Lab #3

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The coefficient of restitution of a bouncing elastic sphere

Equipment list

At the front of the room: The box of different types of spheres and balls.

1 Objectives

This lab involves measuring the coefficient of restitution of a bouncing sphere.

Audio sampling rates are quite high (typically 20 kHz). This means we can make precise time measurements by recording audio. We recommend that you use audio recording to make your measurements, though you could opt to make other types of measurements instead or in addition to audio measurements.

A similar lab was done as a take home lab after Thanksgiving in 2020. This year you have some more options! Your goal is to design and carry out an experiment. Some possible experimental goals:

- Design and carry out experiments that give you an accurate measurement of the coefficient of restitution of a sphere. You can design your experiments to minimize your final error.
- Design and carry out experiments to determine if and how the coefficient of restitution of a sphere depends on impact velocity. You can consider how to quantitatively describe a dependence on velocity and how your experiment can place quantitative constraints on such a dependence.
- Design and carry out experiments to determine how the coefficient of restitution of a sphere depends on the properties of the surface that it impacts. If your experiments are designed to be repeatable, you can argue that two similar experiments, on two different surfaces, are measuring the sensitivity of the coefficient of restitution to the impact surface.

We suggest you focus on one of these goals. However, you could also propose and explore a similar topic. For example with an inflatable ball you could study the relation between a ball's coefficient of restitution and pressure inside the ball.

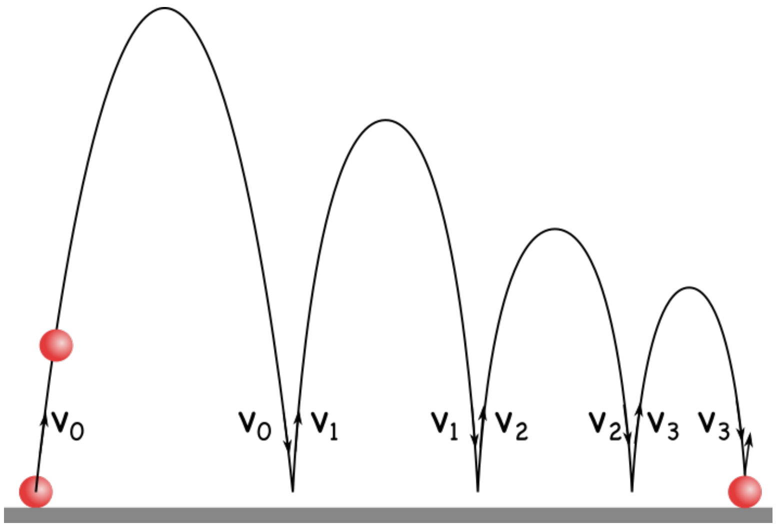

Figure 1: An elastic ball bounces off the floor. The ratio of velocity after and before each bounce is the coefficient of restitution. This figure shows some horizontal motion, but we will only consider a ball that bounces up and down vertically.

1.0.1 Things you need to do this lab

Find a spherical object that will bounce at least four times when you drop it onto a hard flat surface such as a wood, vinyl or cement floor. For example, a marble, a rubber ball or a ping-pong ball would work. We will have some spheres in the lab, but you could also bring your own.

You will need the ability to record sound and look at a sound file. I recommend using the open-source, multi-platform Audacity software which can be installed on a laptop.

2 Physics of a bouncing elastic sphere

2.1 The times and velocities of a series of bounces

Consider a ball moving vertically under downward gravitational acceleration, g. The vertical coordinate is z and $z = 0$ when the ball touches the floor. At $t = 0$ the ball touches the floor and its upward velocity is v_0. The trajectory

$$z(t) = -\frac{gt^2}{2} + v_0 t. \tag{1}$$

The ball returns to impact the ground at time

$$t_g = \frac{2v_0}{g}. \tag{2}$$

We neglect air resistance. The upward velocity at $t = 0$ is the same as the downward velocity at time $t = t_g$. We invert equation 2 to find the upward velocity at $t = 0$

$$v_0 = \frac{g t_g}{2}. \tag{3}$$

The elastic ball undergoes a series of bounces, as shown in Figure 1. We call t_i the time of the i-th bounce and v_i the upward velocity after the bounce at time t_i. Using equation 3 we can find v_i from the time between two bounces

$$v_i = \frac{g(t_{i+1} - t_i)}{2}. \tag{4}$$

We can use this equation to propagate errors. Suppose the error to measure the time is the same for all bounces and has a standard deviation σ_t. The standard deviation in each measurement of v_i is

$$\sigma_v = \sqrt{2}\frac{g\sigma_t}{2}. \tag{5}$$

The factor of $\sqrt{2}$ comes from summing two errors in quadrature.

Using our audio recording of a ball bouncing, we can measure the times of a series of bounces. We can use equation 4 to find the velocity of the ball just after each bounce, and equation 5 can be used to estimate the standard deviation of these velocities.

2.2 The coefficient of restitution

The coefficient of restitution, COR, is defined as the ratio of velocity after and before a collision or bounce,

$$COR \equiv \frac{v_{\text{after}}}{v_{\text{before}}}. \tag{6}$$

If $COR = 1$ the bounce is perfectly elastic and no energy is lost during the collision.

The definition of the coefficient of restitution implies that

$$v_{i+1} = (COR)\, v_i \tag{7}$$

where v_i is the velocity just after the i-th bounce.

With three bounces, we can compute two time intervals and this would give us two impact velocities. That's enough information to compute an estimate for the coefficient of restitution.

Inverting equation 7

$$COR = \frac{v_{i+1}}{v_i}. \tag{8}$$

We can propagate errors to estimate the standard deviation of COR from the standard deviations in σ_v,

$$\frac{\sigma_{COR}}{COR} = \sqrt{\frac{\sigma_v^2}{v_{i+1}^2} + \frac{\sigma_v^2}{v_i^2}}.$$

Equation 8 implies that data points on a v_{i+1} vs v_i plot would lie on a line with slope COR. Such a plot could be made with vertical error bars $\sigma_{v_{i+1}}$ and horizontal error bars σ_{v_i}. If data points don't lie on a line that goes through the origin (and that goes through or near the error-bars) then you would have evidence that the coefficient of restitution is dependent on velocity.

If the coefficient of restitution COR is independent of velocity then

$$v_i = (COR)^i v_0. \tag{9}$$

If we take the natural log of this equation

$$\ln\left(\frac{v_i}{v_0}\right) = i \ln(COR). \tag{10}$$

On a plot of $\ln\left(\frac{v_i}{v_0}\right)$ vs index i, we expect all the points to lie on a line and the slope of the line would be the log of the coefficient of restitution. If the points don't lie on a line then the coefficient of restitution probably depends on velocity. You would need to compute vertical error bars on such a plot to determine whether points were significantly off a line. For $z = \ln v_i$, the standard deviation $\sigma_z = \frac{\sigma_{v_i}}{v_i}$.

What types of plots should you make? I would explore a few different types of plots and then choose the ones that you think are most interesting or descriptive to put in your lab report.

Models for the coefficient of restitution can take into account the spherical shape of the ball, the contact area, the extent of deformation in elastic and plastic regimes for both ball and surface that the ball bounces on. Heat generated during deformation (viscoelastic behavior) can also affect the model. Models often predict a weak dependence on velocity. If you are curious about physical models of the coefficient of restitution for a bouncing elastic sphere, see On predicting the coefficient of restitution and Model for collisions in granular gases.

2.3 How much and what type of data?

If you do experiments multiple times you can improve your measurement of the coefficient of restitution COR by combining measurements.

To measure a dependence of COR on velocity it helps to have measurements at different velocities. This can be done either by measuring many bounces during a single time series or by measuring bounces for a ball dropped from different heights.

3 Procedures

3.1 With the Audacity Audio software

The Audacity software is a multi-platform open source audio recording and analysis software.

- Download and install Audacity.
- Launch the Audacity application. Try recording and playing back a file. Sound clips can be deleted by clicking on the x on the left hand side. See Figure 2 showing the Audacity application with some labels on how to record and play sound files. You can select different regions of the audio with a mouse click and drag. You can delete regions with 'delete' after you have selected a region. You can zoom in and out in the horizontal axis. You can change the format of time using drop down menus.
- Press record. Drop your marble or ball. Wait for it to stop bouncing. Stop the recording. You may need to drop the marble a few times to ensure that it doesn't hit anything after only a few bounces. Each bounce should be seen as a peak in the recorded audio (see Figure 2). Figure 2 shows a marble bouncing and also a rubber ball bouncing. Note that the time between bounces is progressively shorter.
- Zoom in on the horizontal time axis to better see the onset of a single peak which is when the bounce began. Measure the time of the onset in milliseconds. It may not be obvious how to identify exactly when the bounce began. Estimate an uncertainty in the time. Do you feel you are certain of your time measurement to 1 ms? 3 ms? 5 ms? In your lab notebook, record your time measurement for the onset of the bounce and an estimate for your uncertainty in this measurement.
- Repeat the above measurement for each bounce (each peak in the audio file). You should notice that the time between bounces is progressively shorter.

The above procedure is a guideline. You can adjust your procedures according to your experimental design!

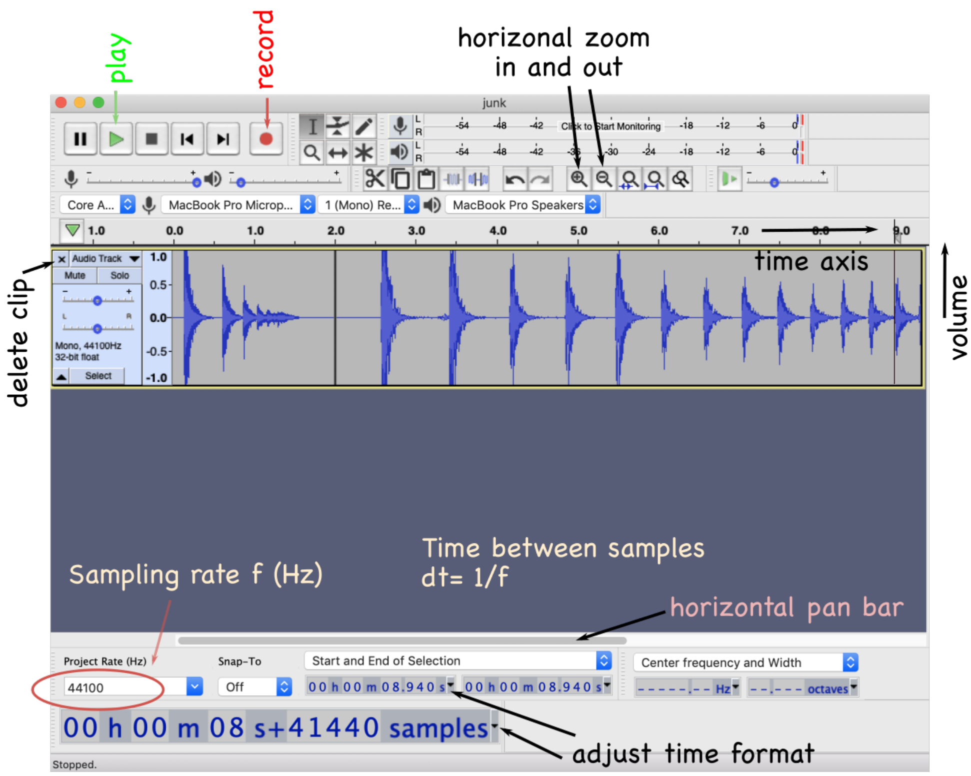

Figure 2: A snapshot of the Audacity audio application. I recorded a marble bouncing (the first two seconds of audio) and a rubber ball bouncing (starting at about 2.5s).

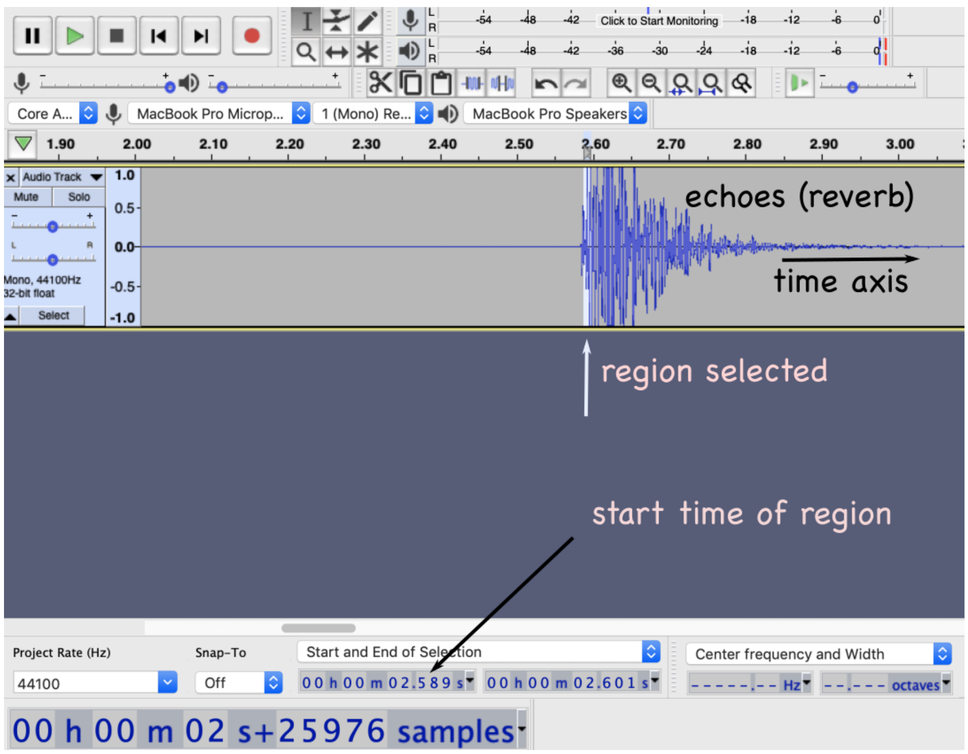

Figure 3: A snapshot of the Audacity audio application. This is zoomed in on the time axis to better show the onset and reverberation sound detected from a single bounce of a rubber ball. After the onset, echos and sound waves from the floor are detected. A small region is selected (in white) at the onset. The time when the bounce took place in milliseconds is approximately 2.589 s from the start of the audio clip.

4 Analysis and Discussion

Include an abstract summarizing your results. Describe your experiments and how you designed them. Justify your choice of the number and types of measurements. We would encourage you to describe your plots, list your measurements, describe how your data is analyzed and summarize your results. You could also mention assumptions and physical processes that are neglected in your analysis and discuss whether you think these issues may have impacted your results.

It is important to include tables of your measurements but tables are not always the most effective way of showing data or results. Plots, figures and diagrams can convey information in a more direct way.

We assume that you will find a way to plot data (matlab, python, excel, google colaboratory).