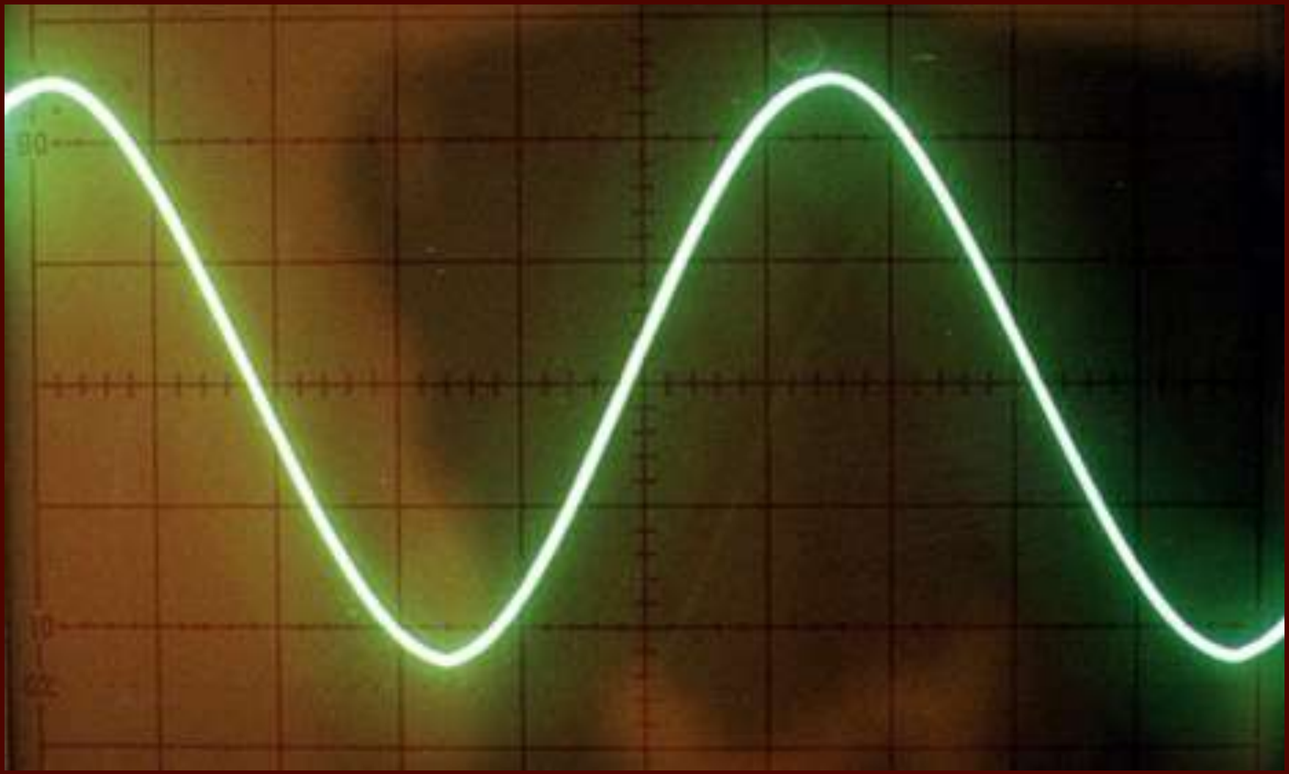
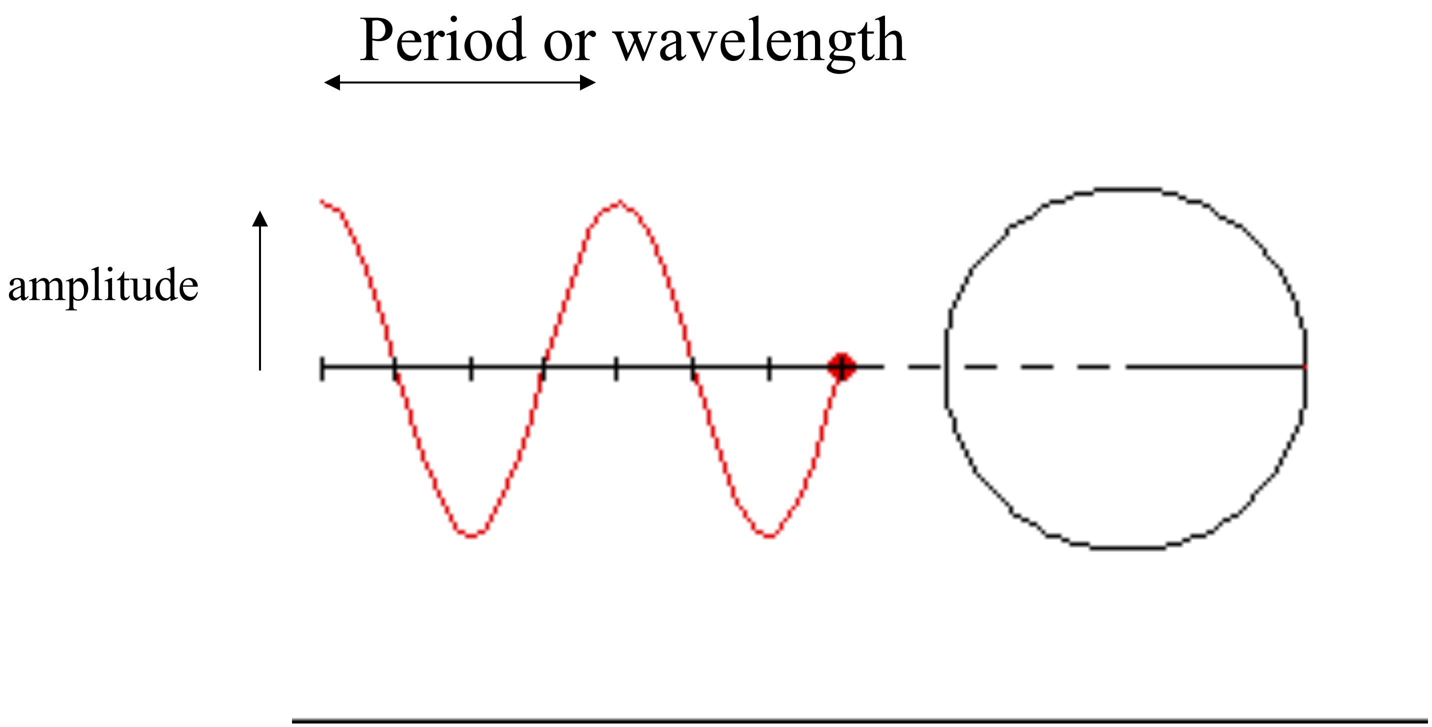
Pure Tones and the Sine Wave



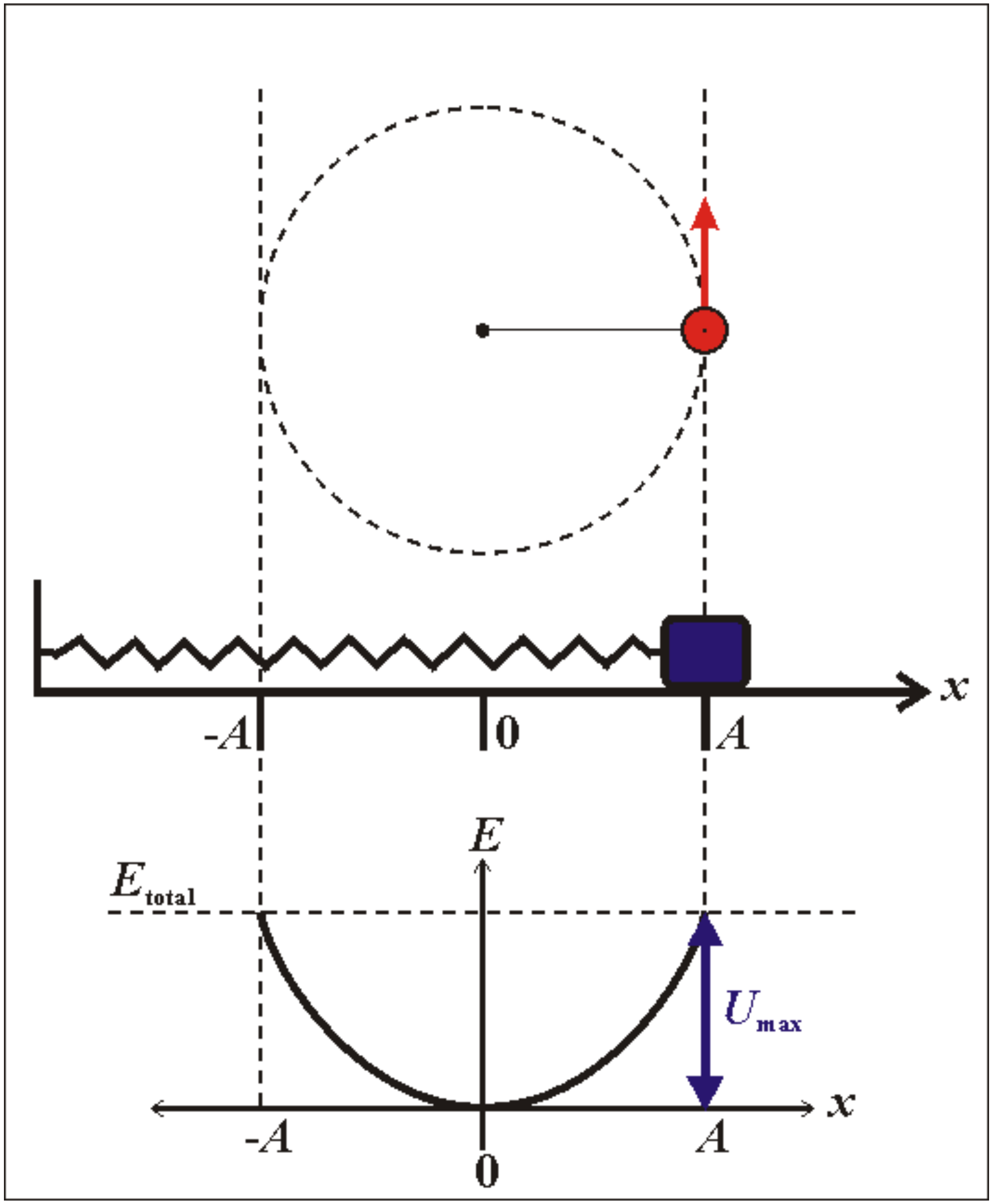
Physics of Music
PHY103 Lecture 2

Image from
www.math.ucdavis.edu/~angela/mathC.html

Trig definition of a sine wave



From math learning service U. Adelaide

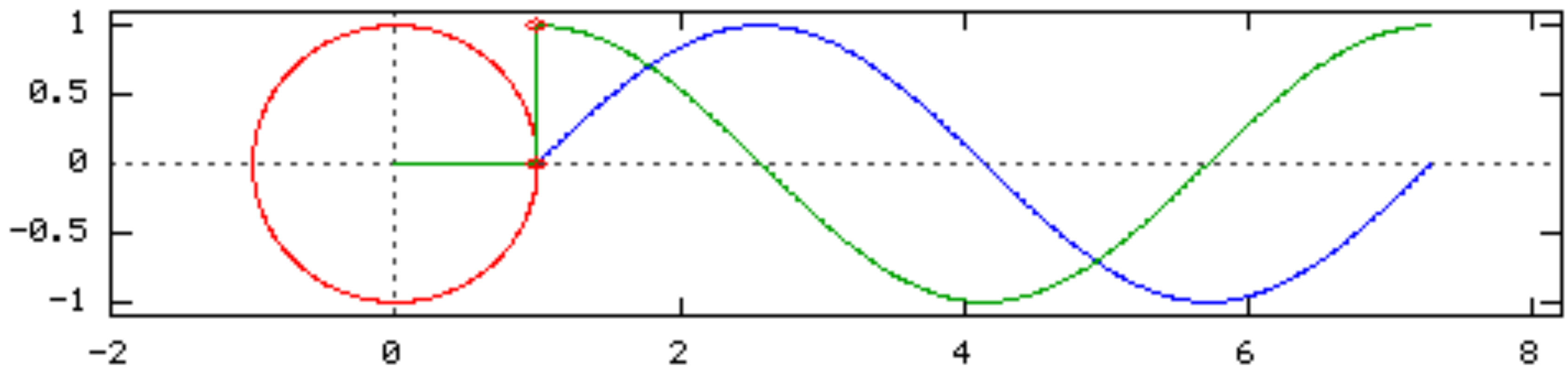


Harmonic motion

From
ecommons.uwinnipeg.ca/ archive/ 00000030/

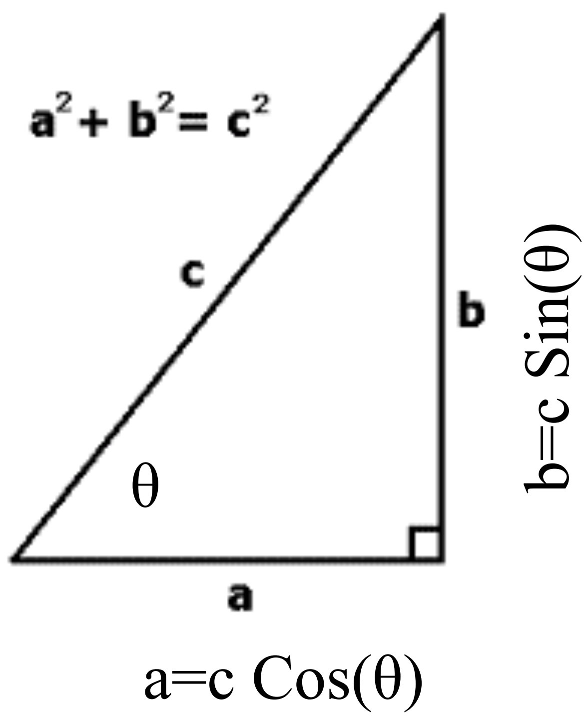
Velocity and Position
Sine and Cosine

Sine and Cosine



From www2.scc-fl.com/lvosbury/AnimationsForTrigono...

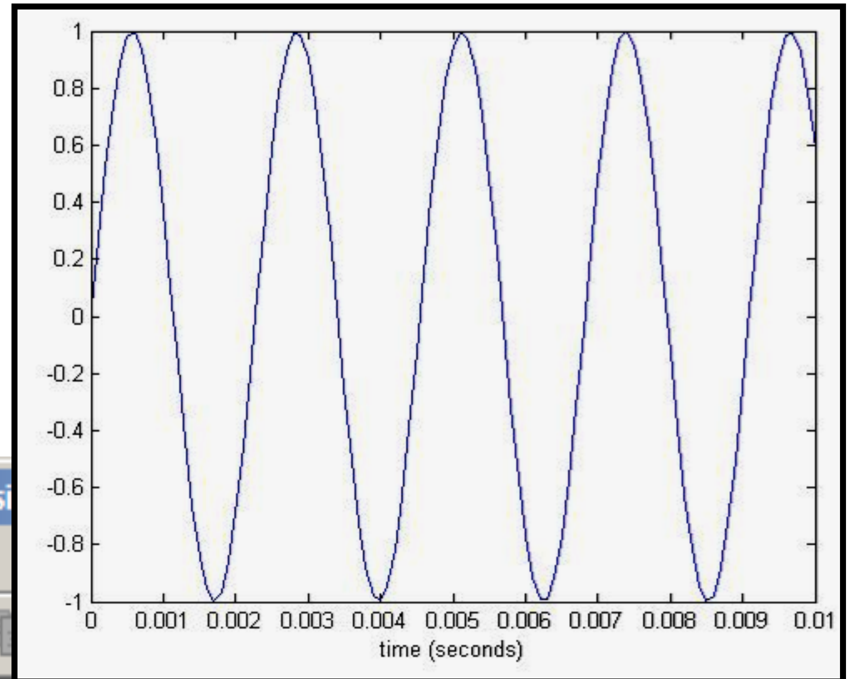
Pythagorean theorem
Conservation of Energy



$$\sin^2\theta + \cos^2\theta = 1$$

When the spring is extended, the velocity is zero. When the spring is in the middle, the velocity is maximum. The position is the sine wave, the velocity is the cosine wave. Kinetic energy (square of velocity) + Potential energy (square of position) is total energy is conserved.

Making a pure tone with Matlab



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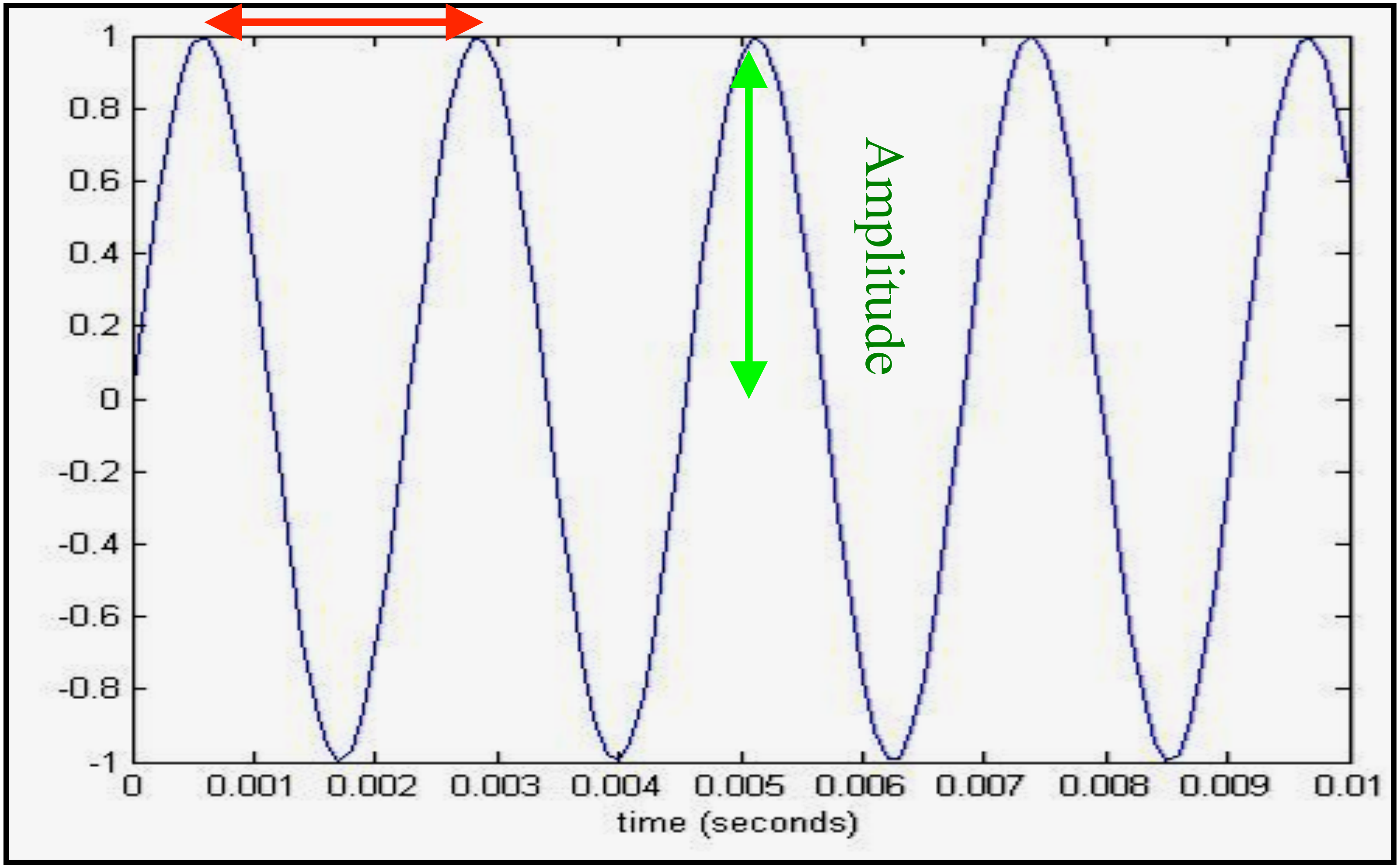
File Edit Text Cell Tools Debug Desktop Window Help

```
% This routine plots a sine wave and also makes its sound
dt = 0.0001;     % sampling
time = 0:dt:2;  % from 0 to 2 seconds total  with sampling interval dt
% Here my sample interval is 0.0001sec or a frequency of 10^4Hz
frequency = 440.0;  % This should be an A note
% note frequency is in units of Hz or cycles/seconds
nu = 2.0*pi*frequency;   % nu is known as the angular frequency
y = sin(nu*time);     % y is the sin function with amplitude 1
period = 1.0/frequency   % period is 1/frequency and is in seconds
plot(time, y)    %plot it out!
axis([0 0.01 -1 1]);   % set the x-axis range so only a small piece shown
xlabel('time (seconds)');
sound(y, 1/dt)  % the second number here is the sampling frequency
```

Sine Wave

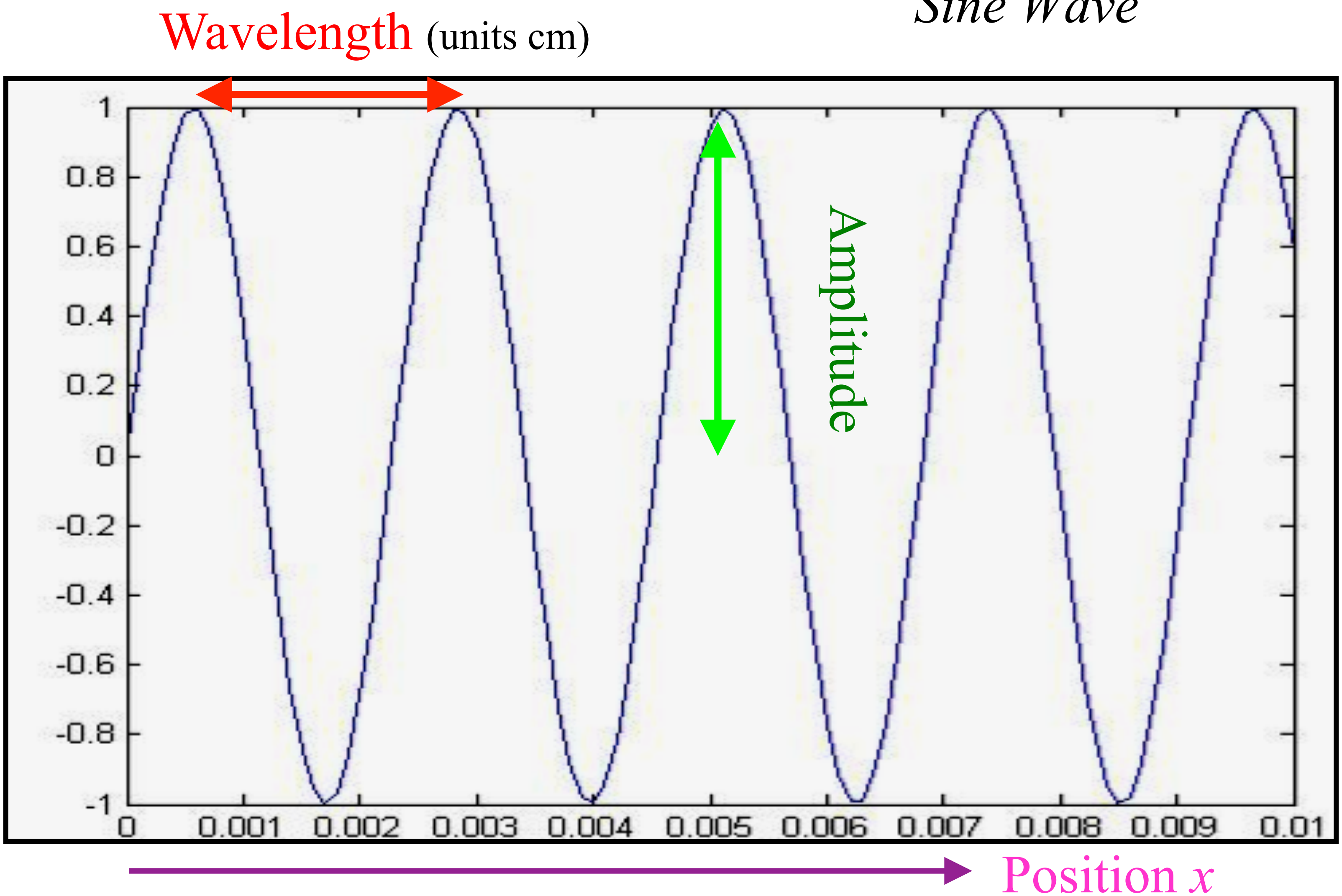
Period (units time or seconds)



Frequency units (1/time) or Hz $f = 1/P$

cycles per second

Sine Wave



For a wave on water or on a string - spatial variation instead of temporal variation

Amplitude

- Units depend on what is measured
- Velocity, pressure, voltage?

Angular frequency

$\sin(\omega t)$ ω angular frequency

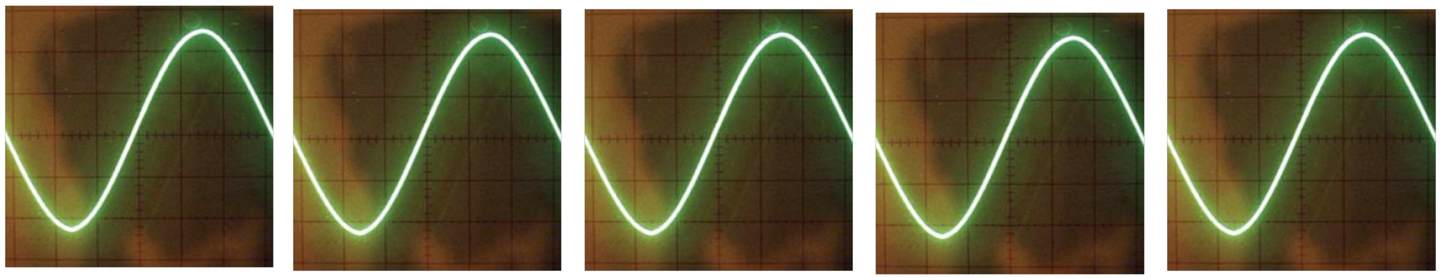
radians per second

$\sin(2\pi f t)$ f frequency in Hz

cycles per second

$$2\pi f = \omega$$

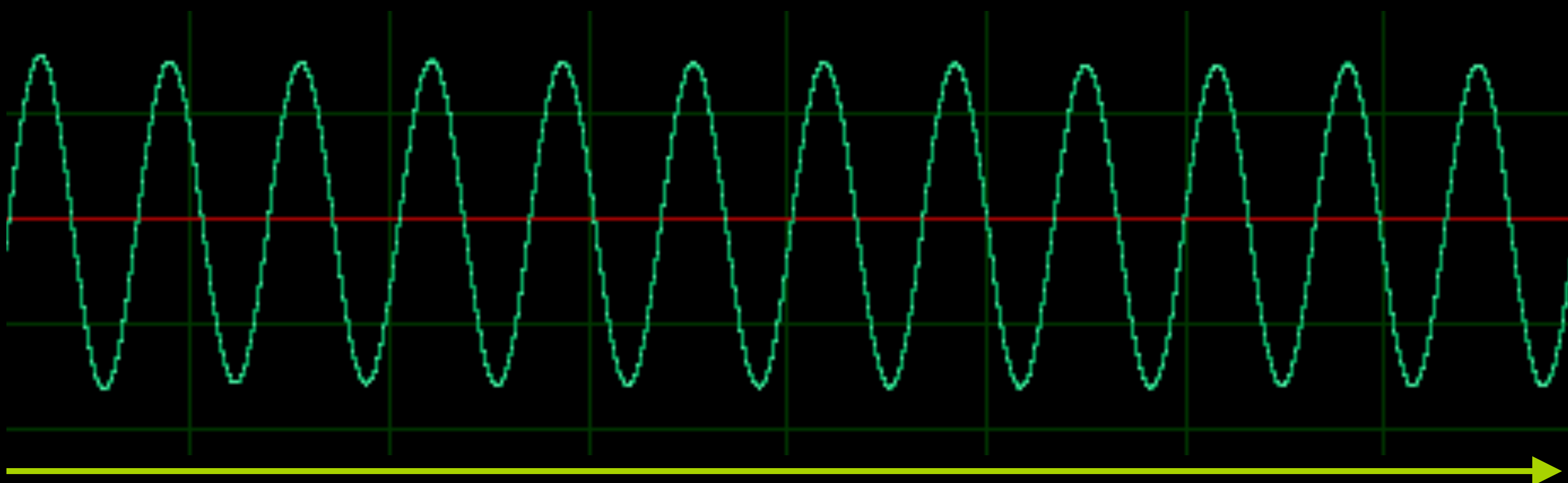
Relation between frequency and period



- Suppose the period is P=0.2s
- I need 5 cycles to add up to 1s
- So the frequency is f=5Hz.
- The number of periods/cycles that add up to 1 second is the frequency

$fP=1$ $f=1/P$

Relation between frequency and period $P=1/f$



12 cycles in 1 second
The frequency is 12 Hz
The period (time between each peak
is 1/12 seconds or 0.083seconds

How does energy/power depend on the amplitude?

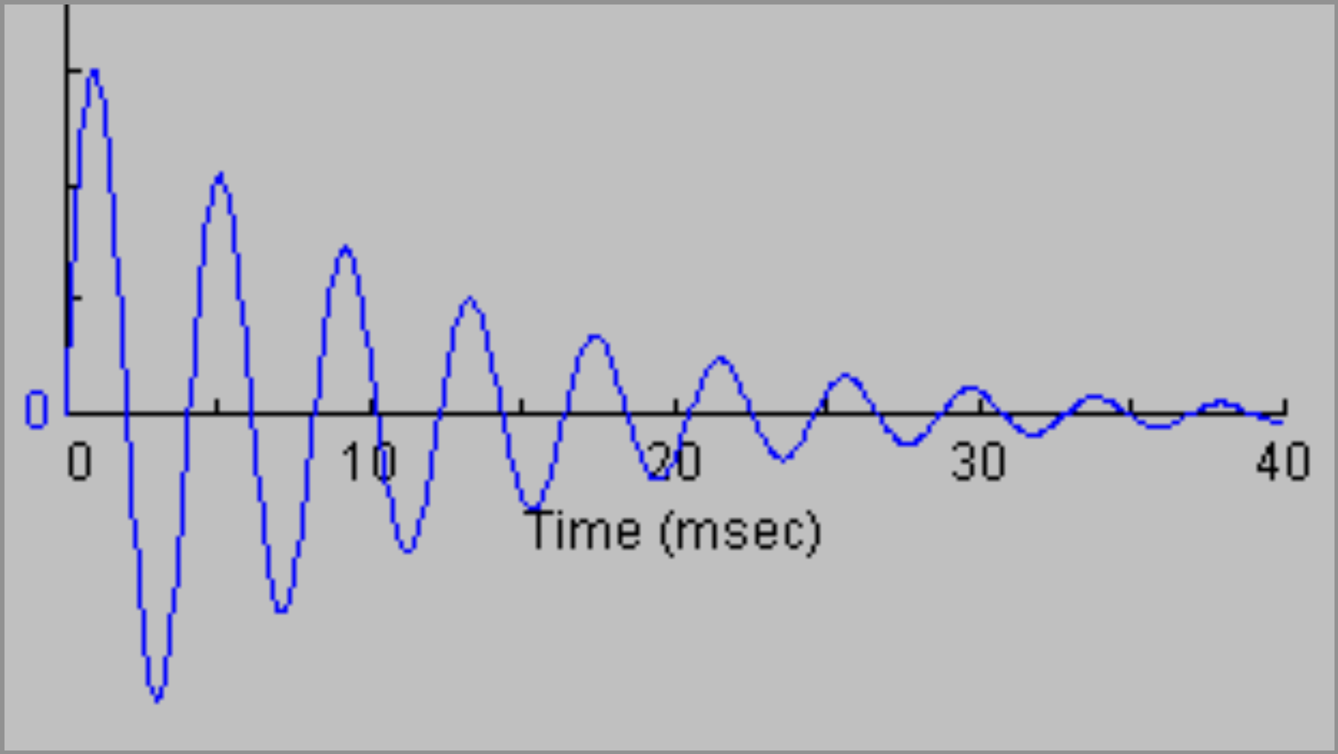
How does energy/power depend on the amplitude?

- Energy depends on the sum of the square of velocity and square of position (from equilibrium)
- We expect that the energy or power (energy per second for a traveling wave) depends on the square of the amplitude.

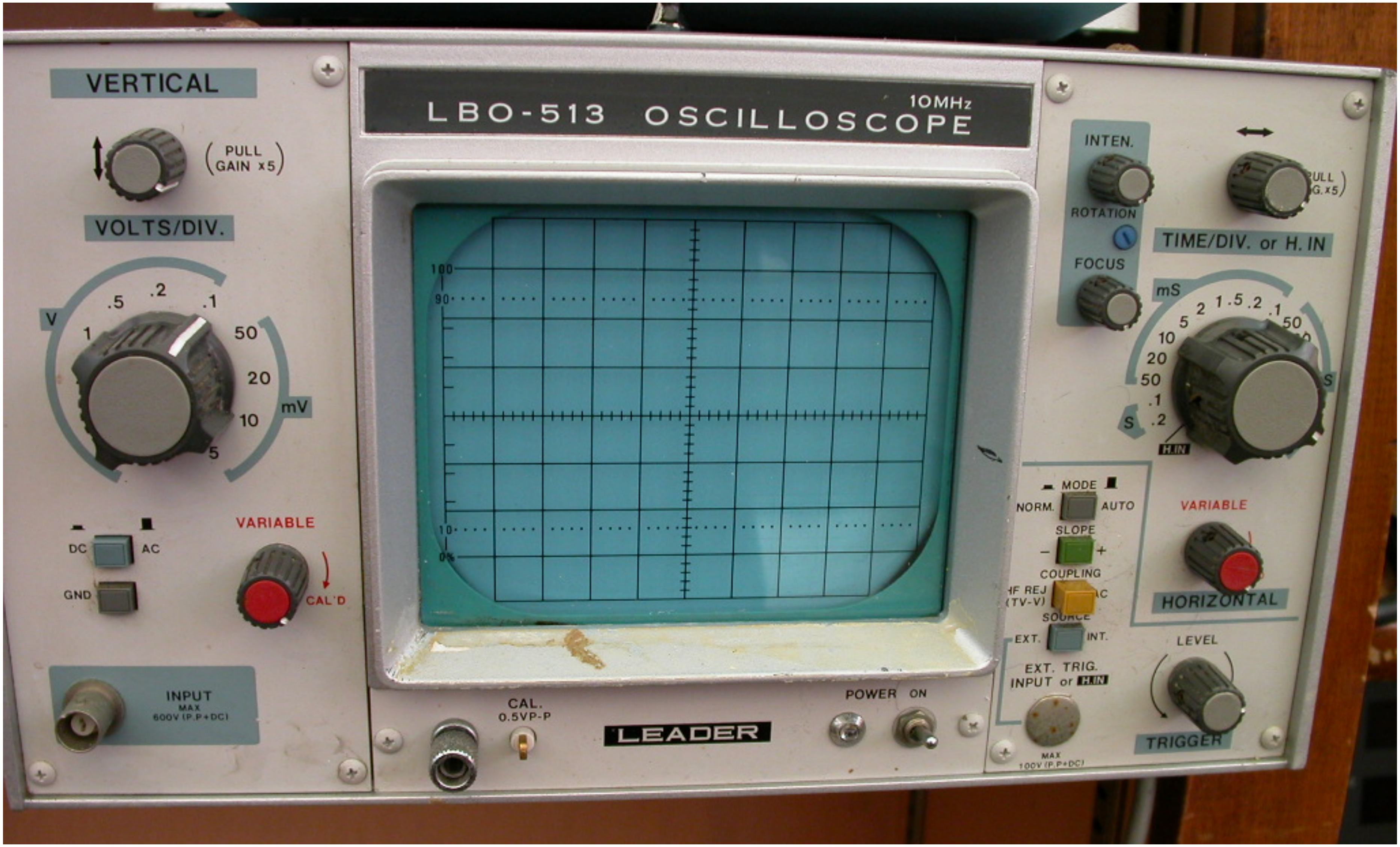
Power proportional to square of Amplitude

$$P \propto A^2$$

Decay – loss of energy



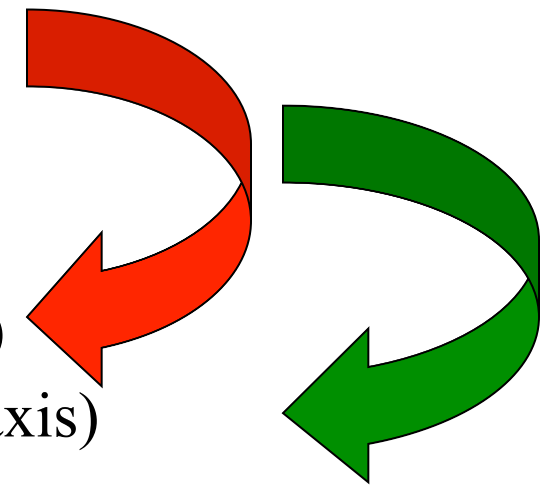
Showing a sine wave on the oscilloscope



Signal or waveform generator

Can adjust

- Shape of wave (sine, triangle, square wave)
- Voltage (amplitude of wave)
- Frequency of wave



Oscilloscope

Adjust voltage of display (y–axis)

Adjust time shown in display (x-axis)

Adjust trigger

Can also place display in x-y mode so can generator Lissajous figures

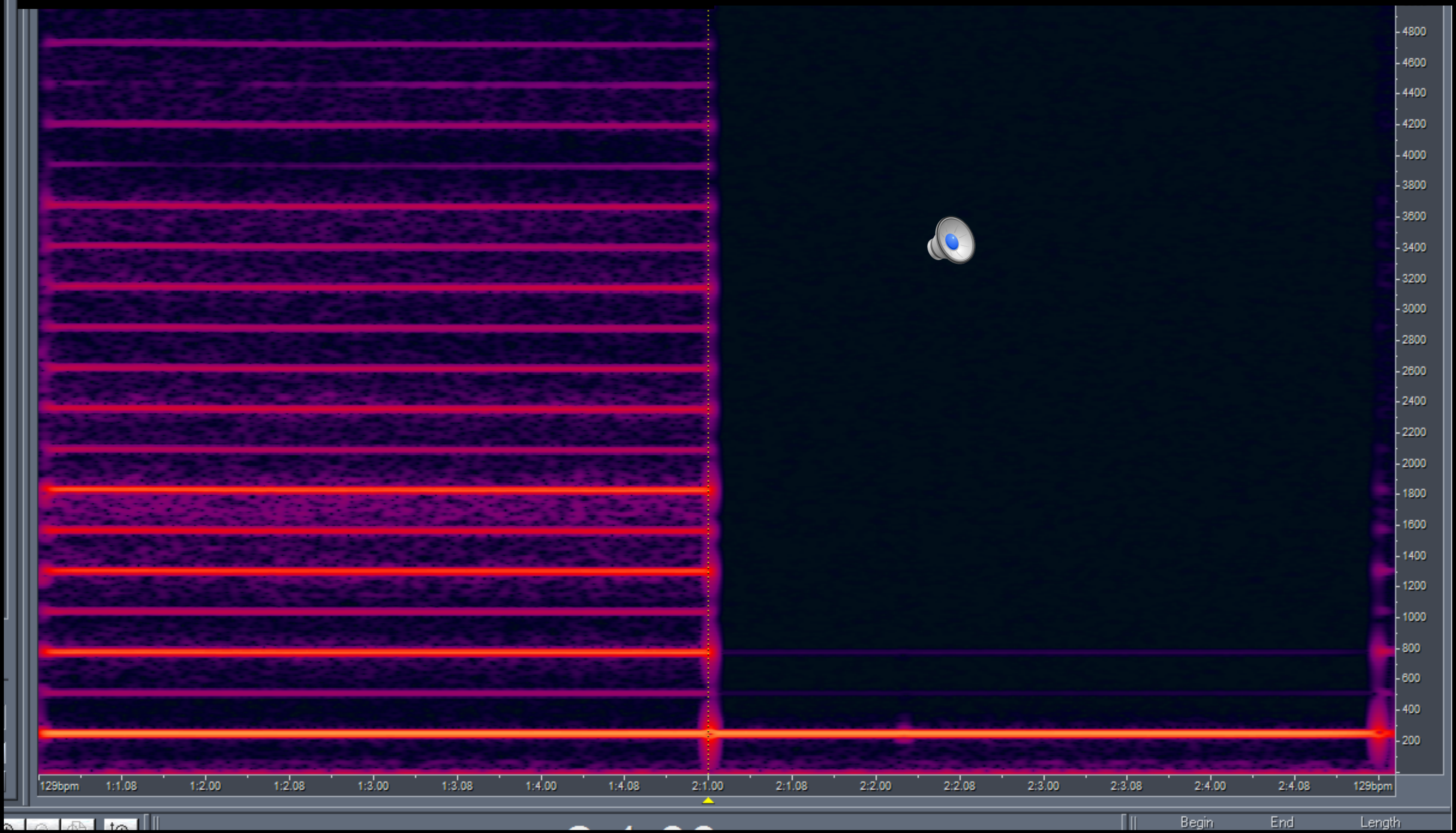
Sine waves – one amplitude/ one frequency

Sounds as a series of pressure or motion variations in air.

Sounds as a sum of different signals each with a different frequency.

Clarinet spectrum

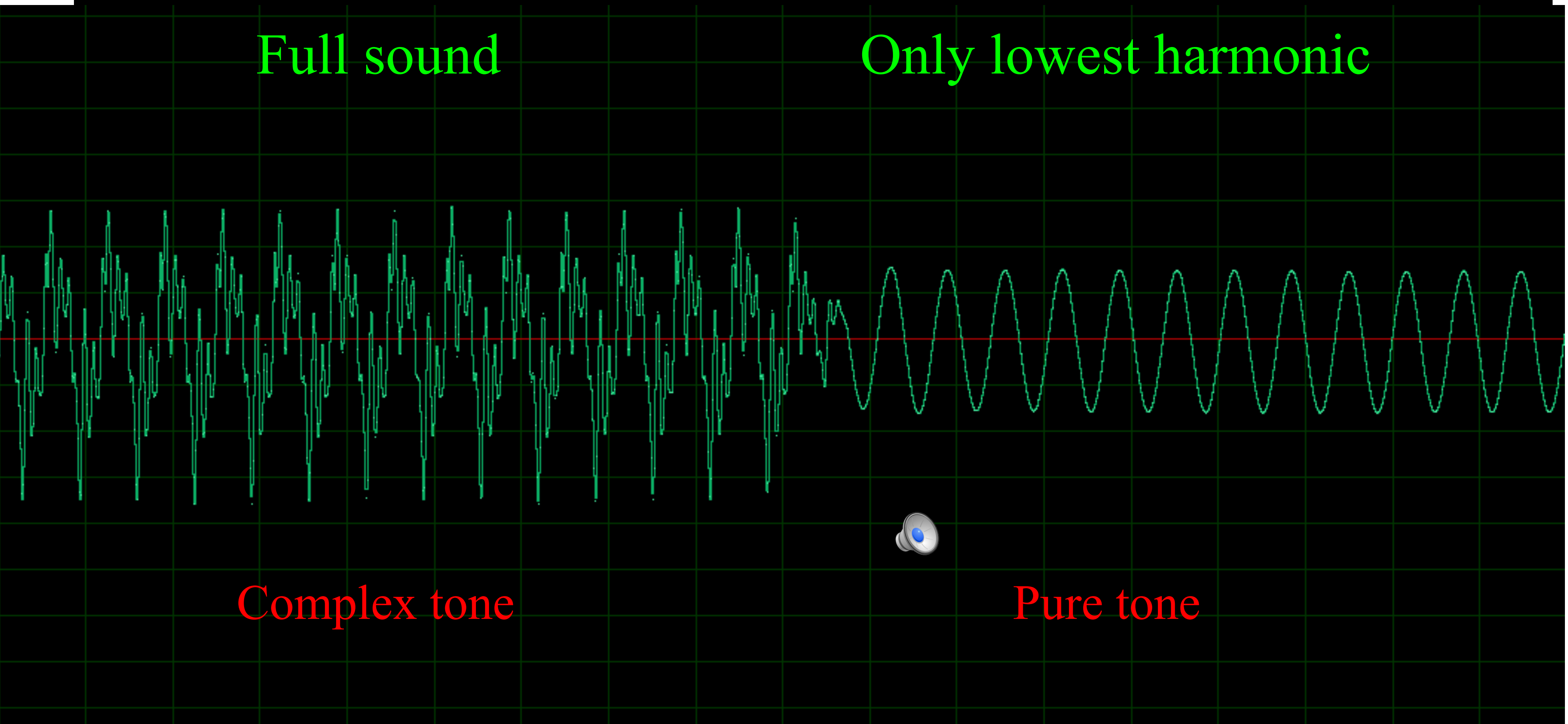
Clarinet spectrum with only the lowest harmonic remaining

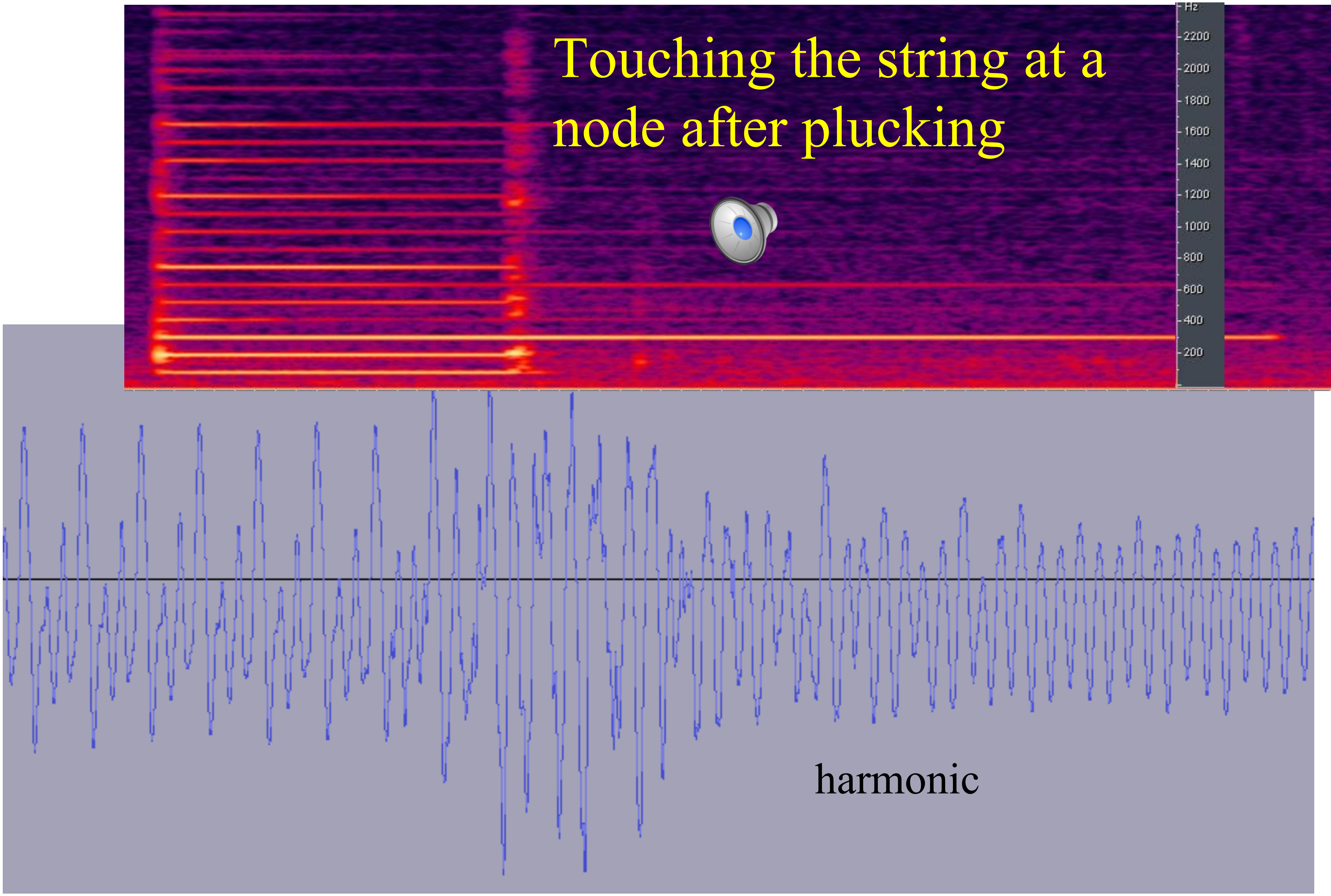


Frequency →

Time →

Waveform view





Touching the string at a node after plucking
Hz
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
harmonic

Decomposition into sine waves

- We can look at a sound in terms of its pressure variations as a function of time

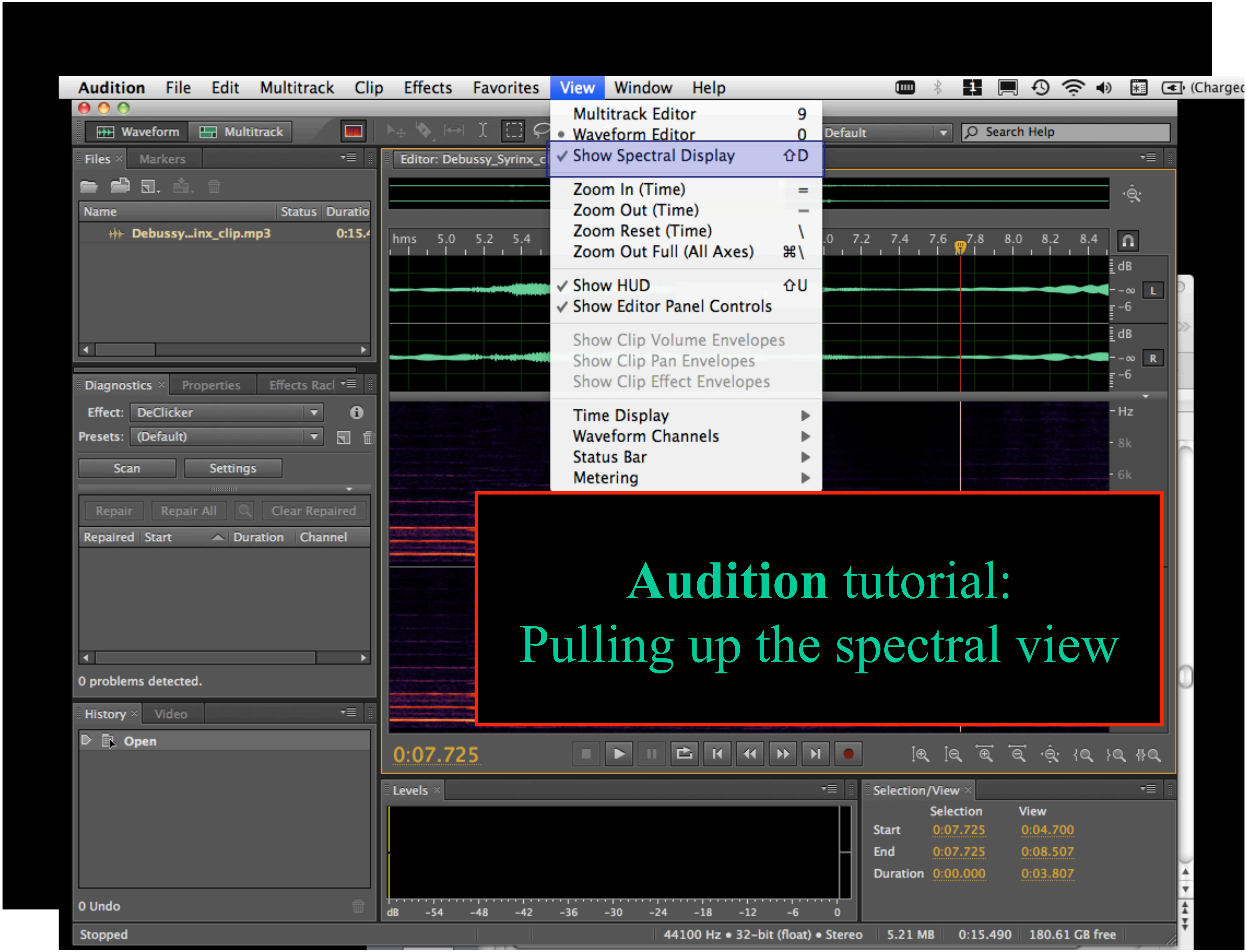
OR

- We can look at a sound in terms of its frequency spectrum

This is equivalent to saying each segment is equivalent to a sum of sine waves.

"Fourier decomposition"

Some of the character or "***timbre***" of different sounds comes from its spectrum: which harmonics are present, how strong they are, and where they are exactly (they can be shifted from integer ratios)


Audition tutorial:
Pulling up the spectral view
Audition File Edit Multitrack Clip Effects Favorites View Window Help
Multitrack Editor 9
Waveform Editor 0
Show Spectral Display ⇧D
Zoom In (Time) =
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Show Clip Pan Envelopes
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Time Display
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Properties
Effects Rack
Effect: DeClicker
Presets: (Default)
Scan
Settings
Repair
Repair All
Clear Repaired
Repaired
Start
Duration
Channel
0 problems detected.
History
Video
Open
0 Undo
0:07.725
Levels
Selection/View
Selection
View
Start 0:07.725 0:04.700
End 0:07.725 0:08.507
Duration 0:00.000 0:03.807
Stopped
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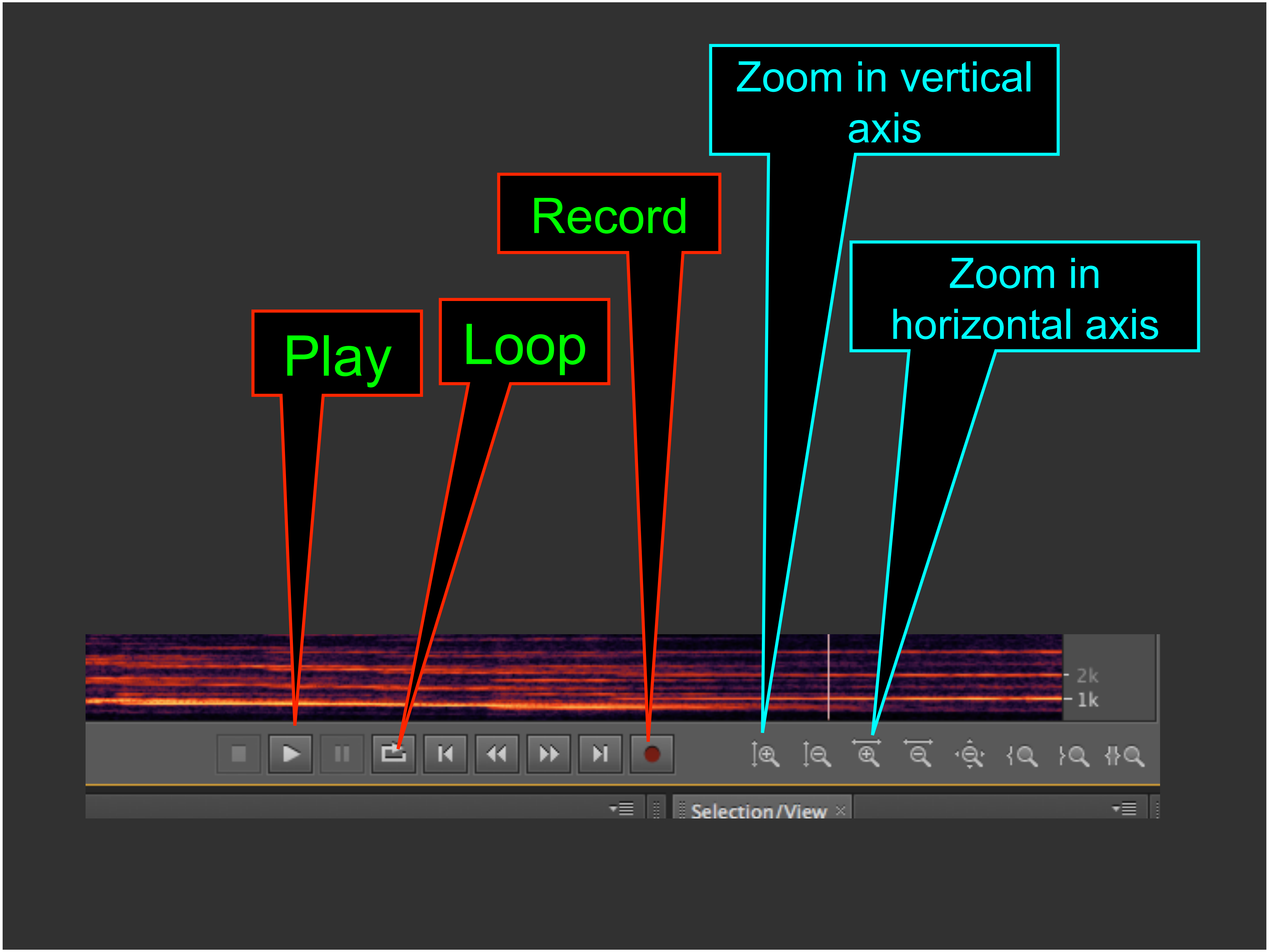
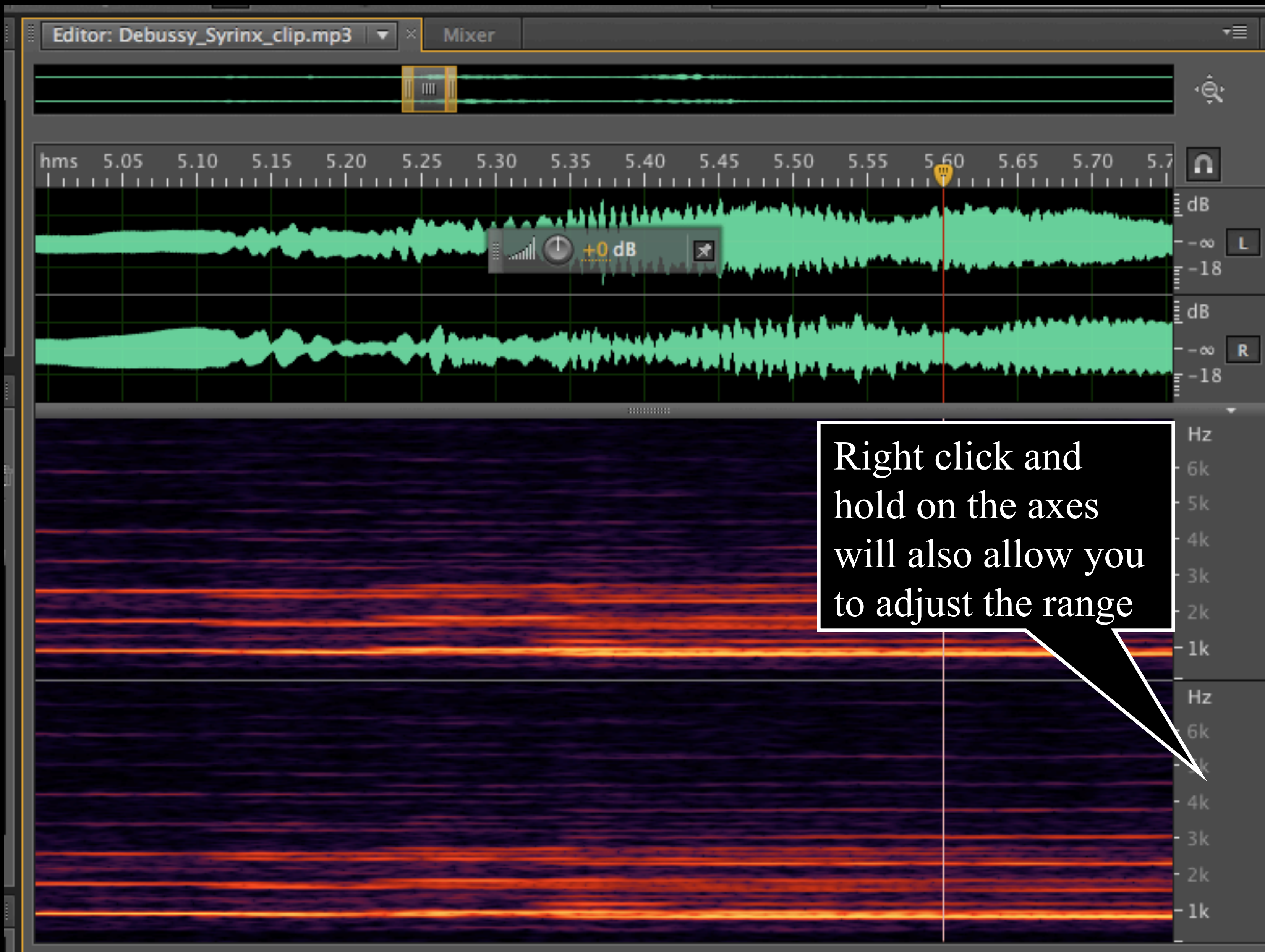
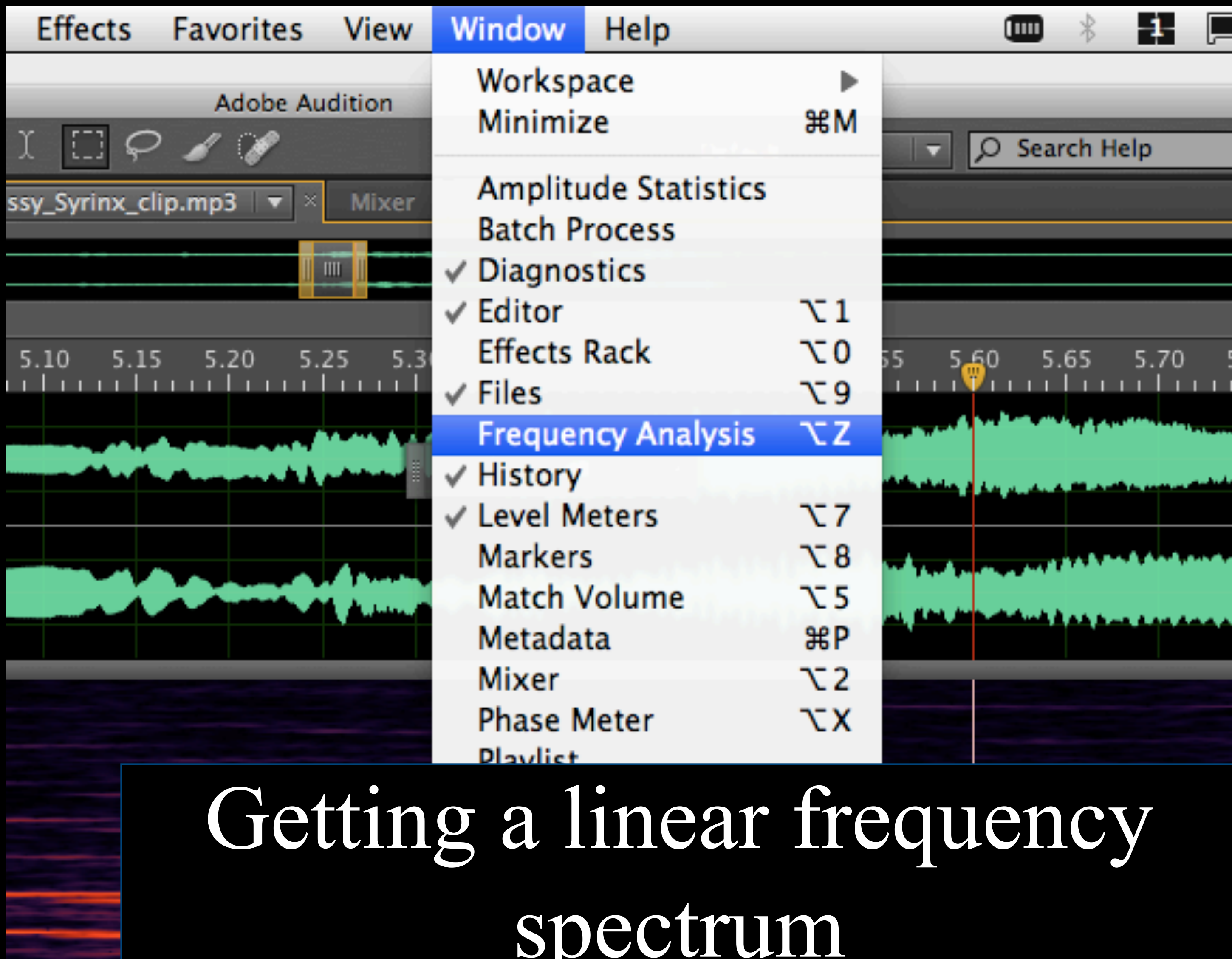
5.21 MB
0:15.490
180.61 GB free


Zoom in vertical axis
Record
Zoom in horizontal axis
Play
Loop
2k
1k
Selection/View


Editor: Debussy_Syrinx_clip.mp3
Mixer
hms 5.05 5.10 5.15 5.20 5.25 5.30 5.35 5.40 5.45 5.50 5.55 5.60 5.65 5.70
+0 dB
dB
-∞
-18
L
R
Hz
6k
5k
4k
3k
2k
1k
Right click and hold on the axes will also allow you to adjust the range



Getting a linear frequency spectrum

Harmonics or Overtones

Wavelengths

$$\lambda_1 = 2L$$

$$\lambda_2 = L = \lambda_1 / 2$$

$$\lambda_3 = 2L/3 = \lambda_1 / 3$$

$$\lambda_4 = 2L/4 = \lambda_1 / 4$$

$$\lambda_5 = 2L/5 = \lambda_1 / 5$$

$$\lambda_6 = 2L/6 = \lambda_1 / 6$$

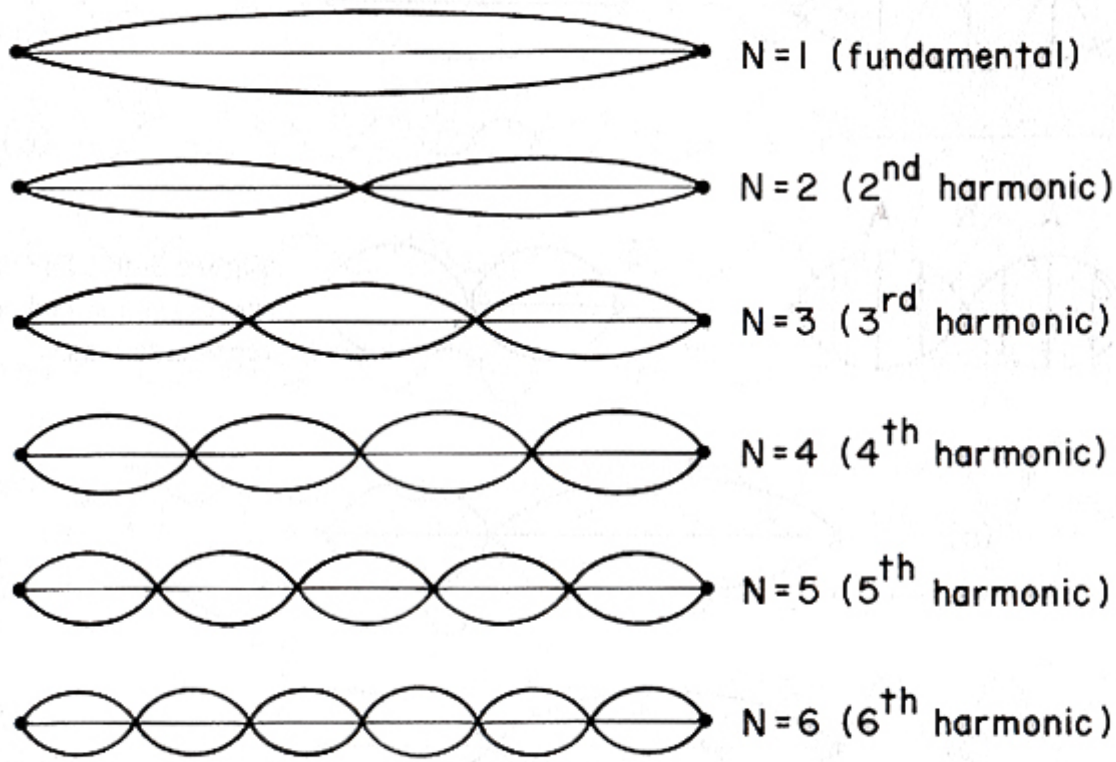


Figure 3–11 The representation of the first six possible standing waves in a stretched wire or rope.

Wavelengths and frequencies of Harmonics

TABLE 3–2 WAVELENGTHS AND FREQUENCIES OF THE FIRST SIX POSSIBLE STANDING WAVES ON A STRING OF LENGTH L

Harmonic number N	Wavelength	Frequency $\left(f = \frac{v}{\lambda}\right)$
1	$\lambda_1 = 2L$	$f_1 = \frac{v}{2L}$
2	$\lambda_2 = L = \frac{1}{2}\lambda_1$	$f_2 = \frac{v}{L} = 2f_1$
3	$\lambda_3 = \frac{2}{3}L = \frac{1}{3}\lambda_1$	$f_3 = \frac{v}{\frac{2}{3}L} = 3f_1$
4	$\lambda_4 = \frac{L}{2} = \frac{1}{4}\lambda_1$	$f_4 = \frac{v}{\frac{1}{2}L} = 4f_1$
5	$\lambda_5 = \frac{2}{5}L = \frac{1}{5}\lambda_1$	$f_5 = \frac{v}{\frac{2}{5}L} = 5f_1$
6	$\lambda_6 = \frac{L}{3} = \frac{1}{6}\lambda_1$	$f_6 = \frac{v}{\frac{1}{3}L} = 6f_1$

Relation between frequency and wavelength

quantity	wavelength	frequency
meaning	distance	cycles per second
units	cm	Hz
symbol	λ	f

longer wavelengths → slower motion

$$\lambda \propto 1/f$$

wavelength of fundamental mode is inversely proportional to frequency

$$\lambda f \sim \text{constant}$$

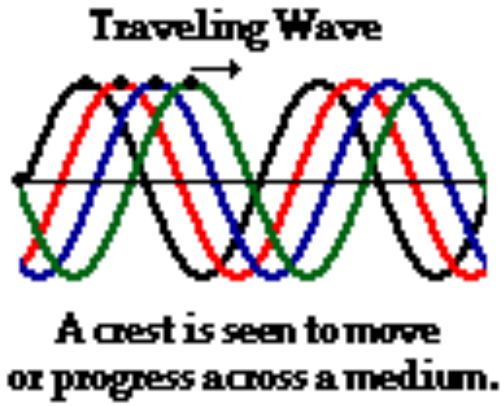
Wavelength/Frequency

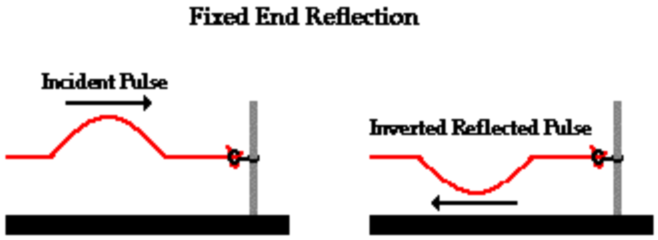
$$\lambda f \sim \text{constant}$$

cm x 1/s = cm/s

frequency is related to wavelength by a speed -- The speed that disturbances travel down a string

Traveling waves





Traveling waves

- Right traveling
- Left traveling $\cos(kx + \omega t)$

Law of cosines

$$\cos(kx + \omega t) = \cos(kx)\cos(\omega t) - \sin(kx)\sin(\omega t)$$

Sums of same amplitude traveling waves gives you standing waves

BOX 10.1 STANDING AND TRAVELING WAVES

Donald E. Hall

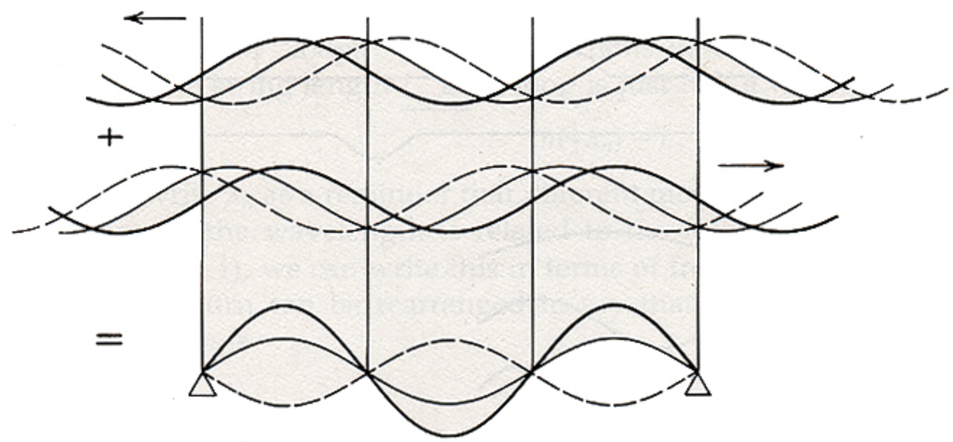
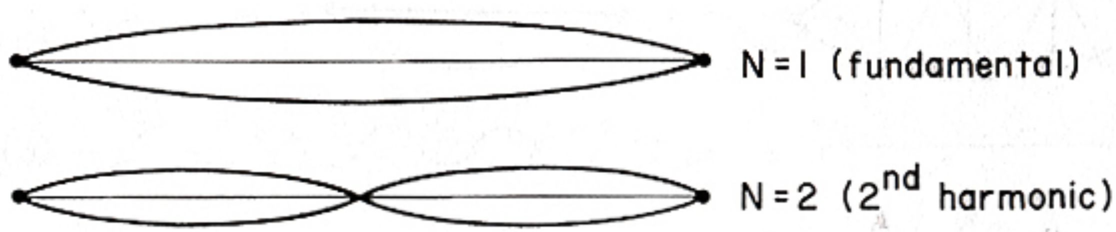


FIGURE 10.2 Multiple-exposure snapshots of string shape for two sinusoidal traveling waves and the standing wave that results when both are present together. Dashed, light solid, and dark solid lines indicate successively later times. The graph for the standing wave at each instant is the sum of the two corresponding graphs for the traveling waves, with the addition being performed the same way as in Figure 9.6 (page 154).

Why the second mode has twice the frequency of the fundamental

- Exciting the fundamental. Excite a pulse and then wait until it goes down the string and comes all the way back.
- Exciting the second harmonic. When the first pulse gets to the end the string, you excite the next pulse. This means you excite pulses twice as often. You must drive at twice the frequency to excite the second mode



Adding two traveling waves one moving left one moving right

$$\cos(kx - \omega t) = \cos(kx)\cos(\omega t) + \sin(kx)\sin(\omega t)$$
$$\cos(kx + \omega t) = \cos(kx)\cos(\omega t) - \sin(kx)\sin(\omega t)$$

$$\cos(kx + \omega t) + \cos(kx - \omega t)$$

$$= 2\cos(kx)\cos(\omega t)$$

Standing wave!

Traveling waves vs standing waves

- Can think of standing waves as sums of left traveling and right traveling waves
- The time to go from zero to max depends on the time for the wave to travel a distance of 1 wavelength→ smaller wavelengths have faster oscillation periods (frequencies)

$$\cos(kx + \omega t) = \cos(2\pi(\frac{x}{\lambda} - ft)) \qquad v = \frac{\lambda}{f}$$

- If the waves move faster on the string then the modes of oscillation (the standing waves) will be higher frequency

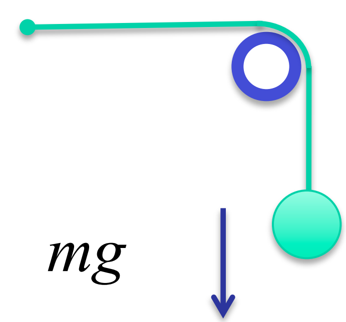
Wave speed dimensional analysis

- Only quantities we have available:
  - String density (mass per unit length) ρ
  - String length L
  - Tension on string T

- Force = mass times acceleration

  $F=ma$ (units g cm/s^2)
- Tension on a string is set by the force pulling on the string

  So T is units of g cm/s^2



Wave speed dimensional analysis continued

- We want a velocity (cm/s). How do we combine the 3 physical quantities to get a velocity?
  - String density ρ (g/cm)
  - String length L (cm)
  - Tension T (g cm/s^2)
- T/ ρ has units cm^2/s^2 so a velocity is given by $\sqrt{T/\rho}$
- To get a quantity in units of frequency we divide a velocity by a length
- When we think about oscillating solids (copper pipes for example) the thickness is also important.

Spring/String

Spring		**String**	
Heavier weight	Slower frequency	Heavier mass string	slower fundamental mode frequency
Stronger spring	Higher frequency	Tenser string	Higher fundamental frequency