On errors and uncertainties.

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1. (1 point) `setPHY141_WW1/Problem_errprop.pg`

On propagation of errors.

Consider a measurement for a variable x giving you -3.0 ± 0.1.
Let $y = x^3$.
What is the uncertainty of y?

Enter σ_y: ________
(Answer needs to be accurate to at least 1 decimal place)

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2. (1 point) `setPHY141_WW1/sum_err.pg`

On summing errors in quadrature.

Consider a measurement for a variable a giving you -1.0 ± 0.1 and a measurement for a variable b giving you 1.0 ± 0.2. We sum the two variables $z = a + b$.
What is the uncertainty of z?

Enter σ_z : ________
(Answer needs to be accurate to at least 1 decimal place)

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3. (1 point) `setPHY141_WW1/error_prop_b.pg`

On propagation of errors.

Consider a measurement for a variable x giving you -5.00 ± 0.10.
Let $y = \frac{1}{x}$.
What is the uncertainty of y?

Enter σ_y : ________
(Answer needs to be accurate to at least 2 decimal place)

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4. (1 point) `setPHY141_WW1/sqrtN.pg`

On averaging N measurements to improve accuracy.

Suppose you run an experiment where you take N measurements and average them to make a final estimate of a physical quantity z. The standard deviation (or uncertainty) of each individual measurement is $\sigma = 0.10$ cm.

How many meaurements do you need for your final estimate of z to have a standard deviation of $\sigma_z = 0.0001$ cm?

Enter N : ________

(The number can be entered in the form 1000 or in the form 1E3 but not 1e3).

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5. (2 points) `setPHY141_WW1/spring_osc.pg`

On fractional error.

We consider a harmonic oscillator.
The period of oscillation for a mass m on a spring with spring constant k is

$$P = 2\pi\sqrt{\frac{m}{k}}$$

We assume we know the mass m very accurately.

a) With a series of data measurements, an experimenter computes an estimate for the oscillator period P_{est} with standard deviation (or uncertainty) σ_P. Here P_{est} refers to the measurement of the period P. Using the measurement of the period, the experimenter will compute the spring constant and an uncertainty in the value of the spring constant.

Using P_{est} and σ_P, the experimenter computes $k_{est} \pm \sigma_k$ where k_{est} is the estimate for the value of the spring constant and σ_k is the uncertainty or standard deviation for that estimate.
After you do this calculation, fill in this formula with a number:

$$\frac{\sigma_k}{k_{est}} = \_\_\_\_\_ \frac{\sigma_P}{P_{est}}$$

b) Suppose the fractional error in P is 10% . In other words, the uncertainty is 0.1 times the period. Equivalently, $\frac{\sigma_P}{P_{est}} = 0.10$.
What is the fractional error in the measurement of k?

Enter: $\frac{\sigma_k}{k_{est}} =$ _____ Give your answer as a natural number. Note this is a percentage!

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6. (1 point) `setPHY141_WW1/error_propc.pg`

On propagation of errors with multiple variables.

Consider a function $z = ax^3y^{-1}$ where x, y are random variables with standard deviations σ_x, σ_y. Here a is a constant.

Compute the variance σ_z^2 and fill in the boxes:

Enter $\frac{\sigma_z^2}{z^2} = \_\_\_\_ \frac{\sigma_x^2}{x^2} + \_\_\_\_ \frac{\sigma_y^2}{y^2}$

(Answers are natural numbers)

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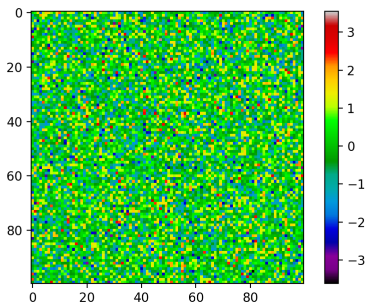
7. **(1 point)** `setPHY141_WW1/camera_pix.pg`

On 3 σ measurements and their significance.



This figure shows a 100×100 pixel image. Each pixel has value chosen from a normal distribution, in other words, a Gaussian probability distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$.

A 1000 by 1000 pixel CCD camera on a telescope has 1 million pixels. An image is taken of blank sky. Each pixel has noise with a standard deviation of $\sigma = 5$ counts. We assume that the value of each pixel is described with a Gaussian distribution.

In the image, about how many pixels are expected to be 3 σ away from the mean value?

Enter a number of pixels: ______

Hint: You want to compute the probability that a Gaussian measurement is above 3 σ away from the mean.
This is given by 1 - erf($3/\sqrt{2}$) = 0.002700.
The point of this problem is that if you have a million measurements then you will get some 3 σ outliers. Some of these might look bright in a CCD image but would not be real astronomical objects.