The Dark Matter of Galactic Halos

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Abstract

The unexplained source of the extended flat portion of rotation curves of galaxies is attributed to dark matter in the galactic halos.

1. **We show it is extremely implausible that any galactic halo is made of particles.**
2. **We show that if halos are made of fluctuations in a simple classic scalar field, one can produce observed rotation curves.**
3. **If we consider a universe filled with abuting scalar field halos, we can define a new average cosmological fluid whose density scales as $\Re^{-2}$, where $\Re$ is the cosmological scale factor. It is possible that this can result in the "quintessence" effect.**

Observations

The rotation curves, V_{rot} versus r, of many galaxies are composite, with baryonic material responsible for the inner portion $r < R_h$ and halo dark matter (HDM) responsible for the extended "flat" portion of the curves at large $r > R_h$. Poisson's equation gives the density structure of a idealized DM halo to be: For $r \leq R_h$ we adopt $\rho(r) \approx \langle\rho\rangle$ and $\rho(r) = {}^1/_3\langle\rho\rangle(R_h/r)^2$ for $R_h \leq r \leq R_\ell$ (for $V_{rot}(r) = V_h$). Here $R_\ell \gtrsim 5 - 10\ R_h$. Often the luminous underlying galaxy extends to greater than ~$1 - 2R_h$. As representative, we use values for the Milky Way, $V_h = 200 - 210$ km/s and $R_h \sim 5$ kpc, giving $\langle\rho\rangle \sim 6 \times 10^{-24}$ g cm^{-3} with the luminous matter's average density clearly exceeding this when $r \leq$ ~2.5 kpc.

- **A scalar field satisfactorily represents observed halos**
  - An (unquantized) scalar field, ϕ, provides an adequate phenomenological description of the halo DM. Steady-state spatial fluctuations in this field form gravitational potential wells into which baryonic matter may flow, possibly forming luminous galaxies in their center regions. The halos correspond to steady state solutions of the generic field equation:

$$c^{-2}\partial_{tt}^{2}\phi - \nabla^{2}\phi = m^{2}\phi(1-\phi^{2}). \tag{1}$$

- **The field's past history**
  - We suggest that these halos arose as very small amplitude unstable field fluctuations, $\delta\phi \sim 10^{-4}$, carried along with the cosmological CDM. With appropriate boundary conditions, the growth of fluctuations is limited by the nonlinear term. Only solutions spherically symmetric in the central regions can grow. From this initial value, it takes ~10 m^{-1}/c ~10^{5} years to develop to finite amplitude, therefore they could not have been important in the very early days of the universe. We assume adequate damping occurs and guess that values of *m* are determined by values of the local Jeans' wavenumber associated with the onset of instability of the CDM at the time of radiation-CDM equipartion.

- **Solutions**
  - To the lowest order in the amplitude a < 1, a halo solution is:

$$\phi \approx \phi_0 = a \sin(mr)/mr \tag{2}$$

  - To order ~a^3, the solution is approximately

$$\phi_1 \approx a \sin\left(m\int\sqrt{(1-\phi_0^2)dr}\right)/mr\sqrt{(1-\phi_0^2)}, \tag{3}$$

    which shows that the effect of the non-linear term, for small *a*, is to produce a slight phase shift.

  - Associated with the field equation is an energy momentum tensor which can be used as the source term of a gravitational field. Using the ϕ_0 solution to represent an isolated halo extending to $r = R_t \gg m^{-1}$, one can calculate the gravitational acceleration on a nonrelativistic particle *f*. The halo mass will be defined observationally by $f \equiv -GM_{halo}/r^2$. One finds for the rotational velocity:

$$\boxed{V_{rot}(r) = V_h[1 - \sin 2mr/(2mr) - \frac{1}{3}(r/R_\ell)^2]^{1/2}} \text{ for } r \le R_\ell\text{, and} \tag{4}$$

    $V_{rot}(r) = (2R_\ell/3r)^{1/2}V_h$ for $r \ge R_\ell$. For $mR_\ell \gg 1$, the mass of the halo is given by

$$\boxed{GM_{halo} = 2V_h^2 R_\ell/3} \tag{5}$$

    (Formally, V_h is defined in terms of the field's amplitude *a* by $(V_h/c)^2 \equiv a^2\alpha^2/2$, where α^2 is a universal coupling constant.)

  - For the Milky Way, other arguments suggest an amplitude of $a_{mw}^2 \approx {}^1/_3$. Since $R_h \simeq 5$ kpc, one determines $m^{-1} \simeq 3.2$ kpc by setting $2mR_h \equiv \pi$. Observations inadequately determine R_ℓ; one can only conclude $R_\ell > 40$ kpc from Figure 1. If $R_\ell = 100$ kpc, $M_{halo} = 6 \times 10^{11}\ M_\odot$. For other galaxies, *m* and *a* will have different values. The figures show that this representation of the observed rotation curves is really very good when the inferred baryonic contributions are small.

Appendix: technical details

The halo gravitational potential wells are specified by the energy momentum tensor T_{ab} determined by $\delta\left[\sqrt{-g}\ L\right]=\sqrt{-g}\,(T_{ab}/2)\delta g^{ab}$ once a Lagrangian $L(\phi)$ is chosen. The associated field equation is given by $\delta L/\delta\phi = 0$. We use this as a Lagrangian $L = -{}^1/_2\alpha^2 L_a^a + \Lambda$ where $L_{ij} = \phi_{,i}\phi_{,j} + m_i m_j[\phi^2(1-{}^1/_2\phi^2)-\lambda]$. Here m_i is an assigned time-like vector, $m^a m_a = m^2$, $x_0 = ct$, and ϕ is dimensionless. We take L to have the same dimension, (length)$^{-2}$, as the Riemann scalar R; it is connected to it by the dimensionless coupling constant α^2. The usual coupling constant $\kappa = 8\pi Gc^{-2}$ is used only in interpreting results, in converting mass density into (wavenumbers)^2 and vice versa. The parameters Λ and λ do not affect the behavior of ϕ; they are introduced to allow us to shape the extent of each individual dark matter halo. Equation (1) is the associated field equation; its form restricts any coupling to dark matter, baryon, and photon fields to occur in the m^2 term. The energy-momentum tensor is $T_{ij} = \alpha^2(L_{ij} - {}^1/_2\, g_{ij}L_a^a) + g_{ij}\,\Lambda$.

For the radially symmetric interior Schwarzschild metric, $d\tau^2 = B(r)dt^2 - A(r)dr^2 - r^2 d\Omega^2$, standard relations give:

$$rA^{-1} = r - \int(\kappa\rho_0 + \Lambda)r^2 dr; \tag{9}$$

$$\ln AB = \int(2\kappa\rho_0)r\ dr + \text{constant; and} \tag{10}$$

$$f \cong -\Gamma^r_{tt} = -(2A)^{-1}\partial_r B. \tag{11}$$

where

$$\kappa\rho_0 = \frac{1}{2}\alpha^2[(m^2\phi^2 + (\partial_r\phi)^2) + (\partial_t\phi)^2 - \lambda m^2] \tag{12}$$

and $c^2 f$ is the radial gravitational acceleration a nonrelativistic particle experiences. The ‘constant’ in equation (5) is determined by boundary conditions; since it is small, on the order of $(V_h/c)^2$, we will ignore it. At a boundary, λ is chosen so that ∂ in $AB/\partial r = 0$. The time evolution of small instabilities, etc., is discussed in a preliminary version of this work, in astro-ph/0308054.

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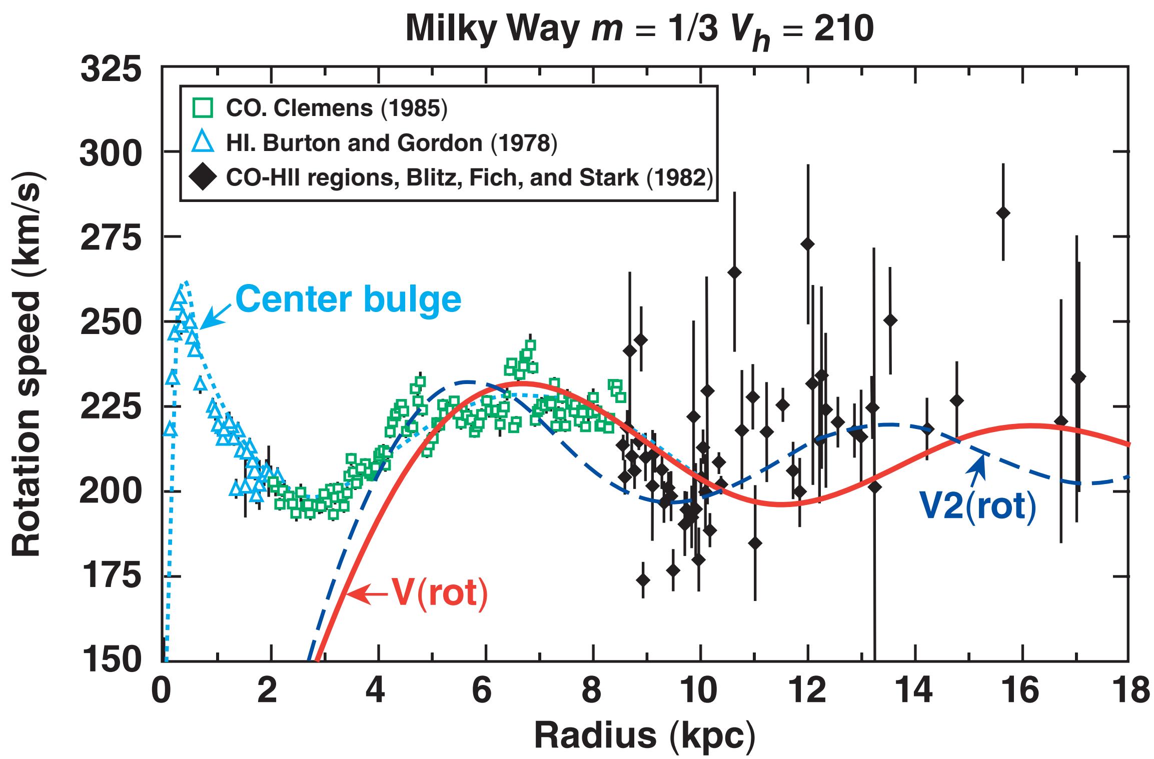
- **Dark halos made of particles are generally unstable.**
  - For a halo's outer region, $r > R_h$, assuming sphericity and hydrostatic equilibrium, one can solve for an equation of state $P = P(\rho)$. Using $\rho \propto r^{-2}$ to replace r as the independent variable, one finds $P = \frac{1}{2}V_h^2\rho + P_0$ (where $P_0 \propto \rho^{V_h^2/2c^2} \cong$ constant) so that for $r > R_h$ we must have an isothermal sphere solution, with a characteristic thermal velocity $\sim V_h$ for the DM. If the equation of state does not change for $r \leq R_h$, the isothermal Bonnor–Ebert solutions [see Alves, Lada, and Lada (2001)] with finite central density are appropriate; these have only a very small range in r in which $\rho \propto r^{-2}$ and are Jeans unstable for $R_t \gtrsim 20$ kpc and large halos will collapse.
- **Halo properties inferred from galaxy–galaxy collisions**
  - If galaxy–galaxy halo collisions have occured, the putative DM particles must be strongly radiative, otherwise, after collision the halos should have relaxed *adiabatically*, contrary to observations.
  - If the particles really do satisfy a collisionless Boltzman equation, then galaxy collisions should produce highly asymmetric halos. Before collision, the particles' average velocity must be of order V_h or less for them to remain bound to a galaxy. Their individual trajectories would then be modified by the nonspherically symmetric gravitational potentials present during the long galaxy collisions and there are no restoring forces to re-establish halo spherical symmetry after collision.
- **Halos must be bounded.**
  - If the halo density profiles $\rho \propto r^{-2}$ are not shaped by gravitational forces, then "edges" of halos must be imposed and defined by a physical mechanism; this cannot be done if the putative halo particles are noninteracting. If edges are not defined because the number of halos in a shell $(r, r + \delta r)$, is $\propto r^2$, the overlap of outer regions of distant unbounded halos will produce an unacceptably high local energy density (Olber's paradox!).

Rotational models for the Milky Way



- A simple model (m^{-1} = 3 kpc, V_h = 210 km/s), shown by the red line, is superimposed on the compilation of gas cloud observations by Clemens (1985, assuming R_0 = 8.5 kpc Θ = 220 km/s) with Clemens' points for $t > R_0$ lowered by 10 km/s (= $\Theta - V_h$). (The vertical lines represent Clemens' estimates of the effects due to uncertainties in the distances. Also, the points for $r < R_0$ show systematic errors, ~10 km/s, because the gas clouds depart from purely circular motions.)
- The dashed curve uses a nonlinear improvement to the simple model, using a^2 = 1/3. It introduces a slight phase shift which might match the "wiggles" in the observations.
- As shown, the curve corresponds to a low stellar galactic disk contribution to the rotation curve. Different assignments of the disk contribution and also of Θ can be accomodated by rescaling the halo field equation in the range V_h = 179 to 220 km/s.

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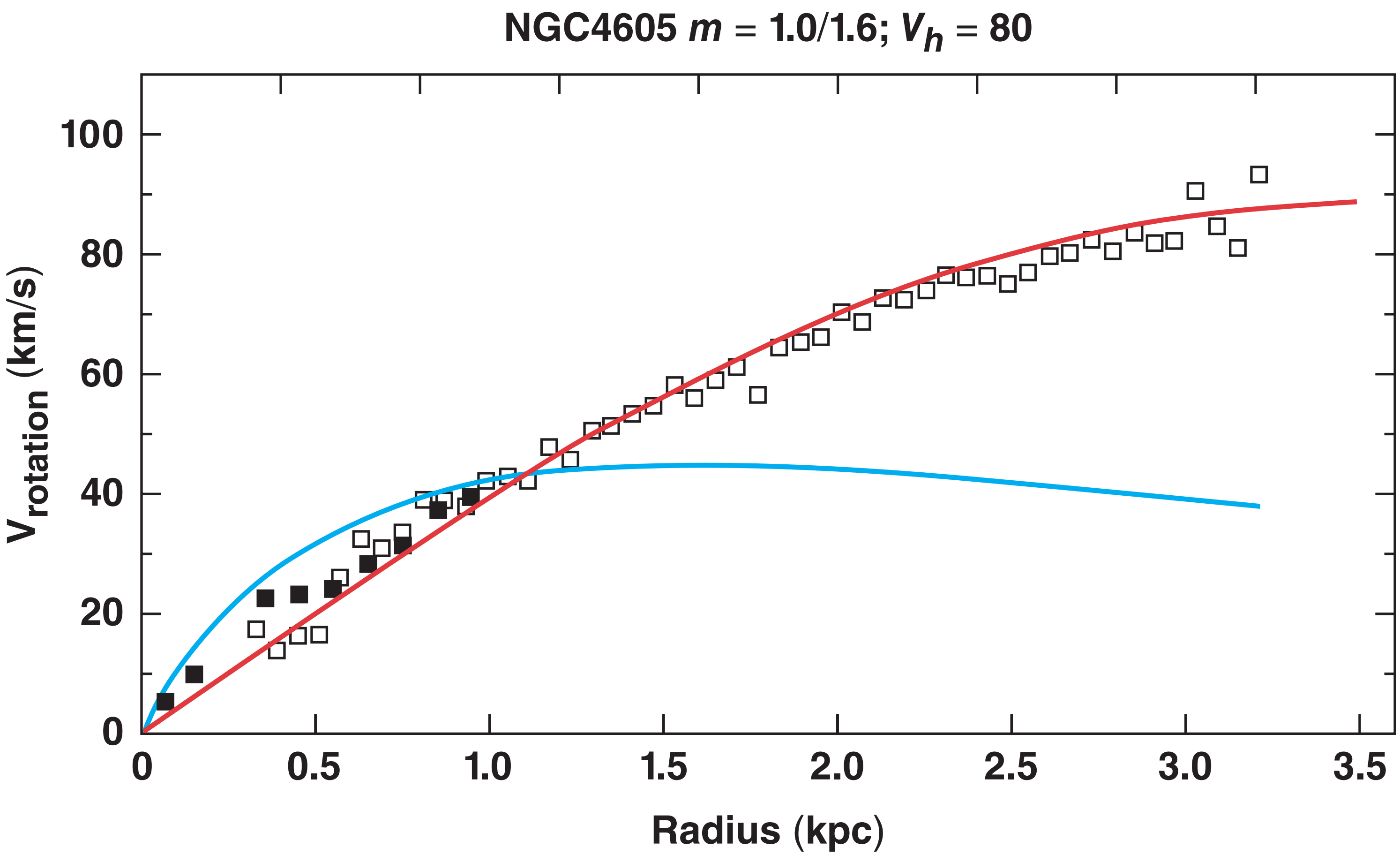
A rotation curve for NGC4605



- The simple m^{-1} = 1.6 kpc, V_h = 80 km/s model is compared to the combined CO and Hα rotation curve for the NGC4603 halo for Bollatto *et al*. (2002). They have removed a luminous disk contribution (blue) determined from K-band observations.

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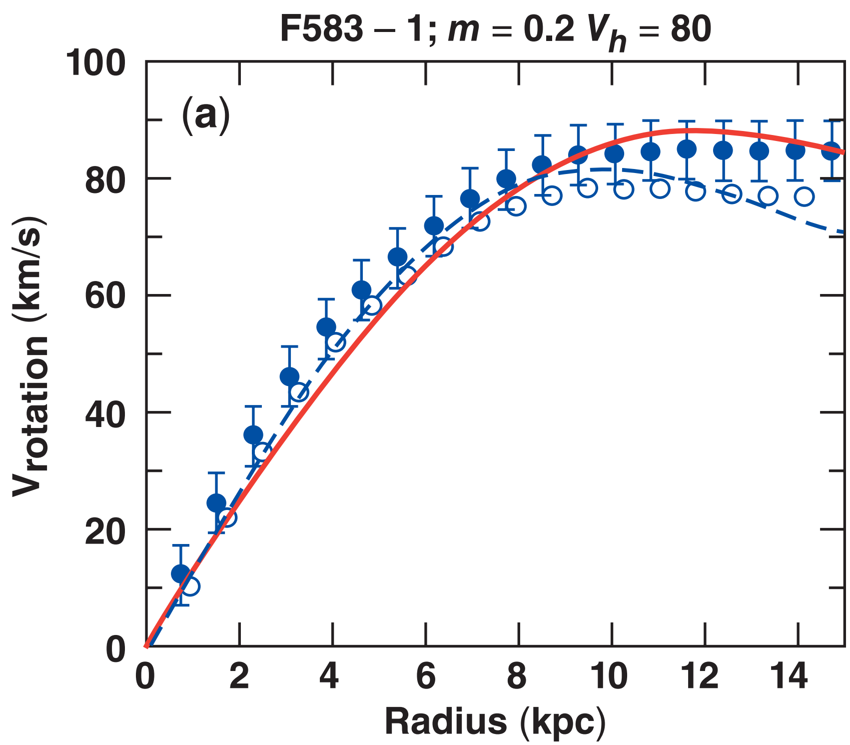
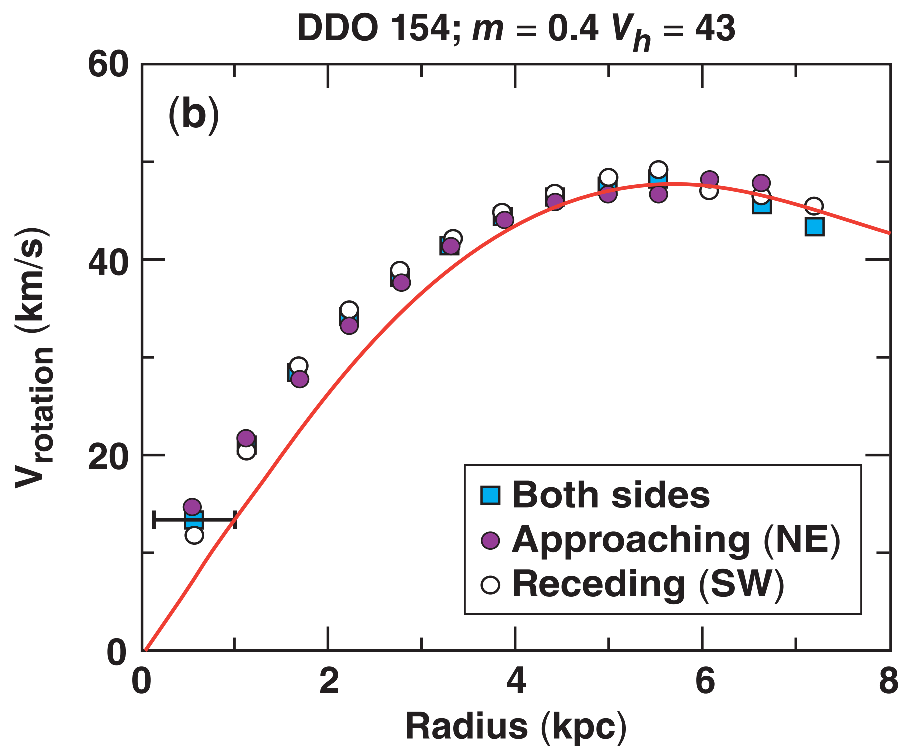
Rotation curves for two low mass galaxies





- Rotation curve model m^{-1} = 5 kpc V_h = 80 km/s compared to the observations (filled circles) of McGaugh and de Blok (1998) for F5 83–1. No disk component has been subtracted. (The open circles are the author's estimates of the halo when a crude disk has been subtracted; it is compared to the halo model m^{-1} = 4.33 kmc V_h = 74 km/s by the dashed curve.)

- Rotation curve model m^{-1} = 2.5 kpc; V_h = 43 km/s compared to the observations of Carignan and Beaulieu (1989) for DDO 154. No disk component has been subtracted.

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The edges of halos and the Λ–term

- It is very important to determine an edge for a halo so that its mass can be defined.

- Formally, the edge of an isolated field galaxy's halo can be described the same way that the edge of an ordinary body is by requiring a discontinuity in the source term ϕ at some $r = R_\ell$. One may then attach an exterior Schwarzschild solution, $\phi = 0$, to the spherical interior solution. The choice of R_ℓ depends upon a halo's past history. Clearly a halo should be terminated when its energy density falls below the critical cosmological density. This places an upper limit $R_\ell \sim 1.6\ h^{-1}$ Mpc on Milky Way-like galaxies.

- Another procedure is possible for a cluster of contiguous halos when one halo runs into another. Take each interior halo's boundary to be the surface on which a test body experiences zero gravitational attraction directed towards its center; this divides the cluster into cells. For the simplest case in which all halos are similar and roughly equispaced with spacing $2R_S$, then within one interior cell for $m^{-1} \ll r \leq R_S$, the gravitational acceleration to a halo's center must have an additional term:

$$f = -(\alpha^2/2)[a^2/r - \lambda m^2 r/3] + \Lambda r/3, \tag{6}$$

  where $\lambda m^2 = a^2 R_S^{-2}$ as before and we need $\Lambda = \alpha^2\ \lambda m^2$ so $f \to 0$ as $r \to R_S$. This neglects the higher-order multipole terms $\sim(r/2R_S)^n R_S^{-1}$; $n \geq 1$.

The edges of halos and the Λ–term (cont.)

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- In each cell the first two terms represent the halo 'bump' itself, while $\Lambda r/3$ represents a differential tidal force present because of the neighboring galaxy halos. The Λ–term causes the entire cluster to experience an expansion force. If the Milky Way halo is typical, the expansion velocity at the edge R_s is $V_h\sqrt{3} \sim 115$ km/s. It is important to realize this is not mysterious. Suppose one had many elastic spheres touching. In any one, a test body would experience a tidal force caused by the neighboring spheres and the sphere itself would experience a distortion. The sum of all such distortions in a given direction would add up, and the entire cluster would appear to have expanded.

- Each cell contains a halo 'bump' with mass of the same form as equation (5). The mean bump density is given by ($\kappa \equiv 8\pi Gc^{-2}$).

$$\kappa\langle\rho_{bump}\rangle = 2\alpha^2 a^2 R_s^{-2} = 2\Lambda\,(mR_s \gg 1) \tag{7}$$

Therefore, the Λ–term always is responsible for one-third of the cluster's mean density.

Cosmology and quintessence

- The halo DM must play an important role in cosmology today. For, suppose we just guess that each galaxy has a halo mass ten times its luminous baryon mass, then Ω_{LDM} ~ $10\Omega_{baryon} \simeq 0.3$. (An arbitrary density ρ is written as $\rho \equiv \Omega\rho_c$.)

- Define a halo cosmological fluid by regarding all dark halos to be part of a large cluster. We assume halos are neither created nor destroyed. For convenience, take the halos to be equidistant from one another with a number density $\psi(t)$ and use mean values of α^2, m^2, and halo mass $M = (c^2/3G)\langle a^2\rangle\alpha^2 R_s$. Here $R_s \approx {}^1/_2\psi^{-1/3} \gg m^{-1}$. Using the results of the previous section, one finds the halo cosmological fluid has a density given by

$$\kappa\rho_{fluid} = \frac{8}{3}\pi\,\psi\langle(V_h/c)^2\rangle\,\psi^{-1/3} + \Lambda \equiv \Omega_{halo}\rho_{crit} \tag{8}$$

- We have explicitly assumed that the very large scale variations in galaxy (or halo) density can be averaged over, with $\psi\langle a^2\rangle$ being our crude approximation to the pair-correlation function $\psi^{-1/3}$. As before, Λ is 1/2 the previous term.

Cosmology and quintessence (cont.)

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- The form of this equation for representing halos so bounded is secure and is remarkable in that both terms scale as $\Re^{-2}$, where $\Re$ is the cosmological scale factor. The pressure associated with this cosmological fluid is $p = -\rho_{\text{fluid}}/3$. (This follows from conservation of energy with this scaling.)

- Suppose the average galaxy is somewhat smaller than the Milky Way with $\langle a^2 \rangle \sim (2/3) a_{mw}^2$ (corresponding to $V_h \sim 170$ km/s); then using for ψ a mass (or luminosity) weighted Schechter function $\psi \simeq 0.014 h^3$ halos Mpc^{-3}, one finds for this halo fluid $\Omega_{\text{halo}} \sim 0.67$, independent of h, with one-third of this due to the Λ/κ term.

- This estimate is quite uncertain but it permits us to point out an interesting scenario. Suppose we take at present $\Omega_{otherCDM} \sim 1/3 \cong (1/2)\Omega_{halo}$, as representing the usual cold DM whose density scales in time as ordinary matter, $\propto \Re(t)^{-3}$. At present, the halo fluid would dominate with $\Re(t) \propto t$. Going back in time to when $\Re = 1/3$, one sees the reverse would have been true, $\Omega_{otherCDM} \sim 2\Omega_{haloDM}$; then the scale factor would have had a different time dependence, $\Re(t) \propto t^{2/3}$. This would mean that in the time between $z = 2$ and the present, the universe would have been observed to experience an accelerated growth rate. Such an acceleration has been observed and is usually referred to as “Quintessence” or “Dark Energy” at work.

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Summary

1. This scalar field theory predicts a universal form for halo rotation curves, equation (4). One finds V_h acts as a scale factor for V_{rot} and m^{-1} acts as the scale factor for r when one is far from the halo edge $r/R_\ell \ll 1$. One expects to find "wiggles" of ~5% to 10% V_{rot} on the onset of the horizontal portion of the curve.

2. The cosmic fluid representing the averaged halo energy density, given by equation (8), scales as $\mathfrak{R}^{-2}$ where $\mathfrak{R}$ is the cosmological scale factor. It is possible that this explains "Quintessence."

For more details, see Chap. 3 in *Progress in Dark Matter Research*, edited by J. Val Blain (Nova Sci Pub, Hauppauge, NY, 2005). A preliminary version is given in astro-ph/0308054. The poster is available at http://astro.pas.rochester.edu/lary/halodm.pdf

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