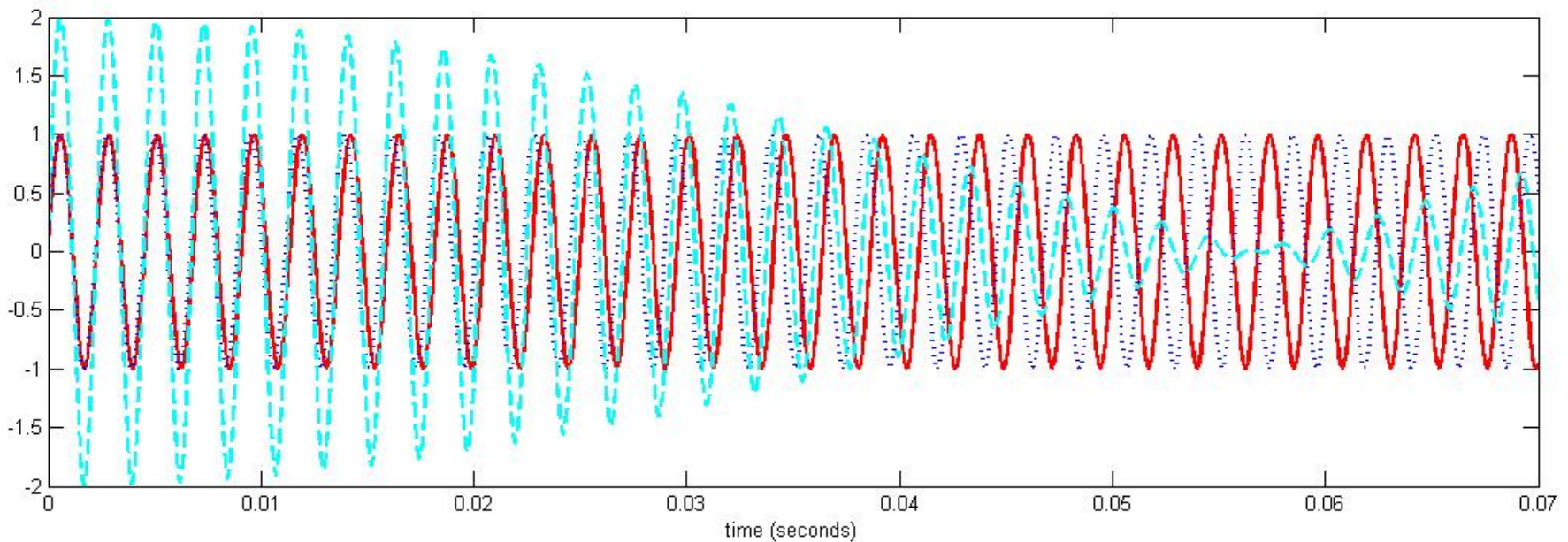# Beats and Tuning
Pitch recognition

Physics of Music PHY103

Sum of two sine waves that differ only slightly in frequency

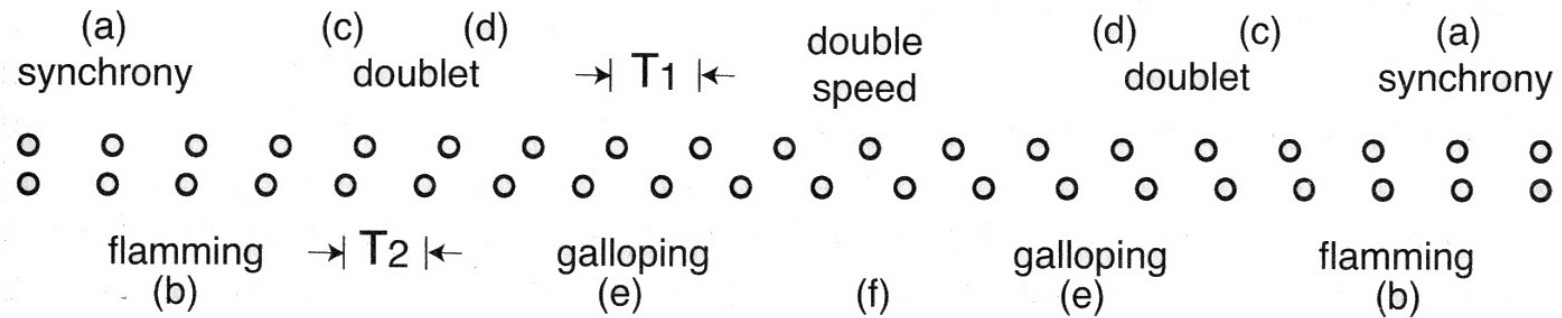Frequency *f* and 1.02*f* and their sum

Patterns emerging from Polyrhythms
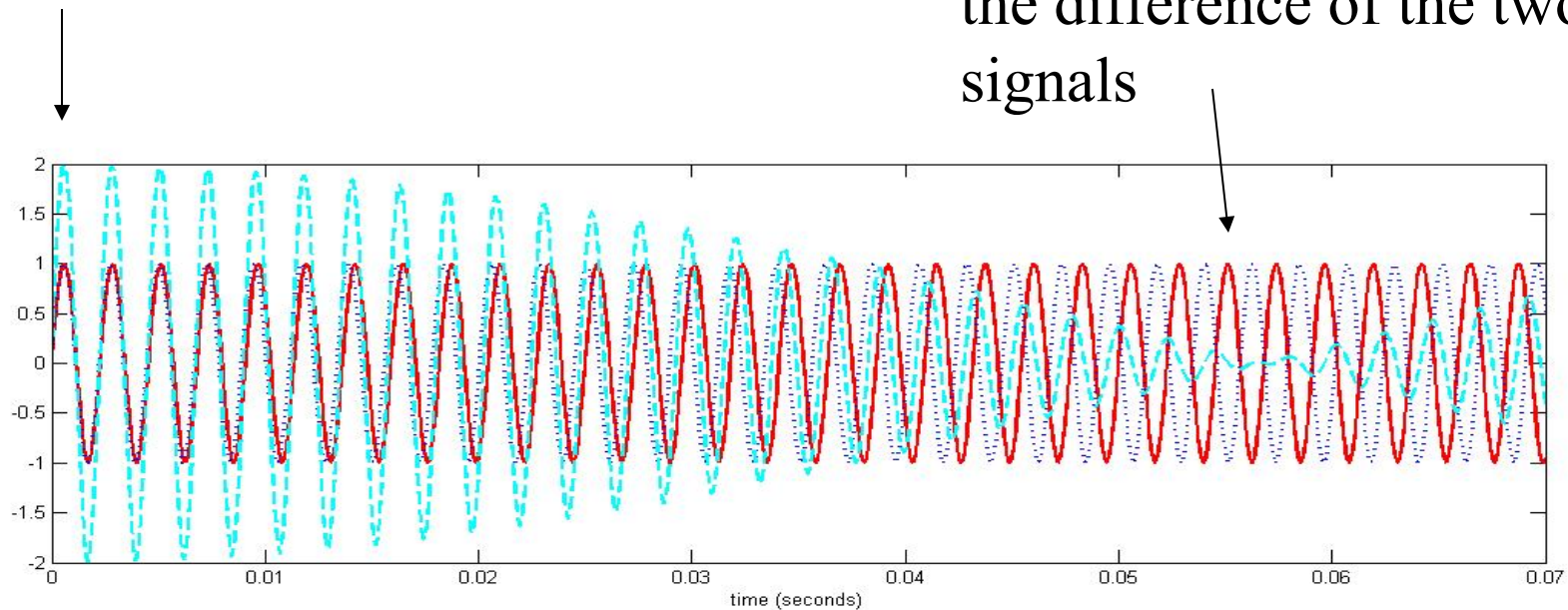

Image from W. Sethare's book Rhythm and Transform

🔊 19 vs 20 beats in 8 seconds

Amplitude variation

Complete cancellation only occurs if the two sines are the same amplitude.

Maximum amplitude is equal to the sum of each signal
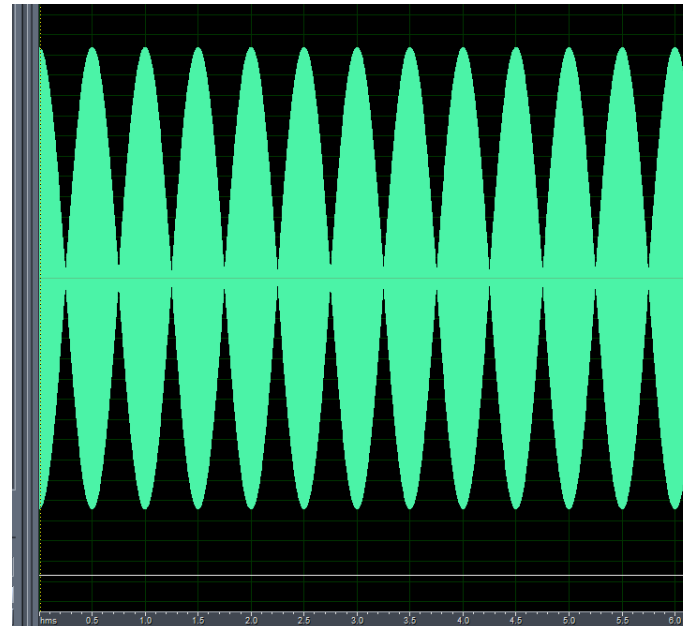
Minimum amplitude is the difference of the two signals

How far apart are the beats?

500 Hz + 502 Hz Sines added together

$P=1/f$

The next beat happens after N Periods for 500 Hz sine and after $N+1$ periods for 502Hz sine

$dT = N/500 = (N+1)/502$

First solve for N: $N[502-500]=500$ $2N=500$ $N=250$

Now find $dT = N/500 = 0.5$ seconds

Can we think of a general formula for the time between beats?

- Adding two sines with frequencies f_1, f_2
- Time to the second beat where the waves add: N Periods of first wave is $N+1$ periods for second wave.
- First solve for N

$$N/f_1 = (N+1)/f_2 \qquad N(f_2 - f_1) = f_1 \qquad N = f_1 / (f_2 - f_1)$$

- The time between beats

$$dT = N/f_1 = 1/(f_2 - f_1)$$

Beat frequency

- Time between beats $dT=1/(f_2-f_1)$
- The closer together the two frequencies, the further apart the beats.
- What is the frequency of the beats?

Beat frequency

- Time between beats $dT = 1/(f_2 - f_1)$
- The closer together the two frequencies, the further apart the beats.
- What is the frequency of the beats?

Answer: $f_2 - f_1$

Amplitude variation

- A sine wave with amplitude varying periodically (beats)

$$A(t) \cos(2\pi f t)$$

- How can we describe $A(t)$?

Amplitude variation

- A sine wave with amplitude varying periodically (beats)

$$A(t) \cos(2\pi f t)$$

- How can we describe *A(t)* ?

$$[1 + \epsilon \cos(2\pi f_b t)] \cos(2\pi f t)$$

f_b beat frequency

Law of CoSines

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos A \cos B = \frac{1}{2}\Big[\cos(A + B) + \cos(A - B)\Big]$$

An amplitude modulated signal

$$[1 + \epsilon \cos(2\pi f_b t)] \cos(2\pi f t) =$$

$$\cos(2\pi f t) + \frac{\epsilon}{2}\big[\cos(2\pi(f + f_b)t) + \cos(2\pi(f - f_b)t)\big]$$

can be written as the sum of signals
with different frequencies

Amplitude Modulation (AM)

- Slow amplitude modulation is equivalent to adding waves with nearby frequencies.

- Adding two signals close in frequency gives beats.

- These two ideas are equivalent.

Practical uses of beat frequencies

- Tuning: to hear minute changes in relative frequency (tuning of guitars, violins, pianos)

- Amplitude measurement with an adjustable reference

Tuning of fifths

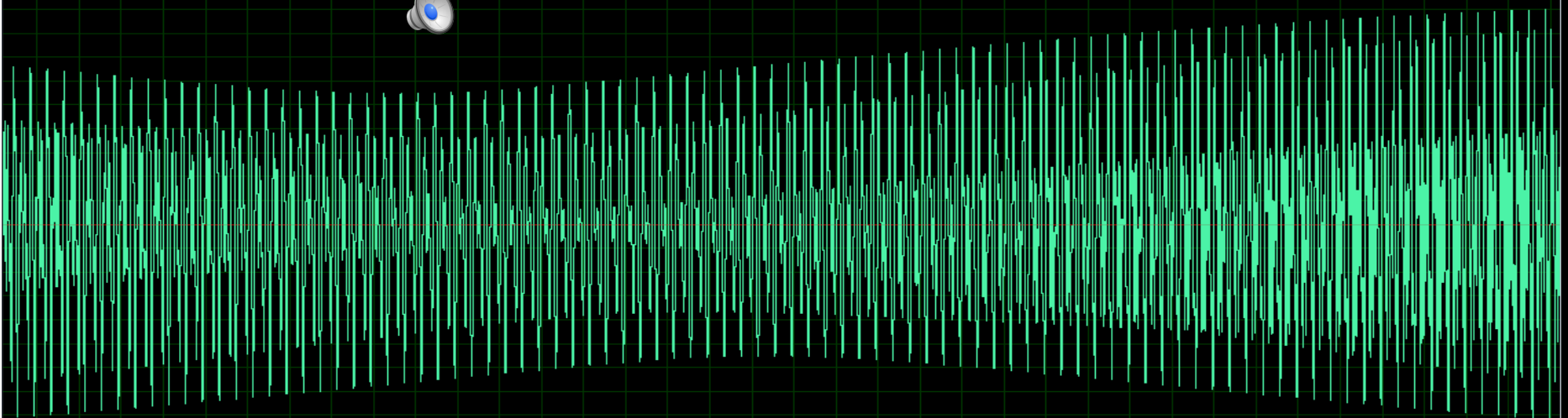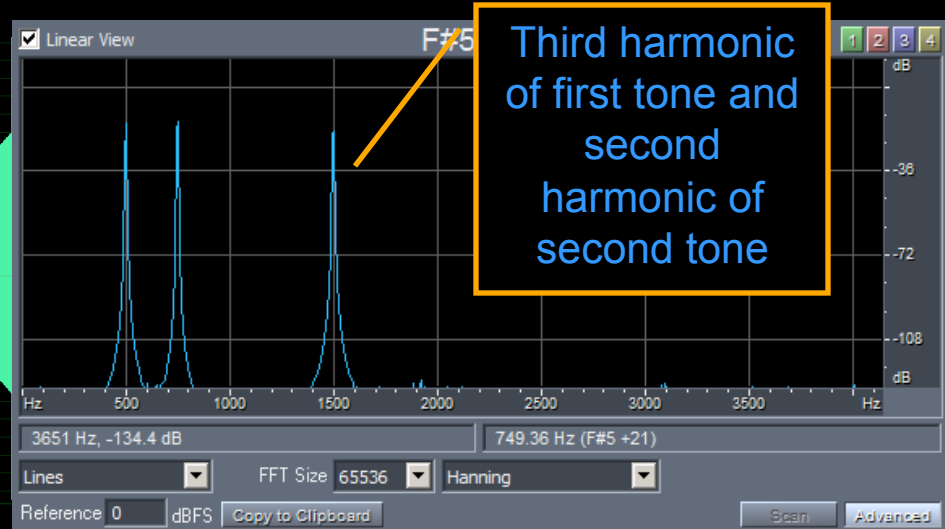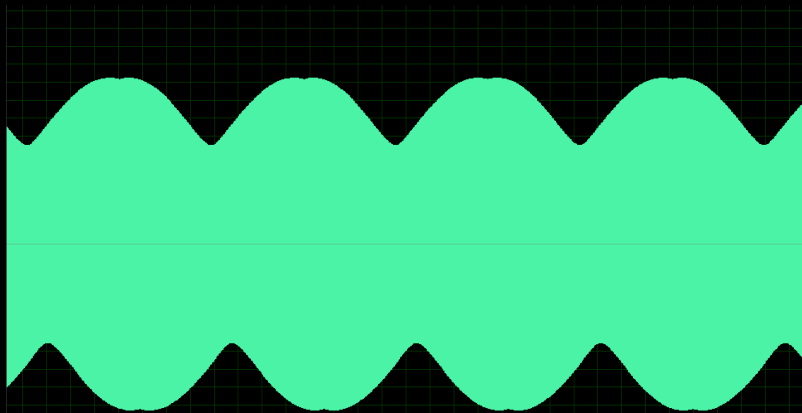Tempered fifth frequency ratio of 1.4987

Tempered fifth without harmonics

  followed by tempered fifth with harmonics

(3rd harmonic of base tone + second harmonic

  of tempered 5th tone both at 80% levels)

Tempered fifth with harmonics

Beat frequency for the tempered fifth

What beat frequency do we predict for the tempered fifth?

- If we tune to this beat frequency we can tune fifths on the piano by ear
- Fundamental $1.0 f_1$ third harmonic $3.0 f_1$
- Tempered fifth $1.4987 f_1$
  second harmonic of this note $2.9974 f_1$.
- Beat frequency is the difference:
  $$(3.0\text{-}2.9974) f_1 = 0.0026 f_1$$
- For f_1 =500Hz, the beat frequency is 0.0026 *500=1.3Hz
  For middle C (C4=262Hz) the beat frequency is 0.68Hz

# Two pure tones beginning at unison and diverging in frequency

- When are beats loud?

- When do you perceive 2 notes rather than one with beats?

- Do you hear a lower pitch?

Butler example 3.10

Periodic Waves

- Both the triangle and square wave cross zero at the beginning and end of the interval.
- We can repeat the signal

Is "Periodic"

- Periodic waves can be decomposed into a sum of harmonics or sine waves with **frequencies** that are **multiples** of the biggest one that fits in the interval.
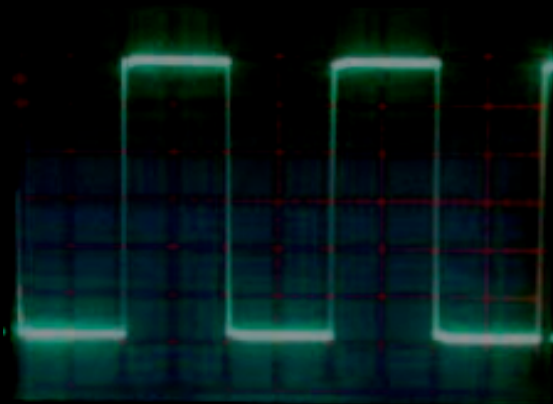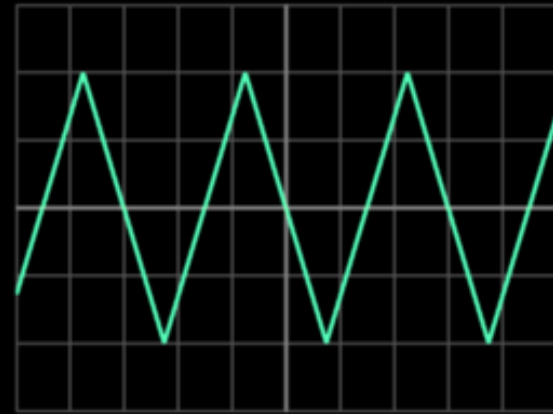
Sum of harmonics

- Also known as the **Fourier series**
- Is a sum of sine and cosine waves which have frequencies *f, 2f, 3f, 4f, 5f, ...*
- Any **periodic** wave can be decomposed in a Fourier series

Complex tones

- Even though the piano is nearly harmonic, the overtones aren't exactly integer multiples of the fundamental.

- Signal is not periodic.

- If you had a periodic signal you would not get beats

- Perhaps richer in harmony when the spectrum is not periodic

- Chorus effect

Pitch discrimination and perception

- Beats are used to tune instruments
- But we probably would not need such exquisite tuning if our ears were not very good at measuring pitch.

Pitch discrimination of pure tones

- **DLF**: <u>D</u>ifference <u>L</u>imen for <u>F</u>requency two tones played, randomly with different frequencies. Subject must identify if they differ

- **FMDL**: <u>F</u>requency <u>M</u>odulation <u>D</u>etection <u>L</u>imen two tones played one is modulated in frequency, subject must identify the one modulated
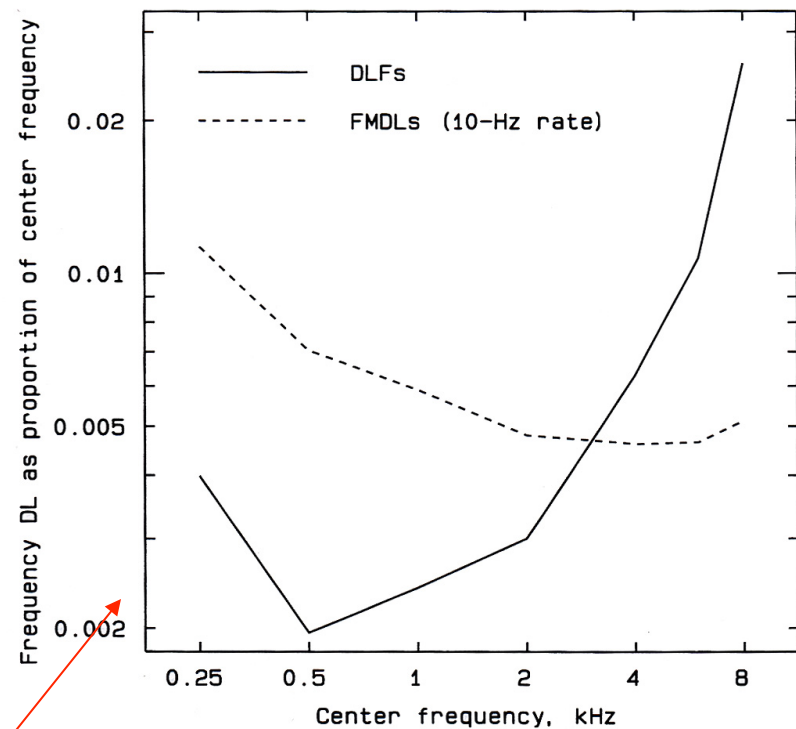
Pitch Perception

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FIGURE 6.2   DLFs and FMDLs for a 10-Hz modulation rate, measured in the same subjects and plotted as a proportion of center frequency. Data from Sek and Moore (1995).

Note the accuracy!

Pitch perception vs masking

- Note our ability to detect pitch changes is at the level of 0.25% well below the width of the critical band.

- This precision requires **active** hair/basal membrane interactions in the cochlea

Pitch Perception
Complex Tones

- Virtual pitch. Pitch is recognized even though the **fundamental is missing**
- ASAdemo20

- Pitch is recognized in the presence of huge operatic vibratos too

Shift of virtual pitch

- A tone with three inexact harmonics will sound as if it had a shifted fundamental
- Tones with shifted harmonics but missing fundamentals can be matched in perceived pitch by tones with exact harmonics

ASA demo 21

Octave matching without harmonics

- ASA demo 15

- Count the pitches! Starting at 985Hz in steps of 5Hz. 4th one should be the octave.

- Which tone is the best perceived octave of 500Hz

- Often people choose a note that is sharper than an exact octave

Theories of pitch perception

- Place only theory: pitch is determined by the region excited on the basal membrane. Excitations on the basal membrane are sorted by frequency.

- Temporal pitch perception. More nerve pulses occur at the maximum of the sound wave. "Phase locking." Pulse distribution in time depends on frequency. Observed in experiments but only at lower frequencies (below about 5kHz).

Masking Spectra and Virtual pitch

- The inability of low frequency noise to mask the virtual pitch points out the inadequacy of the place only theory of pitch perception

- ASADemo 22 a chime melody is played with low-pass then high-pass noise.

Terms and Ideas

- Beat frequency
- Amplitude modulation
- Using beats to tune
- Pitch discrimination and perception

Reading

- Butler Chapter 3 on Pitch
- Moore Chapter 6 on Pitch Perception
- Berg and Stork Chap 2 on Waves and Sound
- Hopkins Chap 10 on Chorusing and Beating effects