0. **(0 points)** /opt/webwork/webwork2/conf/snippets/ASimpleCombinedHeaderFile.pg

Alice Quillen **fall22phy141**

Assignment PHY141_WW12 due 12/14/2022 at 11:59pm EST

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1. **(1 point)** setPHY141_WW12/maxwell_boltzmann.pg

On the Maxwell-Boltzmann velocity distribution

A bottle contains a mixture of Helium and Oxygen molecules (O_2). The root-mean-square velocity for the oxygen molecules is $v_{rms,O_2} = 480$ m/s.

Find the root-mean-square velocity vrms of the Helium molecules.

Enter $v_{rms,\mathrm{He}} =$ ___ m/s.

Find the temperature of the gas.

Enter $T =$ ___ K.

It may be useful to know the Boltzmann constant
$k_B = 1.3806503\text{E-}23$ m^2 kg s^{-2}K^{-1}
and the atomic mass unit 1.66053906660E-27 kg.

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2. **(1 point)** setPHY141_WW12/pnkt.pg

On the Ideal Gas law

A bottle contains argon at a pressure of $P_{init} = 2$ bar and at a temperature of $T_{init} = 285$ K.

The temperature increases by 20 K while the volume remains fixed.

After the temperature increase, what is the pressure inside the bottle?

Enter $P =$ ___ bar

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3. **(1 point)** setPHY141_WW12/adiabatic.pg

On adiabatic expansion

What is the adiabatic index, γ of an ideal monatomic gas?

Enter $\gamma =$ ___

An ideal, monatomic gas is allowed to expand quasi-statically and adiabatically from an initial volume $V_i = 2$ liters and initial temperature $T_i = 300$ K to a final volume of $V_f = 6$ liters.

What is the final temperature?

Enter $T_f =$ _______ K

Hint: $PV = Nk_BT$ for an ideal gas and PV^γ is constant for adiabatic changes. Here P is pressure, k_B is Boltzmann's constant, and N is the number of molecules.

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4. **(1 point)** setPHY141_WW12/col.pg

A collision rate and a mean free path

In space, a very small particle is moving at a velocity $v = 6$ m/s through a sea of hard spherical pebbles.

The pebbles have a radius of $r = 1$ cm and the number of them per cubic meter is $n = 12$ m^{-3}. Here n is the number density of pebbles.

The small particle can collide with and bounce off the pebbles, but it keeps moving at the same speed.

What is the rate that the small particle has collisions?

Enter $\frac{dc}{dt} =$ _______ collisions/second.

What is the small particle's mean free path, λ?

Enter $\lambda =$ _______ m.

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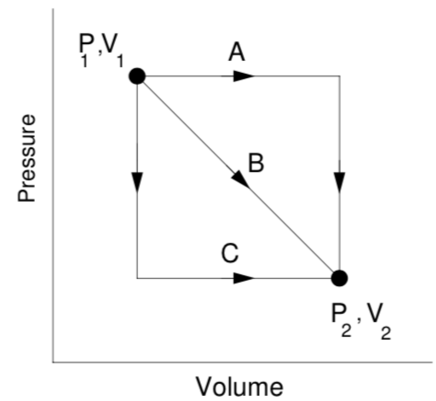
5. **(1 point)** setPHY141_WW12/PVline.pg

Work is the integral of P dV



A sample of gas expands in volume from $V_1 = 1$ m^3 to $V_2 = 2$ m^3 while its pressure decreases from $P_1 = 13$ Pa to $P_2 = 2$ Pa.

How much work is done **by the gas** if its pressure changes with volume according to path A shown in the PV diagram in the Figure?

Enter $W_A =$ _______ J.

How much work is done by the gas if its pressure changes with volume according to path B shown in the PV diagram in the Figure?

Enter $W_B =$ _______ J.

How much work is done by the gas if its pressure changes with volume according to path C shown in the PV diagram in the Figure?

Enter $W_C =$ _______ J.

Hint: work done by the gas is $W = \int_{V_1}^{V_2} P\, dV$.

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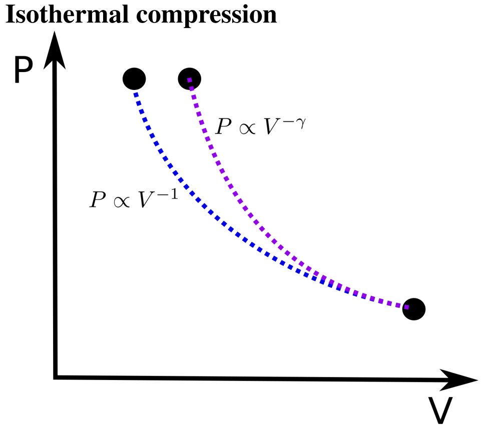
6. **(1 point)** setPHY141_WW12/isothermal_compression.pg

Isothermal compression



Isothermal variation in an ideal gas has pressure $P \propto 1/V$

whereas adiabatic variation (the steeper curve) has $P \propto V^{-\gamma}$. Here V is volume and γ is the adiabatic index. We are considering a fixed number of gas particles but the gas is in thermal contact with a thermal reservoir and can exchange volume with the reservoir. Because the total energy depends depends on $k_B T \propto PV$ (through the ideal gas law), the isothermal curves are also constant energy curves. Adiabatic curves are curves where no heat is transferred but energy is not constant. In both cases work done on the system $W = -\int P(V)dV$ and depends on the area integrated under the curve.

Consider an **isothermal** path that starts at $V_1 = 9$ m^3 with pressure $P_1 = 100000$ Pa and ends at $V_2 = 1$ m^3 and with pressure P_2.

What is the final pressure P_2?

Enter $P_2 =$ _______ Pa.

What is the work W done on the gas during compression?

Enter $W =$ _______ J.

What is the heat transfered Q to the system during compression?

Enter $Q =$ _______ J.

What is the change in total energy $\Delta U = U_2 - U_1$?

Enter $\Delta U =$ _______ J.

Numbers can be entered as 1E5 not as 1e5. Answers can be positive, negative or zero.

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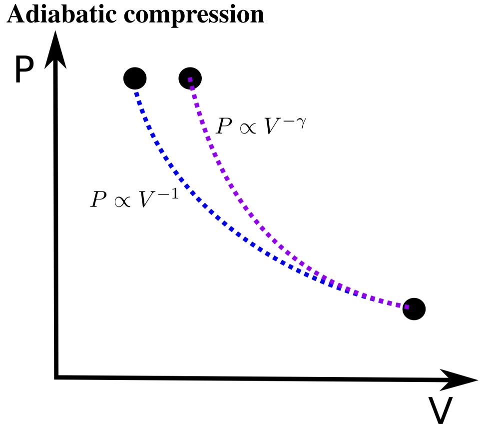
7. **(1 point)** `setPHY141_WW12/adiabatic_compression.pg`

Adiabatic compression



Isothermal variation in an ideal gas has pressure $P \propto 1/V$ whereas adiabatic variation (the steeper curve) has $P \propto V^{-\gamma}$. Here V is volume and γ is the adiabatic index. We are considering a fixed number of gas particles but the gas is in thermal contact with a thermal reservoir and can exchange volume with the reservoir. Because the total energy depends depends on $k_B T \propto PV$ (through the ideal gas law), the isothermal curves are also constant energy curves. Adiabatic curves are curves where no heat is transferred but energy is not constant. In both cases work on the system $W = -\int P(V)dV$ and depends on the area integrated under the curve.

Consider an **adiabatic** path that starts at $V_1 = 6$ m^3 with pressure $P_1 = 100000$ Pa and ends at $V_2 = 1$ m^3 and with pressure P_2. The adiabatic index $\gamma = 5/3$.

What is the final pressure P_2?

Enter $P_2 =$ _______ Pa.

What is the work W done on the system during compression?

Enter $W =$ _______ J.

What is the heat transfered Q to the system during compression?

Enter $Q =$ _______ J.

What is the change in total energy $\Delta U = U_2 - U_1$?

Enter $\Delta U =$ _______ J.

Numbers can be entered as 1E5 not as 1e5.

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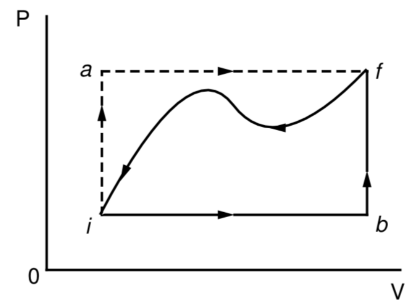
8. **(1 point)** `setPHY141_WW12/PVcurve.pg`

Energy is heat plus work



When a system is taken from state i to state f along the path iaf, as shown in the figure, the heat transferred into the system is $Q_{iaf} = 40$ cal and the work done by the system $W_{iaf} = 30$ cal.

a) If the energy $U_i = 10$ cal, what is the energy at the point f or U_f ?

Enter $U_f =$ _____ cal.

b) Along the path ibf, $Q_{ibf} = 60$ cal.

What is W_{ibf}, the work done by the system along the path ibf?

Enter $W_{ibf} =$ _____ cal.

c) If $W_{fi} = -40$ cal is the work done by the system on the curved return path fi, what is Q_{fi} ?

Enter $Q_{fi} =$ _____ cal.

d) If $U_b = 15$ cal, what is Q_{bf} ?

Enter $Q_{bf} =$ _____ cal.

e) What is Q_{ib} ?

Enter $Q_{ib} =$ _____ cal.

Hints: Internal energy is heat transferred to system + work done on system. $\Delta U = Q + W'$ where work done **on** the system $W' = -\int P\,dV$.

Here we are giving works done **by** the system (W_{iaf}, W_{ibf}, W_{fi}) and these are $W = \int PdV$. This means that we can write

$\Delta U = Q - PdV = Q - W$ with $W = \int PdV$.

Vertical paths do no work.

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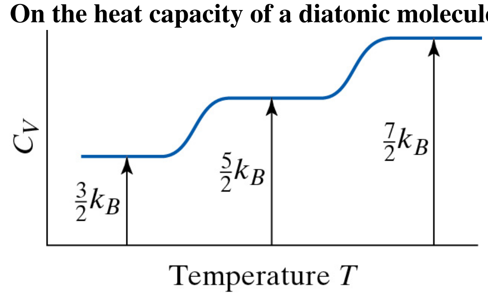
9. **(1 point)** `setPHY141_WW12/CV.pg`

On the heat capacity of a diatonic molecule



In this problem we use the results of measurements of the temperature dependence of the heat capacity of Hydrogen gas, H_2, at constant volume to determine some properties of molecular Hydrogen.

a. At temperatures below 80 K the heat capacity at constant volume is $C_V = \frac{3}{2}k_B$ per molecule, but at higher temperatures the heat capacity increases to $C_V = \frac{5}{2}k_B$ per molecule due to contributions from rotational energy states. Use these observations to estimate the distance d between the hydrogen nuclei in an H_2 molecule.

Enter $d =$ ____ m

b. At about 2000 K the heat capacity at constant volume increases to $C_V = \frac{7}{2}k_B$ per molecule due to contributions from vibrational energy states. Use these observations to estimate the stiffness k of the spring that approximately represents the interatomic force binding the molecule.

Enter $k =$ ____ N / m

Some constants:

Boltzmann constant: $k_B = 1.38065\text{E-}23$ J/K

Planck constant: $h = 6.626\text{E-}34$ J s

Proton mass: $m_p = 1.6726\text{E-}27$ kg

Speed of light: $c = 2.99792\text{E}8$ m/s

You will need to compute the reduced mass μ of a hydrogen molecule.

Vibrational energy spacings $k_BT \sim \hbar\omega$ with $\omega = \sqrt{k/\mu}$ and $\hbar = \frac{h}{2\pi}$

Here ω is the angular frequency, and k is the spring constant.

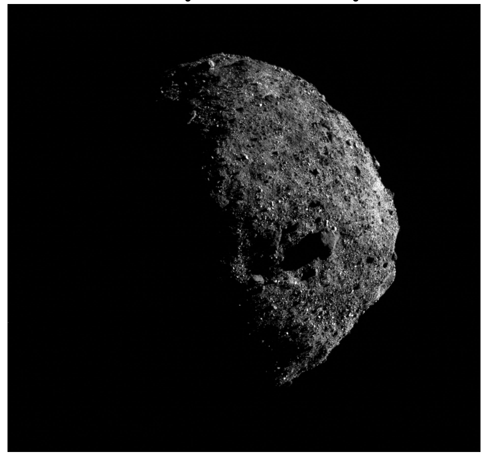
Rotational energy spacings $k_BT \sim hcB$ with rotational constant $B = \frac{h}{8\pi^2 cI}$ and moment of inertia $I = \mu d^2$.

Here d is the distance betweent the two atoms.

Numbers can be entered as 1E-10 not 1e-10.

10. (1 point) `setPHY141_WW12/heatengine.pg`

On the efficiency of a Carnot cycle heat engine in space



A Carnot cycle converts heat flow from a hot reservoir at temperature T_H and to cold reservoir at temperature T_L into work W done by the engine.

The Carnot cycle achieves the maximal possible efficiency where efficiency ε is the ratio of benefit to cost.

Here $\varepsilon = \frac{W}{Q_H}$ where Q_H is the heat absorbed from the hot reservoir in a cycle and W is the work done by the engine in a cycle.

Consider a near-Earth asteroid in outer space, such as Asteroid 101955 Bennu, with one side illuminated by the Sun and with an equilibrium temperature $T_H = 300$ K. The other side is facing away from the Sun and as radiation can escape, it is much colder at $T_L = 30$ K.

What is the maximum efficiency ε of the Carnot cycle that uses these two thermal reservoirs?

Enter $\varepsilon =$ ____