AST233 – Order of Magnitude Dynamics

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1 Order of Magnitude Dynamics

G is the gravitational constant.

Force between masses m_1, m_2 at coordinate positions $\mathbf{r}_1, \mathbf{r}_2$ has size

$$F = \frac{G m_1 m_2}{|\mathbf{r}_1 - \mathbf{r}_2|^2} \tag{1}$$

This equation is correct to order of magnitude. Neglecting sign. Neglecting direction. Neglecting factors of a few.

Force is mass times acceleration so the units of force:

$$F = \frac{MD}{T^2}$$

where M is a mass-scale, D is a distance scale and T is a timescale. Rewriting equation 1 with units

$$\frac{MD}{T^2} = G\frac{M^2}{D^2} \tag{2}$$

We solve for units of G

$$G = M^{-1} D^3 T^{-2}$$

$$\begin{aligned} \mathbf{cgs}: \quad & G = 6.7 \times 10^{-8} \mathrm{g}^{-1} \mathrm{cm}^3 \mathrm{s}^{-2} \\ \mathbf{mks}: \quad & G = 6.7 \times 10^{-11} \mathrm{kg}^{-1} \mathrm{m}^3 \mathrm{s}^{-2} \end{aligned}$$

While **cgs** (centimeters, grams, seconds) is used by many astrophysicists, in planetary science the convention is **mks** (meters, kilograms, seconds).

2 A Point Mass

Three things

1. a mass M.
2. a distance D.
3. the gravitational constant G.

$$GM = \frac{\text{cm}^3}{\text{s}^2}$$

$$\frac{GM}{D} = \frac{\text{cm}^2}{\text{s}^2} = V^2$$

We have a characteristic velocity

$$V = \sqrt{\frac{GM}{D}}$$

This velocity is the sizescale of a bunch of different things

1. **Orbital velocity** of a small particle in a bound orbit around a mass M with average radius of orbit D.
2. **Escape velocity** from a planet of mass M and radius D.
3.
4. The velocity of a **wind or jet** driven from a radius D in an accretion disk around a mass M.
5. **Pericenter velocity** of a highly eccentric comet at pericenter distance D.

2.1 Timescale and frequency

If we take a sizescale and divide by a velocity we get a timescale.

$$t_{grav} = \frac{D}{V} = \sqrt{\frac{D^3}{GM}}$$

This timescale is the sizescale for a bunch of different things.

1. **Orbital period** for a small particle in orbit with radius D about M.
2. **Free-fall time**. The time it takes a small particle to fall into mass M starting at distance D from it.
3. An important *timescale for a fast gravitational encounter*. Describing when the force from gravity toward M is important.

The inverse of timescale is a frequency.

$$\Omega_{grav} = \sqrt{\frac{GM}{R^3}}$$

This frequency is the sizescale for a bunch of different things.

1. **Angular rotation rate** of a particle in orbit with radius D about M. A star in a galaxy. A planet around a star. This is also called the **mean motion**, n, in the celestial mechanics setting. For a circular orbit the mean motion is the same as the angular rotation rate.

2. Angular rotation rate of a particle grazing the surface of a sphere with mass M and radius R.

3. **Maximum spin rate** of a gravitationally bound strengthless body.

4. Angular rotation rate of a star in a galaxy at radius R and with galactic mass inside R that is $M(R)$.

5. Pattern speeds of spiral and bar patterns in galaxies. Again M is enclosed galactic mass within radius R.

Note that the timescale is like M/R^3, so it is a density.

$$t_{grav} = (G\rho)^{-\frac{1}{2}}$$

How much does the density vary for astronomical objects? For asteroids and planets all densities are of order 1 g/cc so these gravitational timescales are all the same. The density of a galaxy is quite different (lower) and that of a neutron star is higher.

Let us put in some numbers

$$\frac{1}{\sqrt{G\rho}} = \frac{1}{\sqrt{7 \times 10^{-8}\mathrm{g}^{-1}\mathrm{cm}^3\mathrm{s}^{-2} \times 1\mathrm{g\ cm}^{-3}}} \sim \frac{1}{3 \times 10^{-4}\mathrm{s}^{-1}} \sim 3000\mathrm{s}$$

$$3000\mathrm{s} \times \frac{1\ \text{minute}}{60\ \text{seconds}} \sim 50\ \text{minutes} \sim 1\ \text{hour}$$

For objects in the solar system using 1g/cc the timescale is about 4000 seconds which is about 60 minutes or an hour. Spin periods of solid bodies in the solar system (and planets) cannot easily be faster than this (except if they are solid rocks or monoliths which hold themselves together via material strength).

2.2 Acceleration

The force per unit mass is an acceleration

$$a_{\rm grav} = \frac{GM}{D^2} \tag{3}$$

On the surface of the Earth and using $D = R_\oplus$ this is the gravitational acceleration g. It may be useful to remember that $a = \frac{\Delta V}{\Delta t}$ which is the velocity change ΔV in a timescale Δt.

While there is a range of masses and radii for planetary bodies, there is not much range in density. Density $\rho \sim M/R^3$ or $R \sim (M/\rho)^{\frac{1}{3}}$ tends to be of order 1g/cc.

$$a = GM^{\frac{1}{3}}\rho^{\frac{2}{3}} \tag{4}$$

Example: Compare surface acceleration for Earth and the moon. The Moon is about 1/10th the mass of the Earth but their densities are about the same. Based on the scaling of equation 4 to estimate the surface gravity of the moon we just take the square root of 10, finding that g_{moon} is about 1/3 of that of the Earth (and ignoring density difference).

For mass $M \sim \rho R^3$ and insertert this into the acceleration (equation 3) with $R = D$

$$a = G\rho R. \tag{5}$$

Example: For a body where we don't know the mass but can estimate the radius (how?)[1] we can estimate the acceleration. The Earth has a radius of 6000 km or so. What is the surface gravity for a body that is only 6 km? It is a thousand times smaller. Instead of Earth's 10 m/s^2 = 1000 cm/s^2 we get 1 cm/s^2 on such a small body.

Scaling from the Earth

$$a \approx 10 \text{ m s}^{-2} \times \left(\frac{R}{R_\oplus}\right)\left(\frac{\rho}{\rho_\oplus}\right)$$
$$\frac{a}{g_\oplus} \approx \left(\frac{R}{R_\oplus}\right)\left(\frac{\rho}{\rho_\oplus}\right)$$

where the density of the Earth is about 3 g/cc and the radius of the Earth is about 6000 km.

Objects vary in mass and radius, but they don't vary much in density. We have rewritten the acceleration to depend on mass and density or density and radius rather than on mass and radius.

2.3 Energy

The quantity $V^2 = GM/D$ is the square of velocity. It is also a kinetic energy per unit mass. It also has units of potential energy per unit mass.

In a star cluster, the kinetic energy can be estimated from the gravitational potential energy. This is because motions are caused by gravitational acceleration. There is a single energy scale. Therefore kinetic energy *must* be approximately equal to potential energy. This statement is the order of magnitude equivalent to the **virial theorem**.

A star cluster of total mass M and radius D. The total potential energy

$$U \sim \frac{GM^2}{D}$$

[1] Brightness

The total kinetic energy

$$K \sim MV^2$$

For these to be equal we arrive at our gravitational velocity scale

$$V = \sigma = \sqrt{GM/D}$$

This is also the scale of the cluster's **velocity dispersion**, σ, characterizing random motions.

Usually what you observe is a sizescale D (from angular extent on the sky) and a velocity dispersion σ (from spectroscopy giving the size of Doppler velocities of stars, or width of an absorption feature for star light in a galaxy bulge).

These two quantities (D, σ) can be combined to give the mass of the cluster (that you don't know, though you can count stars and assume they all are about a solar mass).

$$M_{cluster} \approx \frac{D\sigma^2}{G}$$

and this is the only way to get a mass using parameters G, σ, D.

At a distance R from a point mass, the potential energy per unit mass $\frac{GM}{R}$. We can estimate an energy per unit time

$$L = \frac{GM\dot{M}}{R}$$

which is a power or a luminosity. Here $\dot{M}$ is an accretion rate. The luminosity L would be that of an efficiently accreting object where material is falling to a radius R.

2.4 Energy density and pressure

Potential energy per unit volume

$$e_g = U/V = M\frac{GM}{D}\frac{1}{D^3} = \frac{GM^2}{D^4}$$

Force per unit area

$$M \times \frac{GM}{D^2} \times \frac{1}{D^2} = \frac{GM^2}{D^4}$$

The two are the same.

Force per unit area is a pressure.

GM^2/D^4 is useful to estimate

1. Central pressure of a planet or star with mass M and radius D.

2. Gravitational binding energy of a star or planet or strengthless asteroid or comet.

We notice here that with just a mass and a radius we are saying something about a planet or star's **interior**. The pressure and density are set by an equation of state and that says something about material properties (mechanical strength, compressibility) or temperature.

Planetary objects in the solar system that are above a certain size tend to be round. The yield strength for ductile deformation of rocky or icy material can be estimated from the central pressure

$$P_c \sim \frac{GM^2}{R^4} \sim G\rho^2 R^2. \tag{6}$$

Asteroids and moons above a certain radius tend to be round. The central pressure can be compared to material strength allowing ductile deformation.

Larger objects of similar densities have higher central pressures and would be above a critical value allowing the material to flow. Objects above a certain size tend to be round because the central pressure exceeds the material strength for ductile deformation.

2.5 Sound speed

Density of an object with mass M and sizescale R is

$$\rho = \frac{M}{R^3}$$

Pressure over density

$$\frac{P}{\rho} = \frac{GM^2}{R^4} \times \frac{R^3}{M} = \frac{GM}{R}$$

The sound speed in a gas

$$c_s \sim \sqrt{P}\rho = \sqrt{\frac{GM}{R}}$$

In a gas $c_s \sim \sqrt{kT/m}$ where m is the mass of a molecule or atom. This implies that the central temperature of a star or gas giant planet can be estimated from the central pressure.

Our gravitational velocity scale $V = \sqrt{\frac{GM}{R}}$ is also about the speed of pressure waves traveling through a star or planet.

$\Omega_{grav} = \sqrt{GM/R^3}$ is about the frequency of the lowest vibrational/oscillation mode or the sound travel time through a gaseous planet or star.

2.6 Setting the velocity, finding a radius

We could invert the question using a particular important velocity. Suppose we set $V = c$ with c equal to the speed of light.

$$c = \sqrt{\frac{GM}{R}}$$

solve for radius R

$$R = \frac{GM}{c^2}$$

What is special about this radius?

It is essentially (with a factor of 2) the **Schwarzschild radius**.

Suppose we put a point mass into a gas cloud that has sound speed c_s.

$$R = \frac{GM}{c_s^2}$$

What is this radius?

It is essentially the **Bondi radius**. This is the **transonic radius** for accretion flows onto compact objects or winds from planets and stars.

What if we use σ the velocity associated with random motions of stars near a black hole.

$$R = \frac{GM}{\sigma^2}$$

Is sometimes called the **sphere of influence** of a black hole. Inside this radius the black hole's perturbations are important (stellar orbits obey Keplerian dynamics) and outside it the random motions of the stars are primarily what is seen.

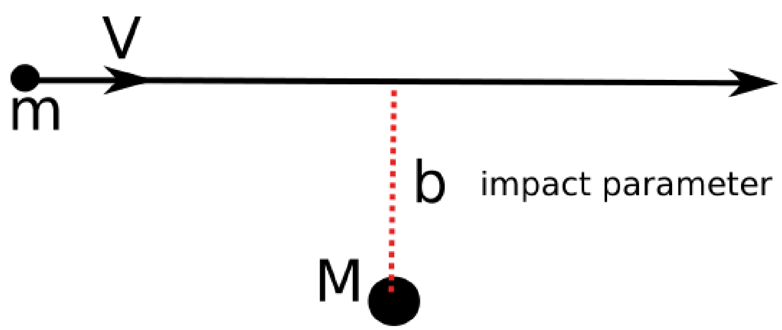

Figure 1: A gravitational encounter with impact parameter b and relative velocity V.

3 Two masses

3.1 Gravitational encounters and the impulse approximation

A small mass encountering a big mass M with impact parameter D and with relative velocity V_{rel}.

Is m deflected?

We compute our gravitational velocity scale $V_g = \sqrt{GM/D}$. We compare V_{rel} to V_g.

$V_{rel} > V_g$	little deflection
$V_{rel} < V_g$	significant deflection

To estimate the angle of the encounter and the velocity change, we would need to go beyond an order of magnitude approach. But we expect that these computations will depend on the ratio V_{rel}/V_g.

What is the size of the velocity change ΔV (on m) caused by the encounter?

Assume the trajectory is a line. That means we are assuming $V_{rel} > V_g$. The force per unit mass on m at closest approach is GM/D^2. This is an acceleration and $a = \Delta V/\Delta t$.

What is Δt?

$\Delta t \sim D/V_{rel}$. Putting this together the change in m's velocity is about

$$\Delta V \sim a\Delta t \sim \frac{GM}{D^2}\frac{D}{V_{rel}} = \frac{GM}{DV_{rel}}$$

Notice that $\Delta V \sim V_g^2/V_{rel}$.

The estimate is approximately the same as an approximation called the **impulse approximation**.

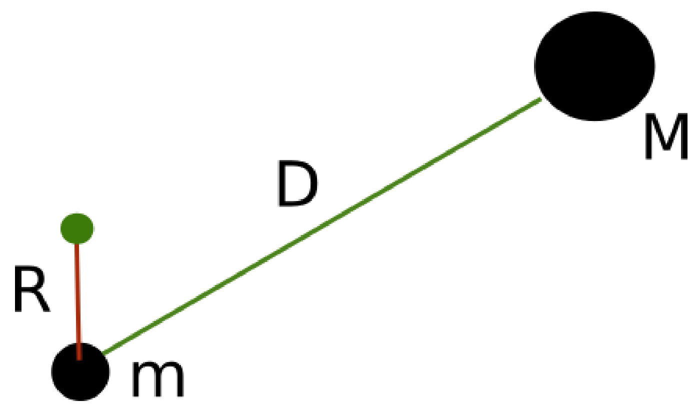

Figure 2: At what distance R is a small body in orbit around m rather than M? Here D is the distance between m and M.

3.2 Tides

Suppose we have two masses m, M and two distances R, D. There multiple ways to combine these quantities and it is not obvious how to predict when one object is tidally disrupted by another or in the gravitational sphere of another without a bit more work.

We introduce some new parameters:

- R/D, ratio of distances.
- m/M, mass ratio.
- Tidal force on object m with radius R

$$F_{tidal} \sim \frac{GMR}{D^3}$$

- Self gravity

$$F_{sg} \sim \frac{Gm}{R^2}$$

When is self-gravity on m more important than the tidal force from M onto m?

$$\begin{aligned} F_{tidal} &> F_{sg} \\ \frac{R^3}{D^3} &> \frac{m}{M} \\ \frac{m}{R^3} &> \frac{M}{D^3}. \end{aligned} \tag{7}$$

The last line is a density ratio that is independent of the gravitational constant G. The transition radius that tells you whether the tidal force is important

$$R_t = \left(\frac{m}{M}\right)^{\frac{1}{3}} D$$

The transition radius is approximately equal to

1. Radius of the Hill sphere. This tells you whether a small additional object orbits m or M.
2. The Roche radius. This tells you whether m is losing mass that would fall onto M.
3. Gap width in a circumstellar disk caused by an exoplanet. This is because the gas or particles in the disk are either in orbit about M or m.
4. Width of the Trojan asteroid population. In this case the asteroids are in orbit about the Sun but because they are locked with Jupiter's orbit, Jupiter's Hill radius is relevant.
5. Distance between L1, L2 Lagrange points about m.

3.3 Tidal disruption

We consider a mass m with radius R that is a distance D from mass M. Self gravity is exceeded by the tidal force when (reversing equation 7)

$$\frac{m}{R^3} < \frac{M}{D^3}. \tag{8}$$

The quantity on the left is approximately the density ρ_m of mass m, giving a condition for disruption

$$\rho_m < \frac{M}{D^3}. \tag{9}$$

The quantity on the right can be considered an effective density, but with the mass of M distributed into a sphere of radius D. Tidal disruption arguments are often written in terms of densities.

4 Eccentricity

With G, M, D we get a velocity and a time. When we considered encounters we introduced introduced a ratio V_{rel}/V_g. Instead of introducing a comparison velocity we can introduce a comparison length scale. For an orbit of mean radius R_0, a length scale δR gives an orbital eccentricity of

$$e = \frac{\delta R}{R_0}.$$

Eccentricity is a unitless ratio, $e < 1$,

$$e \sim \frac{|R_{min} - R_0|}{R_0} \sim \frac{|R_{max} - R_0|}{R_0} \sim \frac{|R_{apocenter} - R_0|}{R_0}$$

Let us estimate the size scale of velocity changes in the orbit δV.

$$e \sim \frac{\delta V}{V_g} \sim \frac{|V_{min} - V_g|}{V_g} \sim \frac{|V_{max} - V_g|}{V_g}$$

Consider a particulate thin disk in orbit about a point mass M. Some random motions in the particles: σ, a velocity dispersion at radius R: What is the eccentricity dispersion? What is the inclination dispersion? What is the disk thickness? We assume an isotropic velocity distribution.

Answers:

$$e \sim i \sim \frac{\sigma}{\sqrt{GM/R}}$$

where i is the inclination. Disk thickness $h \sim iR_0$ where i is the inclination.

5 Extended Mass distributions

Instead of working with a point mass, we consider a mass density ρ. Units

$$G\rho = M^{-1}D^3T^{-2} \times MD^{-3} = T^{-2}$$

$$t_g = \frac{1}{\sqrt{G\rho}}$$

is a free fall collapse timescale for a molecular cloud.

To say anything more about the cloud we need to provide more information. A temperature or a sound speed, c_s.

$t_g \times c_s$ is a distance. It is approximately the **Jeans wavelength**, giving a sizescale for things to tend to collapse.

$\rho(t_g c_s)^3$ is a mass. It is approximately the **Jeans mass**, giving a sizescale for clumps to form under gravitational collapse in a molecular cloud.

Sound waves traveling through a molecular cloud would have a dispersion relation

$$w^2 = k^2 c_s^2$$

How would self gravity change this? Can we make a velocity in another way using G and ρ? Recall $G\rho$ has units of T^{-2}. We might guess that we add another term to the dispersion relation that looks like $G\rho/k^2$. The transition between the two terms gives us waves at small scales and instability at large scales with a growth timescale of order the free-fall time.

Suppose we have a flat mass distribution like a Galactic, circumstellar or circumplanetary disk. The relevant parameter is Σ the density per unit area.

$$G\Sigma = M^{-1}D^3T^{-2} \times MD^{-2} = DT^{-2}$$

We can combine $G\Sigma/k$ to make give a velocity so a dispersion relation for spiral density waves might look like

$$w^2 = k^2 c_s^2 - G\Sigma k$$

where the minus sign arises because gravity might cause instability. The sound speed gives us a particular distance $c_s^2/(G\Sigma) = D$.

We can make a unitless parameter

$$\frac{c_s^2}{G\Sigma R}$$

with $R \sim 1/k$. This unitless parameter tells you whether a gaseous or stellar disk or ring system would grow spiral arms or collapse into clumps.

We don't quite get the Toomre Q parameter here for stability of a disk to growth of spiral density waves as we are missing mass from another source such as dark matter or a central star.

6 Harmonic Oscillators and Pendula

A spring has force kx where x is a displacement from equilibrium. We have a spring constant and a mass m. Units of k are force/distance $= ma/d = M/T^2$. Units of k/m are T^{-2}. So $\sqrt{k/m}$ is a frequency.

With a pendulum we start with g, L, m, acceleration, length and mass. Our force is mg. The only way to combine our parameters to get a frequency is $\sqrt{g/L}$. Once we have a frequency we can think about an angular velocity $\dot{\theta}$ which is also in units of $1/T$. Starting with an initial $\dot{\theta}$ we can ask: is it larger or smaller than $\sqrt{g/L}$? A kinetic energy is $mL^2(\dot{\theta})^2$. An energy associated with the pendulum is $mL^2 \times g/L$. If $\dot{\theta}$ much larger than $\sqrt{g/L}$ then kinetic energy dominates potential energy by a lot and we would have a different type of behavior than if we were in the regime of small oscillations. Libration vs. oscillation regimes.

7 N-body Units

Of when running numerical explorations we work in different sets of units. For orbital problems a common choice is $G = 1$.

What happens if we set $G = 1$? We need a length scale and a mass scale. $M = 1, D = 1$. If D is the orbital semi-major axis, then the mean motion $n = \sqrt{GM/D^3} = 1$. But the orbital period is $P = 2\pi/n = 2\pi$.

For example, suppose we are doing simulations near a single planet like the Earth which is in orbit around the Sun. We would set distances in AU, mass in units of solar masses and set G to 1. In this case time is in units of the inverse of Earth's mean motion.

Example Suppose the orbital semi-major axis is in AU and the mass is in solar masses. What is the speed of light in these N-body units?

Answer:

The relevant velocity is that of the Earth in orbit which is

$$\begin{aligned} v_\oplus &= \sqrt{GM_\odot/AU} \\ &\sim \sqrt{\frac{7 \times 10^{-8}\text{cgs} \times 2 \times 10^{33}\text{g}}{10^{13}\text{cm}}} \\ &\sim \sqrt{10^{-7} \times 10^{20}} \sim \sqrt{10^{-13}} \\ &\sim 10^6\text{cm/s} \sim 10\text{km/s} \end{aligned}$$

We take the ratio of this and the speed of light

$$\frac{c}{10^6\text{cm/s}} \sim \frac{3 \times 10^{10}}{10} \sim 10^4$$

This is the speed of light in these N-body units.

8 The Buckingham Pi theorem

Given n physical variables and k types of dimensions (think units), there are $p = n - k$ independent dimensionless numbers.

8.1 A short dynamical example

Suppose we have velocity V, gravitational constant G, mass M and a distance D.

The number of variables is 4.

The number of dimensions is $k = 3$ (length, time, mass) which we write as $[L], [T], [M]$.

There is $p = 4 - 3 = 1$ one dimensionless number which must be a power of

$$\Pi \equiv \frac{V^2 D}{GM}.$$

Variable	V	G	M	D
Units	$[L][T]^{-1}$	$[M][L]^3[T]^{-2}$	$[M]$	$[L]$

We write this in terms of a matrix of exponents

$$\log \begin{pmatrix} V \\ G \\ M \\ D \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 3 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \log \begin{pmatrix} [M] \\ [L] \\ [T] \end{pmatrix} + \text{constant} \tag{10}$$

8.2 An example with water waves

Another example is water waves. We are interested in the dispersion relation. Variables are angular frequency ω, wave vector k, wave height A, gravitational acceleration g, depth of water h, surface tension γ (units energy per unit area) and density ρ.

Variable	ω	k	g	h	A	γ	ρ
Units	$[T]^{-1}$	$[L]^{-1}$	$[L][T]^{-2}$	$[L]$	$[L]$	$[M][T]^{-2}$	$[M][L]^{-3}$

We write this in terms of a matrix of exponents

$$\log \begin{pmatrix} \omega \\ k \\ g \\ h \\ A \\ \gamma \\ \rho \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -3 \end{pmatrix} \log \begin{pmatrix} [M] \\ [T] \\ [L] \end{pmatrix} + \text{constant}. \tag{11}$$

We have 7 variables and three units (length, time, mass). So we can construct 4 dimensionless variables. The combination

$$\frac{g\rho}{\gamma k^2} \tag{12}$$

is known as the Bond number and is small when the surface tension is high. The Bond number is small when wavelength is small, which makes sense as we would expect that surface tension is only important when the wavelength is small. Only when the Bond number is low would we worry about surface tension.

There are multiple choices for length ratios $kh, kA, h/A$. Only two of these are independent. Lastly use g to construct a fourth dimensionless parameter. We construct a ratio gk/ω^2.

To construct dispersion relations, we can discard variables. If the wavelength is small compared to the depth and the amplitude is small compared to the depth, and the surface tension is irrelevant (the bond number is high) then we ignore γ, h, A and $\omega = \sqrt{gk}$ is the only way to construct a dispersion relation. This is relevant for deep water waves.

If surface tension is important than gravity and we don't care about depth because the wavelength is small then the only way to construct a dispersion relation is $\omega = \sqrt{\gamma/\rho} k^{\frac{3}{2}}$. These are known as capillary waves.

If the water depth is relevant then we can construct a velocity $\sqrt{gh}$. If we use that to construct a dispersion relation we get $\omega = \sqrt{gh}k$ which is relevant for shallow water waves.

8.3 Outline of derivation

We have n variables q_i with $i \in 1, ...n$ and k dimensions x_j with $j \in 1, ..., k$. Each variable q_i can be written as a production of dimensions

$$q_i \propto \prod_j x_j^{\alpha_{i,j}} \tag{13}$$

with a set of powers $\alpha_{i,j}$. We desire dimensionless quantities π_i in the form of a product of variables

$$\pi_m = \prod_i q_i^{\beta_{i,m}} \tag{14}$$

with exponents $\beta_{i,m}$. Inserting equation 13

$$\pi_m \propto \prod_i \prod_j x_j^{\alpha_{i,j}\beta_{i,m}}. \tag{15}$$

We take the log of both sides

$$\text{constant} = \sum_j \log x_j \sum_i (\alpha_{i,j}\beta_{i,m}) \tag{16}$$

For the dimensionless variables to be unitless, we require that $\sum_i \alpha_{ij}\beta_{im} = 0$. The coefficients α_{ij} can be considered an $n \times k$ matrix, as shown in the above examples. Finding independent combinations for the exponents β_{im} are equivalent to finding independent vectors in the null space of the matrix.

9 Dynamical Size-scales in Astronomy

1. **Lengths:** Planetary systems 10s of AU, galaxies 10s of kpc, Planets thousands of km.
2. **Spin periods:** Giant planets, asteroids: hours, days.
3. **Orbital periods:** planets years, hot Jupiters: days. Binary stars: hours to years. Moons of Uranus: days.
4. **Age of Universe:** 10^{10} years about.
5. **Age of Solar system:** 4 billion years.
6. **How long does it take planets to form?** million to 10 million years.
7. **Galactic rotation period:** 200 million years.

The number of rotation periods in the age of the solar system ranges from 10^8 or so for Pluto to 10^{11} for some moons.

The number of rotation periods for the Sun around the Galaxy is only around 40.

10 Some order of magnitude problems

1. The Sun is 8 kpc from the Galactic center and the circular velocity of its orbit is about 220 km/s. How much mass is interior to the Sun's orbit in solar masses? What is the rotation period?
2. How charged does a 0.1 micron dust particle need to be for Jupiter's magnetic field to push it around? At this charge, can the particle hold itself together with the typical size-scale of atomic bonds?
3. In the early universe an absence of metals might makes it difficult to cool below 10^4K. Compare the Jeans mass at 30K to that at 10^4 K.
4. Could the Jeans mass be as big as a galaxy?
5. How often is there a conjunction between Venus and the Earth? What size velocity kick does the Earth get during each conjunction?

6. Pulsars can have spin rotation periods of milliseconds. What can you infer about their density from this alone? What size magnetic field would be in **equipartion** with this density? (you can assume a sound speed that is approximately equal to the speed of light). Equipartition means the magnetic pressure ($B^2/(8\pi)$) is equal to the pressure.

7. Why is the maximum spin of an asteroid approximately independent of body size?

8. On crater scaling and the Buckingham pi theorem. A projectile has velocity v_{pj}, radius a_{pj} and density ρ_{pj}. It impacts an asteroid with density ρ_a, gravitational acceleration g_a and strength Y_a. The strength has units of energy per unit volume. How many independent dimensionless parameters can you construct with these variables? Construct a dimensionless parameter that lacks Y_a but contains g_a. This parameter is important in what is called the *gravity regime* (large craters or small craters in weak materials like rubble). Construct a dimensionless parameter that lacks g_a but contains Y_a. This parameter is important in what is called the *strength regime.*

9. For a binary asteroid with total mass m_B and semi-major axis a_B how close does it need to get to a planet of mass M_p to be tidally pulled apart?

10. A dwarf galaxy with velocity dispersion 10 km/s approaches a Milky Way sized galaxy with a flat rotation curve and rotational velocity about 200 km/s. At what distance is the dwarf galaxy tidally disrupted?

11. An exo-planet is seen in transit against a star. Does parallax affect timing of an exoplanet transit? Consider how the viewpoint from the Earth changes as the Earth rotates about the Sun. How large is the change δt in the time of the mid-point of the transit as a function of season (our fall, winter, spring, summer)? Assume the transit period is P (this is the orbit of the planet around the star) and the distance to the star is D. Is this transit time detectable? Would it tell you anything interesting? What types of systems are most likely to have detectable δt? (See paper by Scharf 07) `https://ui.adsabs.harvard.edu/abs/2007ApJ...661.1218S/abstract`

12. A comet approaches Saturn and is on an orbit that would graze but not touch the planet's surface. As the comet nears Saturn, it is tidally disrupted. What can you say about the density and strength of the comet?

13. Consider a debris disk (bunch of dust particles or asteroids) in orbit about a star. How is the collision rate related to the area filling factor? Does the collision rate depend on the disk's velocity dispersion? Does the energy of collisions depend on the velocity dispersion?

14. Consider a particulate disk about a point mass of M at radius R and with velocity dispersion σ. What is the relative velocity of collisions between particles?

15. The inverse of the Hubble constant gives an expansion timescale. For what density is the gravitational collapse timescale similar to the Hubble constant? Is this related to the density required to close the universe?

16. Show that the gravitational torque on a spinning body due to a point mass of mass M and at distance D is of order $T \sim n^2 I$ where I is the moment of inertia and $n = \sqrt{GM/D^3}$ is the mean motion.

17. Compare angular momentum in spin to that in orbital motion for a spinning body of mass m in orbit around a point mass M. The spinning body has radius R and is at a distance a from M.

18. Compute the ratio of the radius of Jupiter to its Hill radius. How does this compare to the same ratio for a hot Jupiter. What can you say about the existence of exo-moons around hot Jupiters? (How much room is there?)

19. On N-body units. An N-body simulation of galaxies adopts the following units. $G = 1$. Distances in units of kpc. Velocities in units of km/s. What are the units of mass? What are the units of time?

20. What is the gravitational collapse timescale for a molecular cloud? Molecular clouds tend to have velocity dispersion of a few km/s and sizes of a few pc and masses of a few thousand to a few million solar masses.

21. What is the sphere of influence for a 1000 $M_\odot$ black hole in a stellar cluster? How does this compare to the sphere of influence in the Galactic center? Stellar clusters tend to have velocity dispersions of a few km/s. Bulges of galaxies tend to have velocity dispersions of a few hundred km/s.

22. What is the speed of light in units of the Earth's orbital velocity about the Sun? This is relevant to radiative drag forces on dust.

23. Protostellar disks can be 10 a few hundred AU large in radius. Their mass tends to be low compared to their host star mass. At what distance from the Galactic center black hole ($M_{bh} \sim 4 \times 10^6 M_\odot$) is a protostellar disk tidally disrupted? Low mass young stars have recently been detected within a few pc of the central black hole in the Galactic center.

24. Rock has a yield strength that is a factor of less than a percent of its elastic modulus which is of order 50 GPa. A Pa (Pascal) is an mks unit of pressure which is an energy per unit volume. The elastic modulus of an icy material is about an order of magnitude lower than that of a rocky material. We observe that large asteroids and icy bodies like Pluto tend to be spherical. Compare the central pressure of an object to the yield strength of a rocky or icy material. What is the maximum size that a

non-spherical object can be before the central pressure exceeds the yield stress and the body deforms so that it would become a sphere?