MKS units of Pa (pascals) which is equivalent to $\mathrm{N/m^2}$ or $\mathrm{J/m^3}$.

5.2 Deviations from linear elasticity

When pulled too far, the material can flow or be ductile or plastic instead of elastic. This is why large solid asteroids or planets tend to be round. The material can also break. Tensile yield strengths are often not the same as compressive yield strengths.

A material can heat up when stretched or compressed. With a velocity dependent or strain rate dependent force, the material is described as *viscoelastic*.

6 Summary

- Hooke's law $F = -kx$ with positive spring constant k.
- Harmonic motion. Angular frequency $\omega = \sqrt{k/m}$. Period $P = 2\pi/\omega$.
- How to find the solution for harmonic motion at later times from initial conditions.

- The pendulum's angular frequency $\omega = \sqrt{g/L}$ for small amplitude motions.
- Driven and damped harmonic motion.
- Springs in parallel and in series.
- Elastic force in a solid and how this is related to interatomic forces.
- Practicing making force diagrams!

7 Props

Silly putty and a marble. Two slinkies.

Force chain movie called `fromsolidtol.gif`