Handout 18: Convection

- The energy generation rate in the center of the sun is ~ 16 erg/gm/s
  - Homework, sum of p-p and CNO production
- The average energy generation needed to supply the sun's luminosity is 2 erg/gm/s
  - Only about 1/8 the mass of the sun is needed to supply its luminosity
  - The Luminosity is all generated in a small core region
  - Outside of this region, $L = L_{sun}$, constant
  - If this energy is carried by radiation, this implies a T-gradient

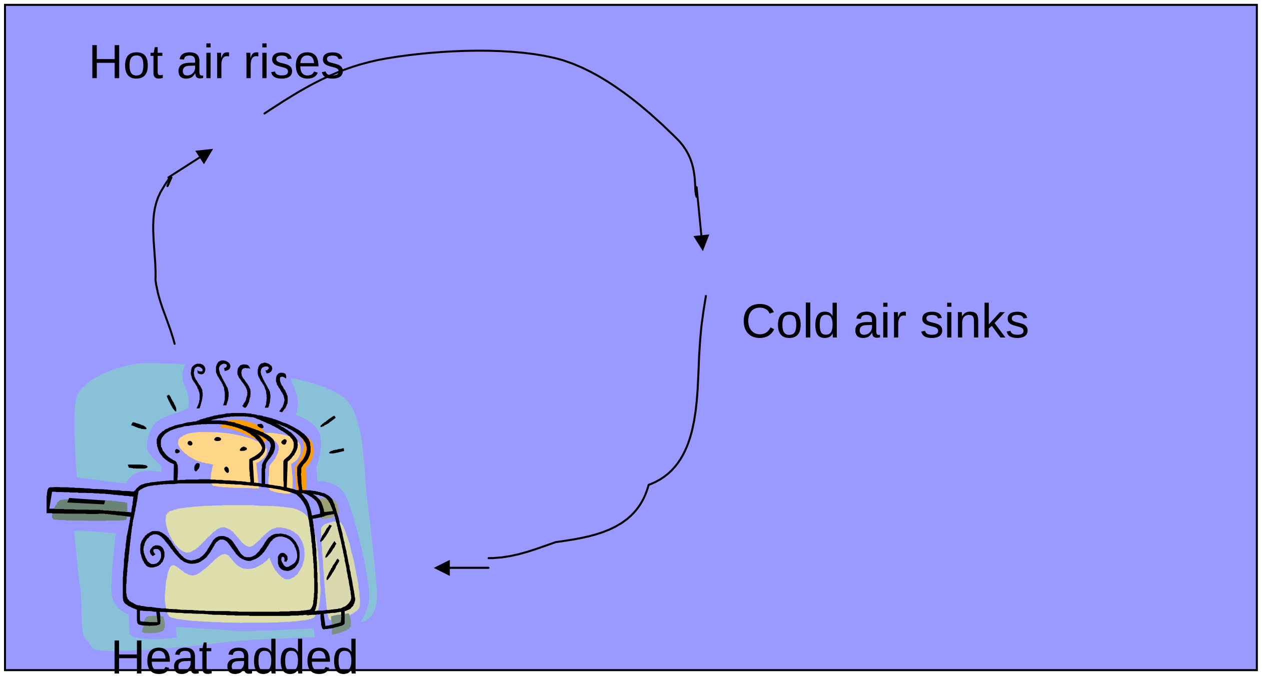
T-gradient, convection

$$\frac{d}{dr}T = \frac{-L_{rad}}{4\cdot\pi\cdot r^2}\cdot\frac{3}{4}\cdot\frac{1}{4\cdot\sigma\cdot T^3}\cdot\kappa\cdot\rho$$

- A high $\kappa\rho$ implies a high |dT/dr| T-gradient
    - Recall (fig. 9.10) as T decreases, κ increases
        - More and more ions are available to give f-bound opacity
    - Eventually, too high a T-gradient leads to convection
        - In a room, convection will occur if a parcel of gas which is moved up without adding heat (adiabatically), finds itself **less dense** than its neighbors.
            - Then it will rise further

Unstable to convection

- Heat circulation will establish the adiabatic temperature gradient



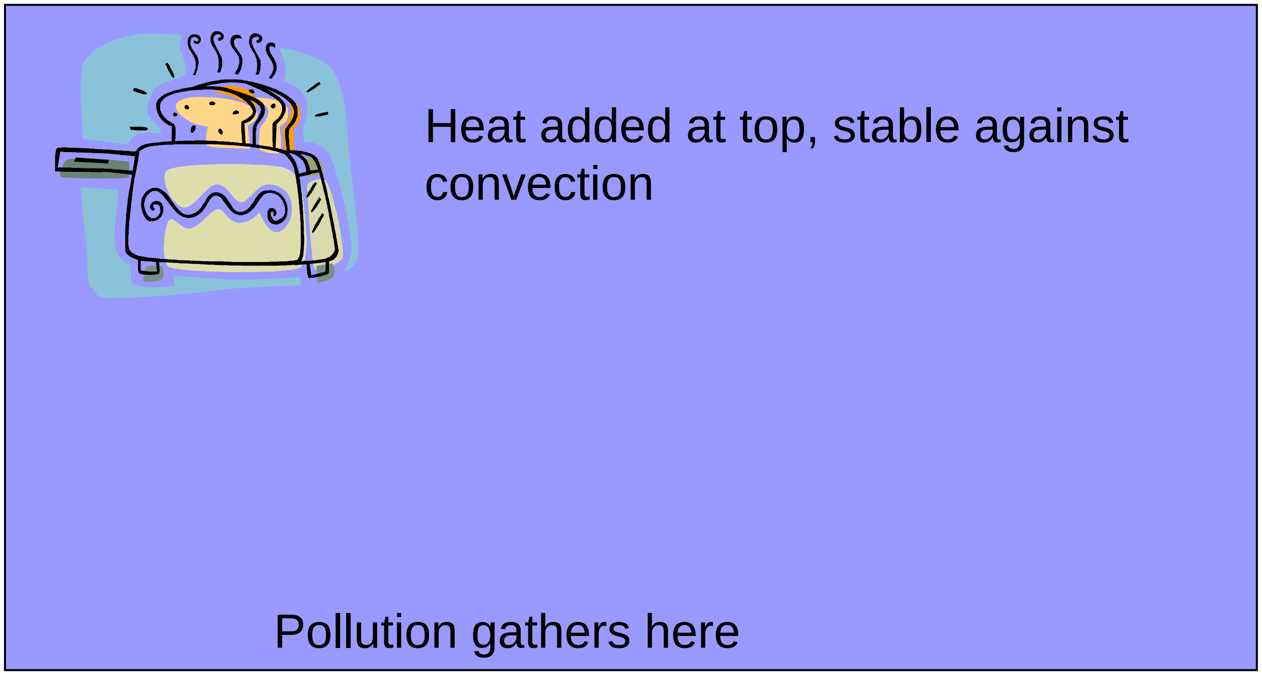
Hot air rises

Cold air sinks

Heat added

Stable against convection, heat added at the top

This situation occurs in the L.A. basin. A temperature inversion inhibits Convection, and pollution is trapped near the ground.



Heat added at top, stable against convection

Pollution gathers here

Efficiency of convection

- Convection can be very efficient *cf*. radiation
  - Even though the speed of convection ~ sound speed ~ 100 to 300 km/s
  - Speed of light 300,000 km/s
- Advantage, convection knows what direction to go in, i.e. up.
  - m.f.path ~ scale height ~ fraction of radius
    - *cf*. radiation m.f.path ~ 1 cm

Radiation *vs*. Convection

- Radiation diffusing out
  - ☐ From homework, m.f.p. ~ 1 cm, implies it takes 10,000 yr. for radiation to leak out of the sun.
    - Why is this so much shorter than the Kelvin-Helmholtz time?
- Convection, knows which way is up
  - ☐ Timescale = R/v ~ 1 hour!
- Only a small amount of convective instability can carry all the Luminosity
  - ☐ T-gradient adjusts to just a bit steeper than stable (which is the adiabatic gradient)
  - ☐ Convection “short-circuits” the radiative gradient

Condition for convection

- Convection occurs if the T-gradient exceeds the adiabatic gradient
  - $|dT/dr|_{rad} > |dT/dr|_{ad.}$ ➔ convection occurs
  - Consider a (zero-strength) balloon of gas rising **adiabatically** from r to r + Δr
    - It's new pressure, T, and density are P', T', ρ'
    - It will be in pressure equilibrium so P' = P(r + Δr)
    - If $\rho' > \rho(r + \Delta r)$, stable
      - Balloon falls back where it belongs
    - If $\rho' < \rho(r + \Delta r)$, unstable
      - Continues to rise, carrying thermal energy to higher levels

Causes of convective instability

- Steep |dT/dr| T gradient
  - Then $T(r + \Delta r) < T'$
    - But $P(r + \Delta r) = P'$, so $\rho(r + \Delta r) > \rho'$
    - Balloon is buoyant, keeps rising
      - i.e. a hot air balloon situation
- High heat capacity
  - Consider limit, infinite heat capacity
  - $T' = T(r) > T(r + \Delta r)$
    - Again $\rho(r + \Delta r) > \rho'$
    - Hot air balloon effect, keeps rising

Causes of causes of convection

- A large |dT/dr| T-gradient caused by
  - ☐ Usually, large κ
    - Sometimes, large energy generation within small radius r
- Large heat capacity when very abundant H and He are ionizing
  - ☐ Extra heat needed to
    - Ionize the atom
    - Energize the liberated e- to (3/2)kT K.E.
  - ☐ Adiabatic gradient tends to zero

Derivation of adiabatic T-gradient

- The condition for convective instability best analyzed by comparing T with P, rather than r
  - Note: as r increases, P decreases
    - i.e. dT/dP is positive
  - Thermodynamics relate P to T
    - The equation of state, and the heat capacity
- Consider first an ideal gas
  - Infinitesimal point particles rushing around
    - P = nkT
    - Only internal energy is K.E. = (3/2)kT per particle

Adiabatic T-gradient for ideal gas

- Heat capacity at constant volume c_V
  - ☐ Per particle, $c_V = (3/2)k$
    - k = Boltzmann's constant, ~ 10^{-4} eV/K
- Heat capacity at constant P $c_P > c_V$
  - ☐ Need to add PdV work done by the gas as it expands
  - ☐ Per particle, $c_P = (3/2)k + k$

Adiabatic T gradient: $\frac{P}{T} \cdot \frac{dT}{dP} = \frac{d\ln(T)}{d\ln(P)} = \frac{c_P - c_V}{c_P} = \frac{k}{c_V + k}$

For ideal gas $\frac{d\ln(T)}{d\ln(P)} = \frac{k}{\frac{3}{2} \cdot k + k} = \frac{2}{5} = 0.4$

Condition for convection

- For an ideal gas
  - If the radiative T-gradient is too steep
      - i.e. $d\ln(T)/d\ln(P) > 0.4$
    - Convection will commence
    - The T-gradient will be just a smidgin steeper than
      - $d\ln(T)/d\ln(P) = 0.4$
  - Compare average Virial gradient (1/3) to 0.4
    - Convection is lurking around
- For any gas
  - Extra heat capacity will raise c_V above (3/2)k
  - The adiabatic gradient will tend toward zero
    - Convection will commence even with a **small** $d\ln(T)/d\ln(P)$ T-gradient