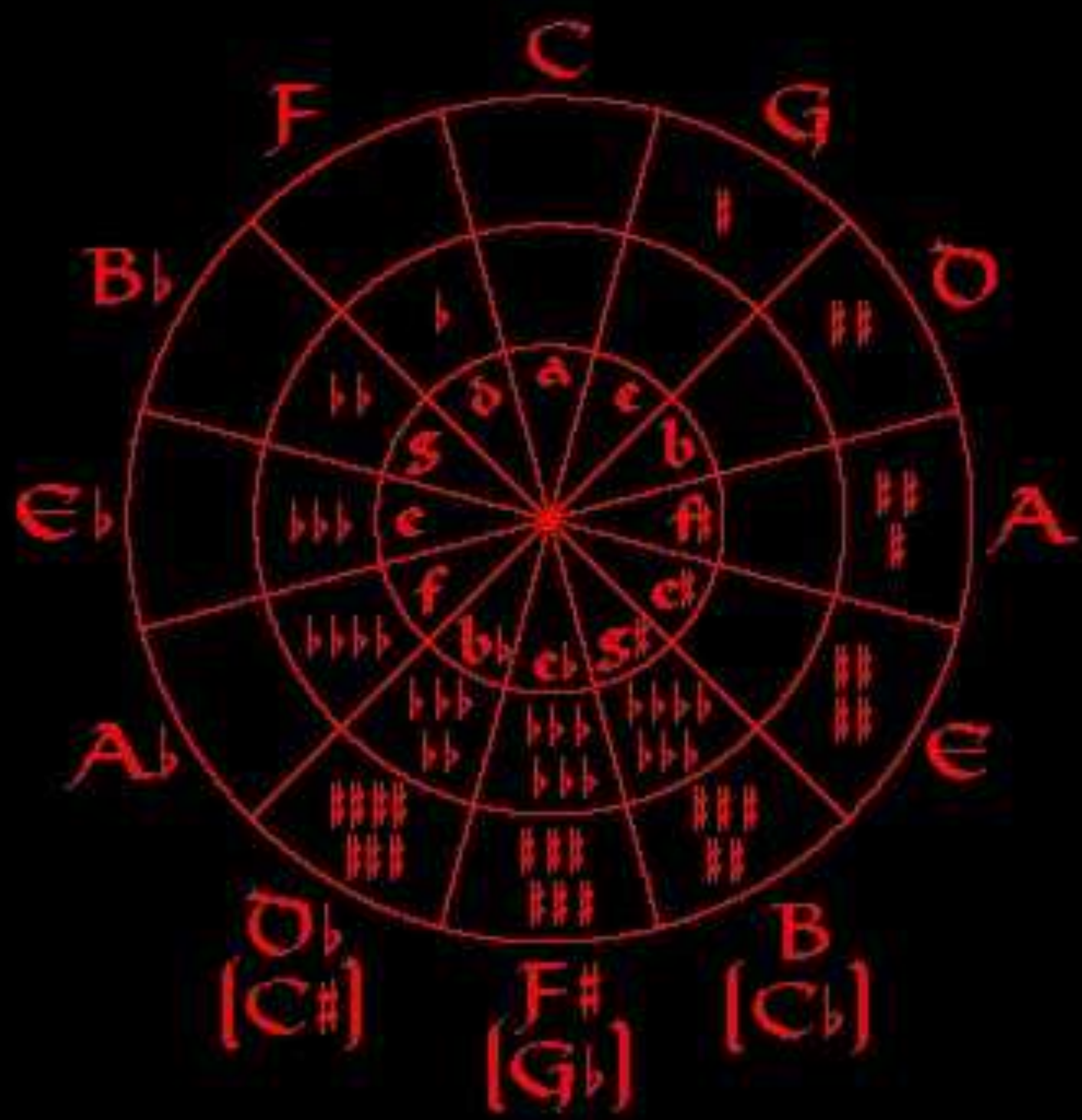
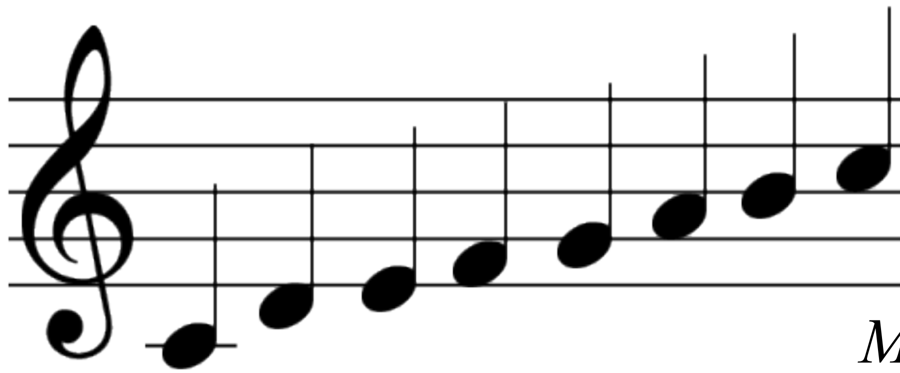


Scales

Physics
of Music
PHY103

Image from www.guitargrimoire.com/

Diatonic scale



Major scale

1	2	3	4	5	6	7	1
do	**re**	**mi**	**fa**	**sol**	**la**	**ti**	**do**

W	W	H	W	W	W	H

How universal is this scale?

9,000 Year Old

Chinese Flutes

JUZHONG ZHANG et al. Nature 1999

Excavations at the nearly Neolithic site of Jiahu in Henan Province China have found the earliest completely playable tightly dated multinote musical instruments

One of these flutes can be played: check the file K-9KChineseFlutes.ram

Neanderthal flute

- This bear bone flute, found in Slovenia in 1995, is believed to be about 50,000 years old.

Differing hole spacing suggest a scale with whole and half note spacings

Overtones of the string

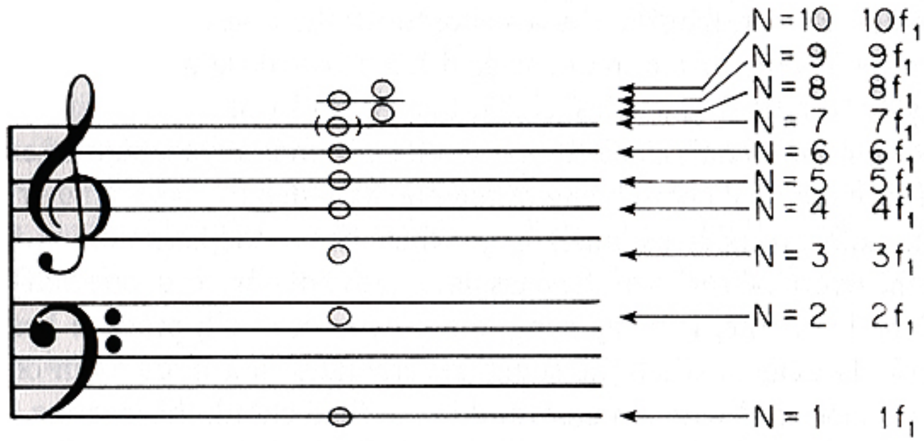


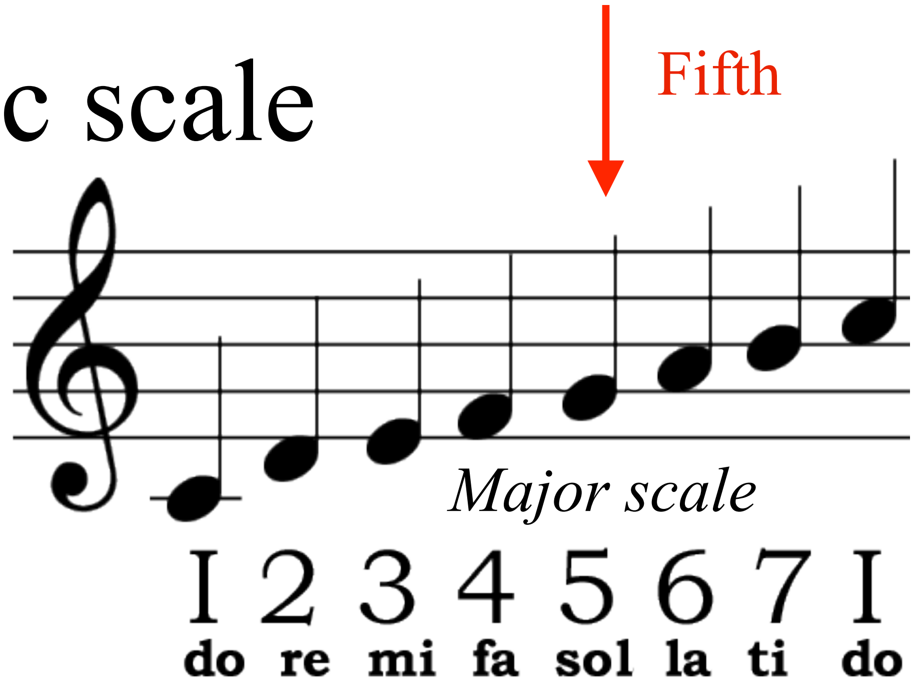
Figure 3–12 Notes on the musical staff having frequencies closest to the notes in the overtone series of G_2; each note is labeled by its harmonic number and the frequency of the corresponding harmonic.

TABLE 3–3 MUSICAL INTERVALS BETWEEN THE FUNDAMENTAL AND OTHER NOTES OF THE OVERTONE SERIES

N	f	Interval
1	f_1	Unison
2	$2f_1$	One octave
3	$3f_1$	One octave + one perfect fifth
4	$4f_1$	Two octaves
5	$5f_1$	Two octaves + one major third
6	$6f_1$	Two octaves + one perfect fifth
7	$7f_1$	Two octaves + one minor seventh
8	$8f_1$	Three octaves

Diatonic scale

- Notes that are octaves apart are considered the same note. x2 in frequency
- An octave+ 1fifth is the 3rd harmonic of a string, made by plucking at (1/3) of the string
- If f is the frequency of the fundamental then the third harmonic is $3f$



W	W	H	W	W	W	H

- Drop this down an octave (divide by 2) and we find that a fifth should have a frequency of $3/2f$

Pythagorus and the circle of fifths

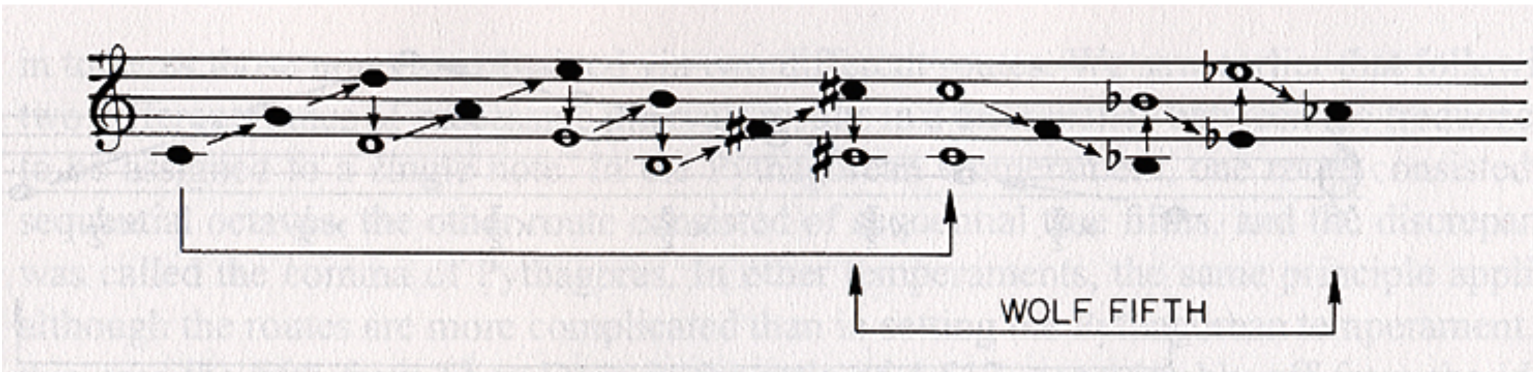
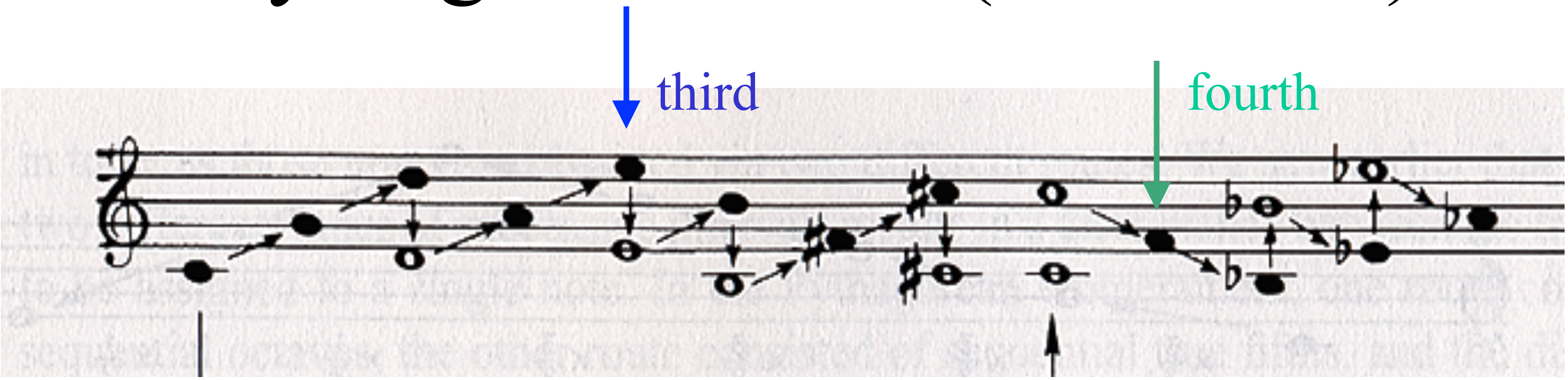


Figure 9–4 Sequence for setting the Pythagorean temperament. Blackened notes are the first tuning of a note with that letter name.

Image from Berg + Stork

Every time you go up a fifth you multiply by 3/2.
Every time you go down an octave you divide by 2.

Pythagorean scale (continued)



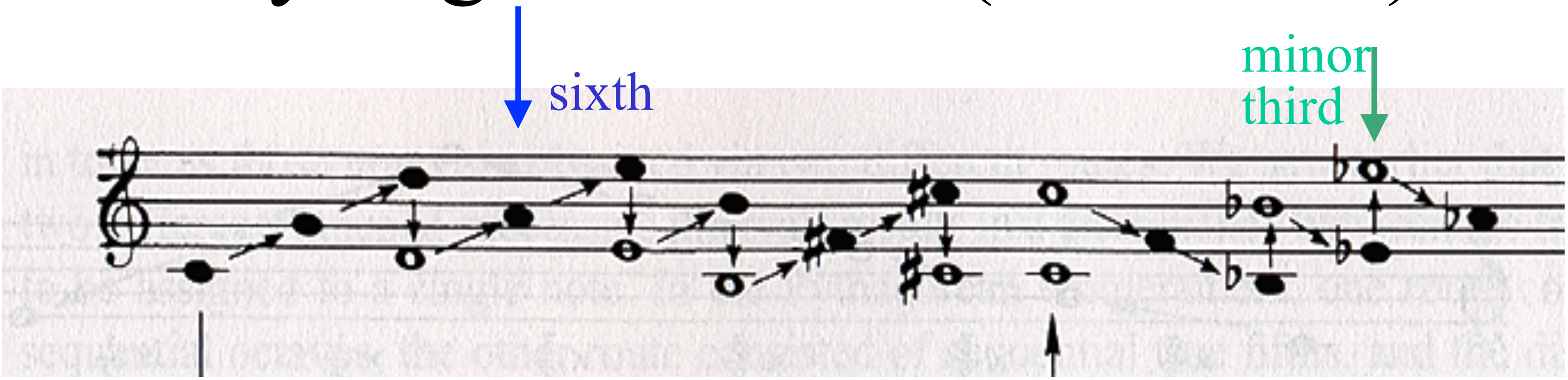
The major third is up a fifth 4 times and down an octave twice

$$\times\left(\frac{3}{2}\right)^4\times\left(\frac{1}{2}\right)^2=\frac{3^4}{2^6}=1.2656$$

The fourth is up an octave and down a fifth

$$\times 2\times\left(\frac{2}{3}\right)=\frac{4}{3}=1.3333$$

Pythagorean scale (continued)



The sixth is up a fifth 3 times and down an octave once

$$\times\left(\frac{3}{2}\right)^{3}\times\left(\frac{1}{2}\right)^{1}=\frac{3^{3}}{2^{4}}=1.6875$$

The minor third is down a fifth three times and up and octave twice.

$$\times 2^{2}\times\left(\frac{2}{3}\right)^{3}=\frac{2^{5}}{3^{3}}=1.1852$$

Pythagorean scale (continued)

		third	fourth	fifth	sixth		octave
1		$\frac{3^4}{2^6}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{3^3}{2^4}$		2

The musical scale related to pure fractions!
Mathematical beauty is found in music.

The circle doesn't exactly close

What happens if you go up by a fifth 12 times? The circle of fifths leads us to expect that we will get back to the same note.

$(3/2)^{12}$=129.746337890625

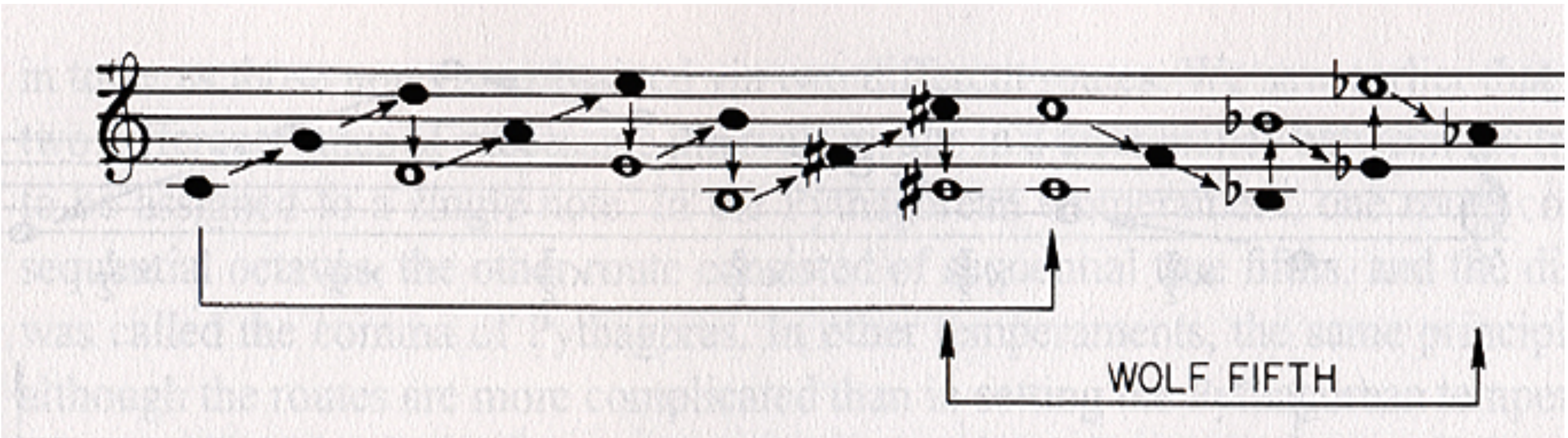
An octave is 2 times the base frequency

7 octaves above is 2^7 times the base frequency.

2^7=128.0000

However $2^7 \neq (3/2)^{12}$

Getting all the notes



One fifth has to be bad. The Wolf fifth.

So the subdominant chords can be played the bad fifth is placed between C# and A ♭

Circle of fifths

Guides harmonic structure and key signatures for baroque, romantic and much folk music.

Keeping chords in tune in one key is sufficient for much of baroque and folk music.

Equivalency over the octave

We associate notes an octave apart as the same note

It is difficult to determine the octave of a note. Some people sing an octave off by mistake.

Other animals (birds) with not recognize songs played in a different octave

Perfect fifths and thirds

What's so perfect about the perfect fifth?

- We started with a **harmonic** $3f$ and then took it down an octave to $3f/2$ Frequency ratio 1:3/2

Is the Pythagorean third perfect?

- Fifth **harmonic** $5f$ is 2octaves +a major third
- Take this down two octaves and we find a major third should be $5f/4 = 1.25f$.

  Frequency ratio 1:5/4
- However the Pythagorean major third is $1{:}(3^4/2^6)$ =1.2656

What do they sound like?

Perfect fifth with frequency ratio 1:3/2
Wolf fifth
(between C# and A♭)
1:218/311=1.4798

Perfect third
1:5/4
Pythagorean third
1:34/26=1.2656

I used the *generate* tones function in the old Audition

Sequence:

Sum of 2 sines prefect, interval followed by Pythagorean interval

Then sum of 2 sets of 5 harmonics, perfect inteval followed by Pythagorean interval

What do the waveforms look like?

One of these is a perfect third, the other is the Pythagorean third. Which one is which?

Perfect intervals and periodic waveforms

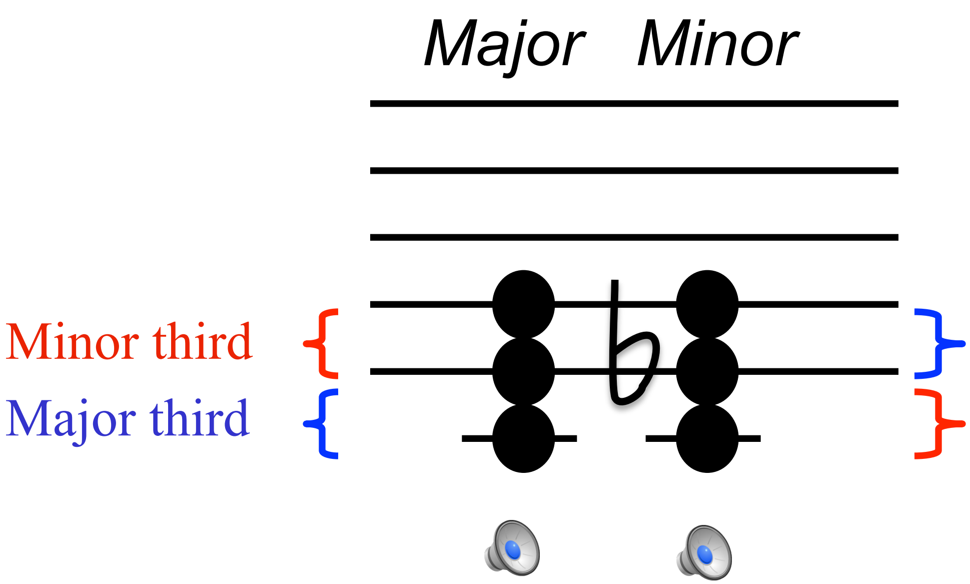
- A sum of perfect harmonics adds to a periodic wave --- for sum of frequencies that are integer ratios - Fourier series
- However 5/4 (prefect third) is not an integer times 1 --- so why does the sum of two frequencies in the ratio 1:5/4 produce a periodic waveform?

Major and minor chords

Perfect **major** third and fifth

Frequency ratios 1:5/4:3/2

For the triad



Minor chord: set the major third.

Divide the G frequency by 5/4 so that the G and E♭ give a perfect third.

3/2*4/5=6/5

Frequency ratios 1:6/5:3/2 for the triad

Triads

- Major triad with perfect major third and fifth 1:5/4:3/2 The minor third in the triad has ratio 3/2 divided by 5/4 = 6/5 and so is perfect or true.
- However other thirds and fifths in the scale will not be true.

Just temperament

More than one tuning system which use as many perfect ratios as possible for one diatonic scale.

TABLE 9–6 DERIVATION OF FREQUENCY RATIOS FOR THE NOTES OF THE JUST MAJOR SCALE

C_4	$= 1.000$ (start)	$= 1.0000$
$C_4^\sharp$	$= \frac{5}{4} A_3 = \frac{5}{4} \times \left(\frac{5}{4} F_3\right) = \frac{5}{4} \times \frac{5}{4} \times \left(\frac{2}{3} C_4\right) = \frac{25}{34} C_4$	$= 1.0417$
D_4	$= \frac{1}{2} D_5 = \frac{1}{2} \times \left(\frac{3}{2} G_4\right) = \frac{1}{2} \times \frac{3}{2} \times \left(\frac{3}{2} C_4\right) = \frac{9}{8} C_4$	$= 1.1250$
$E_4^\flat$	$= \frac{4}{5} G_4 = \frac{4}{5} \times \left(\frac{3}{2} C_4\right) = \frac{6}{5} C_4$	$= 1.2000$
E_4	$= \frac{5}{4} C_4$	$= 1.2500$
F_4	$= 2F_3 = 2 \times \left(\frac{2}{3} C_4\right) = \frac{4}{3} C_4$	$= 1.3333$
$F_4^\sharp$	$= \frac{5}{4} D_4 = \frac{5}{4} \times \left(\frac{9}{8} C_4\right) = \frac{45}{32} C_4$	$= 1.4063$
G_4	$= \frac{3}{2} C_4$	$= 1.5000$
$A_4^\flat$	$= \frac{4}{5} C_5 = \frac{4}{5} \times (2\ C_4) = \frac{8}{5} C_4$	$= 1.6000$
A_4	$= \frac{5}{4} F_4 = \frac{5}{4} \times \left(\frac{4}{3} C_4\right) = \frac{5}{3} C_4$	$= 1.6667$
$B_4^\flat$	$= \frac{4}{5} D_5 = \frac{4}{5} \times \left(\frac{3}{2} G_4\right) = \frac{4}{5} \times \frac{3}{2} \times \left(\frac{3}{2} C_4\right) = \frac{9}{5} C_4$	$= 1.8000$
B_4	$= \frac{5}{4} G_4 = \frac{5}{4} \times \left(\frac{3}{2} C_4\right) = \frac{15}{8} C_4$	$= 1.8750$
C_5	$= 2C_4$	$= 2.0000$

Image From Berg and Stork

The Just Scale

$$\underbrace{1 \quad 9/8}_{T1} \quad \underbrace{\quad 5/4}_{T2} \quad \underbrace{\quad 4/3}_{S} \quad \underbrace{\quad 3/2}_{T1} \quad \underbrace{\quad 5/3}_{T2} \quad \underbrace{\quad 15/8}_{T2} \quad \underbrace{\quad 2}_{S}$$

Two different whole tones

T1	=	9/8	=	1.125
T2	=	10/9	=	1.111

One semitone

S	=	16/15	=	1.067

Listening example (from Butler)

a) An ascending major scale beginning on C4, **equal temperament** followed by the chord progression I-IV-V7 in Cmajor and in F# major.

b) The same scale but with **Pythagorean tuning**, followed by the same chord progression

c) The same scale and chords, **quarter comma meantone temperament** (errors distributed in a few tones)

d) Just intonation

Meantone temperament

In general, a meantone is constructed the same way as Pythagorean tuning, as a chain of perfect fifths, but in a meantone, each fifth is narrowed in order to make the other intervals like the major third closer to their ideal or perfect just ratios.

Quarter-comma meantone is the most well known type of meantone temperament, and the term *meantone temperament* is often used to refer to it specifically.

Uses exact 5:4s for major thirds, but flattens each of the fifths by a quarter of a syntonic comma where a syntonic comma is equal to the frequency ratio 81:80, or around 21.51 cents.

Just intonation

- Desirable if only a few instruments are playing
- You want that ringing baroque quality
- You are only going to play in one or two keys.

Equal temperament

Desirable properties of this scale:

- Sounds okay for all triads for all keys.
- Allows instruments in different keys to play together more easily

Give up perfect fifths and thirds for a scale that is good for any key

Makes rapid key changes possible. Led to the development of a different type of music.

12 tones

To make a tempered scale we need to fit 12 notes within an octave -- between *f* and *2f*

How can we do this?

1) Linearly $f(1+n/12)$? where *n* are integers

C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
1	13/12	7/6	5/4	4/3	17/12	3/2	19/12	5/3	7/4	11/6	23/12	2

Problems with this: 1) it sounds bad, 2) Makes no sense in the next octave where interval would have to be $2f/12$

Equal Temperament

How else can we make 12 even divisions between 1 and 2?

2) With multiplicative factors: A^n where A is a factor and n are integers → Even in log space

A^0=1.0 and we want A^{12}=2.0.

Solve this (take the 12th root of both sides of the equation). We find $A = 2^{1/12}$=1.05946

C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
1	1.059	1.1225	1.1893	1.2601	1.3351	1.4145	1.4987	1.5978	1.6823	1.7824	1.8885	2

Not too far from a perfect third

Nearly a perfect fourth

Pretty close to a perfect fifth

Equally spaced by multiplicative factors

The closer to the bridge the closer the frets

How big is $2^{1/12}$?

Listening examples, log and linear spacings

ASADemo18

- 8 note diatonic scale
- 13 note chromatic scale

Cents

The cents scale

- Cents=1200 $\log_2(f/f_1)$
- If $f/f_1 = 2$, then
- The octave = 1200 cents
- The interval between two scale notes that are a half step apart is 100 cents

Turning notes and cents into frequencies

What is the frequency of C#4 +25cents

- C#4 is 277.18Hz
- $2^{25/1200}$=1.0145453
- 277.18 x $2^{25/1200}$ = 281.21 Hz

What if I want C#4 -25 cents?

Equal temperament

- Intervals are NOT exact integer ratios.
- What does it mean to add two sine waves that are not integer multiples of each other?

------ What difference is there in the waveform?

Attempts at new scales
Harry Partch's tuning

- American composer Harry Partch (1901-1974) based all of his mature musical compositions on a musical scale tuned in just intonation and based on the 11-limit tonality diamond, played on instruments he designed and built himself to be played in that tuning. The number of notes varied for a time from about 29 to 55, until he finally settled on a scale of 43 tones to the octave (his identity interval, which Partch always referred to by its frequency ratio 2/1).
- Partch's tuning is often called "extended just intonation", because traditional just intonation used ratios with only the prime-factors 2, 3, and 5, whereas Partch added 7 and 11 to these.

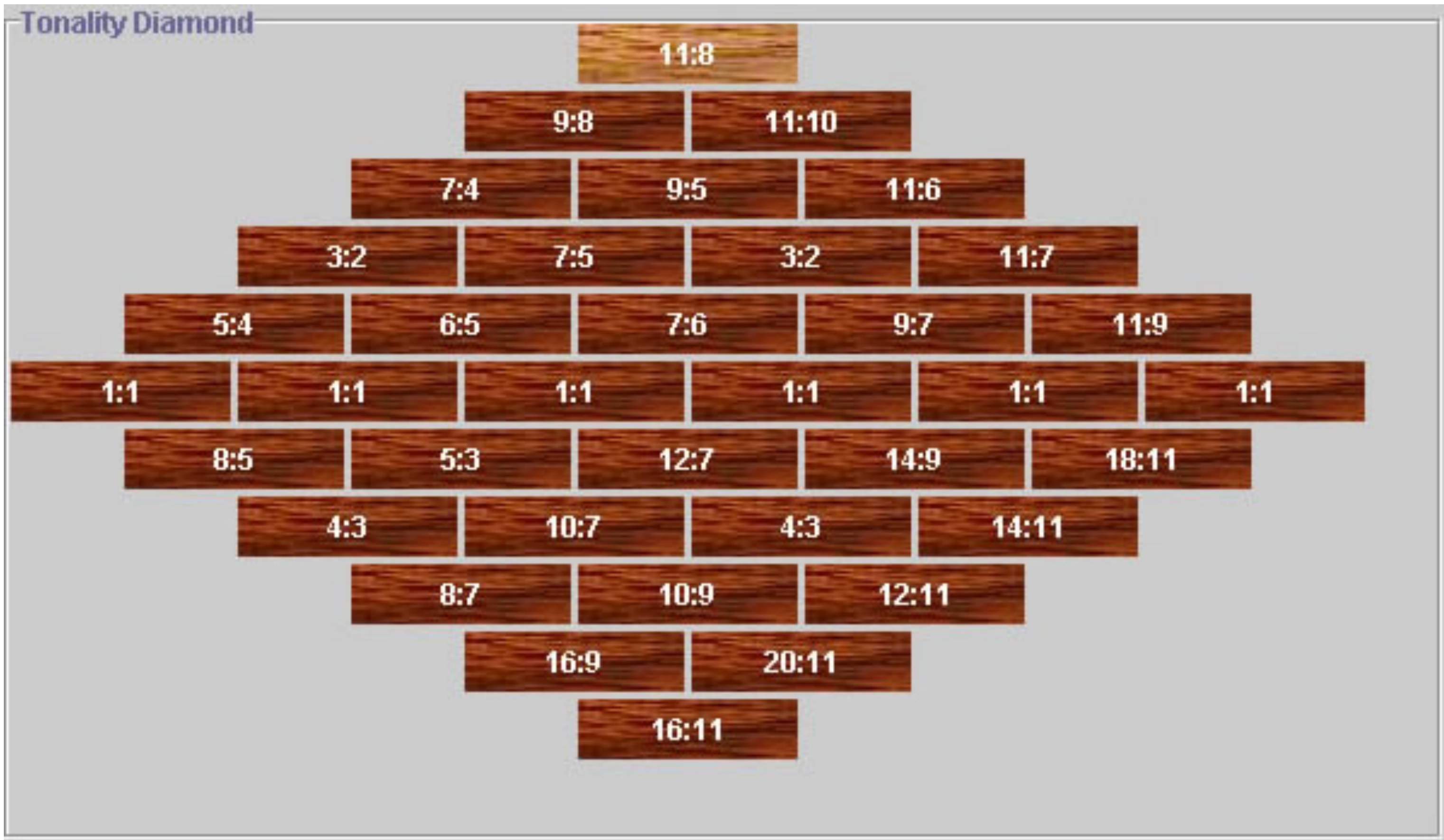
As described by
Joe Monzo

Harry Partch 43 tone Just Intonation scale

Note degree	Interval	Value in cents	Interval Name
1	1/1	0	unison
2	81/80	21.50629	syntonic comma
3	33/32	53.27296	undecimal comma
4	21/20	84.46723	minor semitone
5	16/15	111.7313	minor diatonic semitone
6	12/11	150.6371	3/4-tone, undecimal neutral second
7	11/10	165.0043	4/5-tone
8	10/9	182.4038	minor whole tone
9	9/8	203.9100	major whole tone
10	8/7	231.1741	septimal whole tone
11	7/6	266.8710	septimal minor third
12	32/27	294.1351	Pythagorean minor third
13	6/5	315.6414	minor third
14	11/9	347.4080	undecimal neutral third
15	5/4	386.3139	major third
16	14/11	417.5081	-
17	9/7	435.0843	septimal major third
18	21/16	470.7811	narrow fourth
19	4/3	498.0452	perfect fourth
20	27/20	519.5515	acute fourth
21	11/8	551.3181	harmonic augmented fourth
22	7/5	582.5125	septimal tritone
23	10/7	617.4880	Euler's tritone
24	16/11	648.6823	harmonic diminished fifth
25	40/27	680.4490	narrow fifth
26	3/2	701.9553	perfect fifth

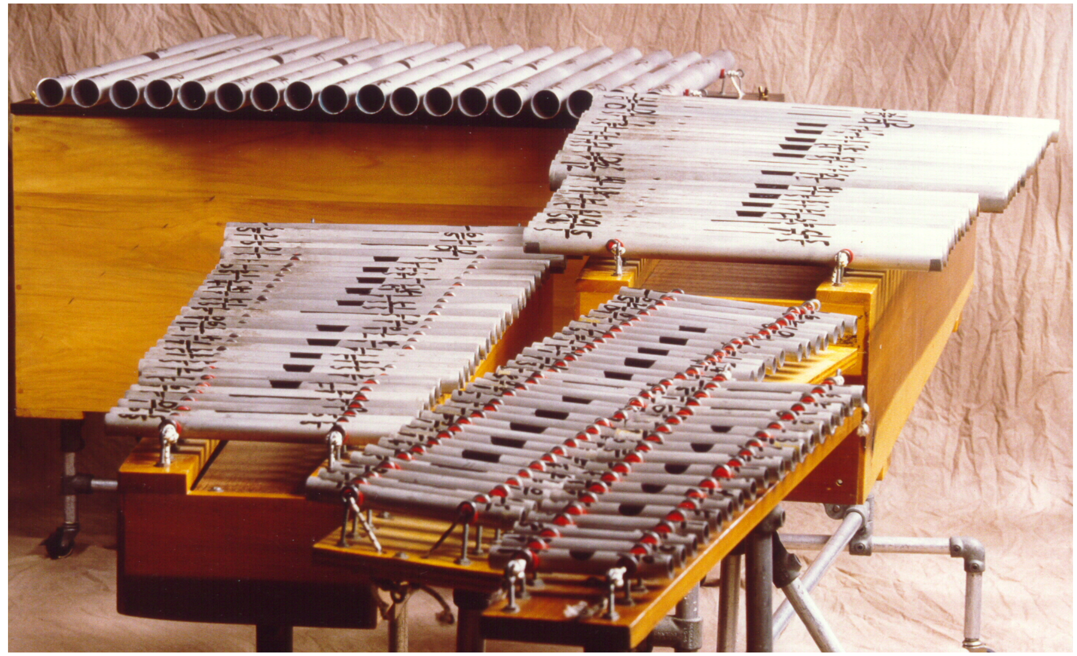
from http://www.microtonal-synthesis.com

Partch's tonality diamond



- Check out a version on this web site that lets you listen to the notes http://prodgers13.home.comcast.net/Sub_Pages/DiamondMarimba.html

Partch's and Partch-era compositions



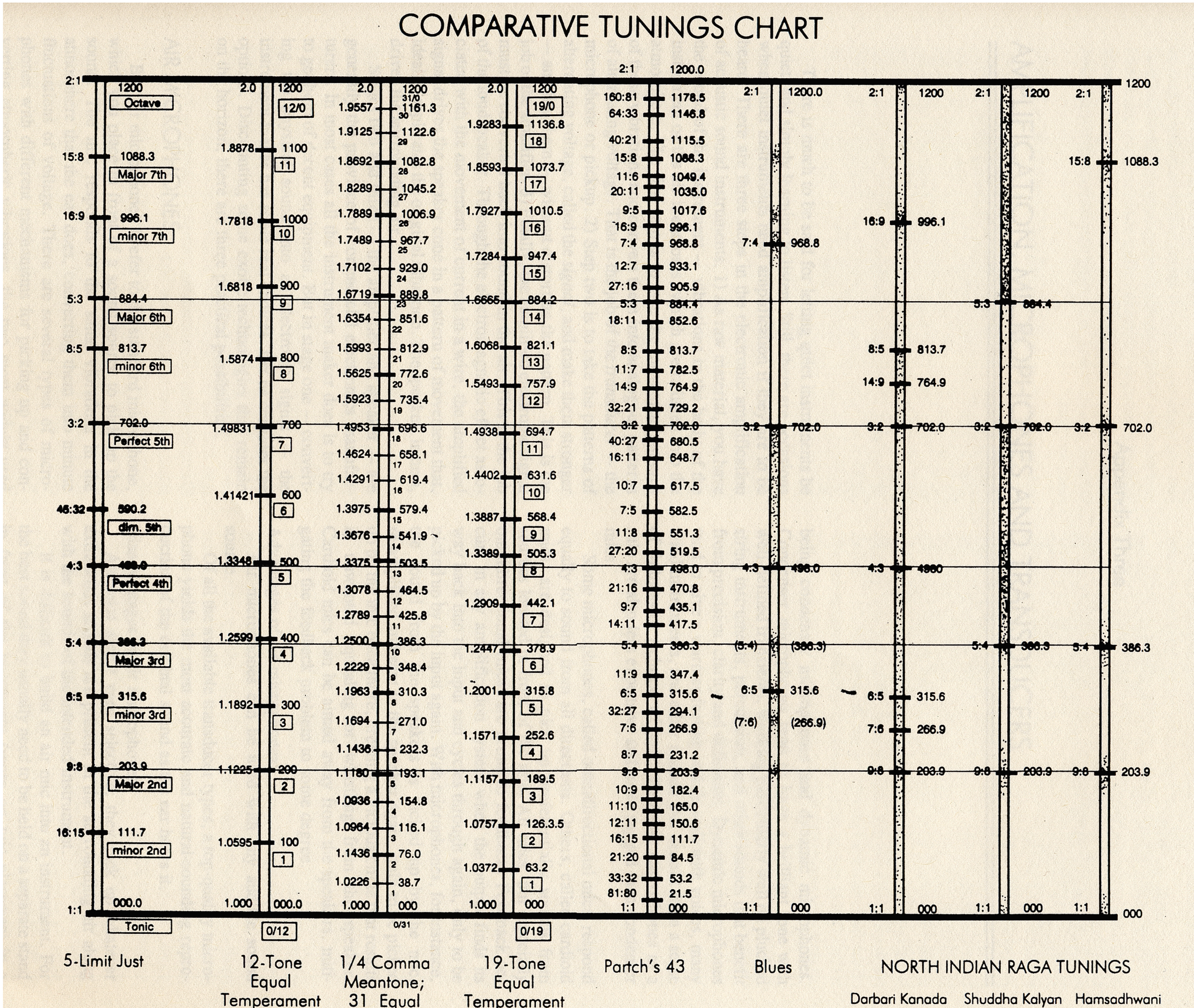
- Two Studies on Ancient Greek Scales
- `Columbus' by Dean Drummond

  For flute (Stefi Starrin)+zoomoozophone (Dean Drummond, Dominic Donato, James Pugliese) -- percussion instrument with 31 tones

From the CD Microtonal Works by Partch, Cage, LaBarbara, Drummond

Microtonal music

- This is a piece for flute, clarinet, finger piano, guitar, and percussion. It's based on the 10:11:12:13:14:16:18:20 septany.
- By Prent Rodgers -for the downridders


COMPARATIVE TUNINGS CHART
Octave
Major 7th
minor 7th
Major 6th
minor 6th
Perfect 5th
dim. 5th
Perfect 4th
Major 3rd
minor 3rd
Major 2nd
minor 2nd
Tonic
5-Limit Just
12-Tone Equal Temperament
1/4 Comma Meantone; 31 Equal
19-Tone Equal Temperament
Partch's 43
Blues
NORTH INDIAN RAGA TUNINGS
Darbari Kanada
Shuddha Kalyan
Hamsadhwani

From Hopkin

19 tone tempered scale

"What happens when you play simultaneously in different tunings? Each note in this 19-tet melody is ``harmonized" by a note from 12-tet, resulting in some unusual nonharmonic sound textures. The distinction between ``timbre" and ``harmony" becomes confused, although the piece is by no means confusing."

- *Incidence Coincidence* **by William A. Sethares**
- Played with an astral guitar built with 19 tones in each octave. *Sympathetic metaphor*

Adaptive tunings

- As each new note sounds, its pitch (and that of all currently sounding notes) is adjusted microtonally (based on its spectrum) to maximize consonance. The adaptation causes interesting glides and microtonal pitch adjustments in a perceptually sensible fashion.
- Three Ears by **William A. Sethares**

Gamelan tones

Cook demo 68 (From the book edited by Perry Cook)

- Gamelon music example
- Sample tune in Bali tuning
- probe tone
- Repeat of tune
- another probe tone

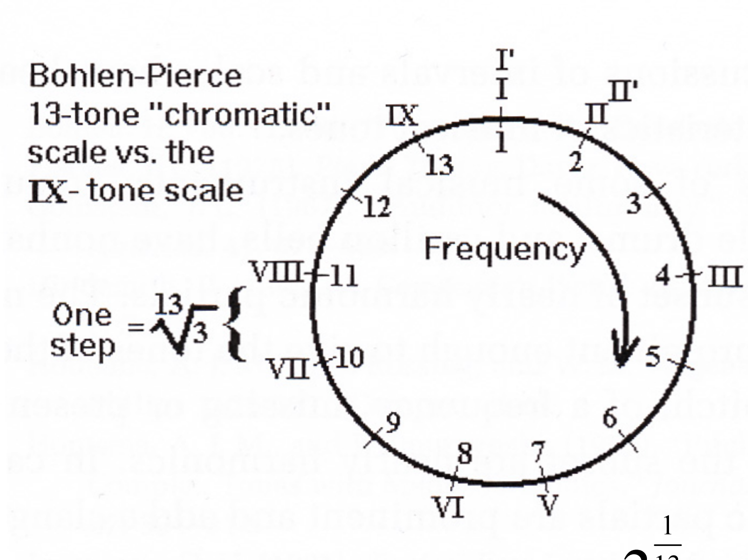
Notes are out of tune compared to the tempered scale

Pierce Scale

- The Bohlen-Pierce scale repeats every tritave (1:3; 1: octave+5th) rather than every octave.
- The tetrad 3:5:7:9 is fundamental to the scale.
- 13 chromatic tones separated by a factor
- 9 main scale notes from the 13

Cook demo 62

Figure 14.6 from Cook



$3^{\frac{1}{13}}$

Intervals and chords of the Bohlen–Pierce scale.

Ragas

Based on the Raga guide (edited by Joep Bor)

- 7 basic tones or scale degrees are used in classic Indian music

Sa Re Ga Ma Oa Dha and Ni.

- Twelve semitones are available. However typically only 7 are in the scale at a time. The first and fifth scale degree cannot be altered. However the other 5 can be adjusted. Lowered by a semi-tone (*komal*) Re Ga Dha Ni

  Like flats

- Raised by a semi-tone only Ma – *tivra* Like a sharp

Indian music and pitch

- While Indian musicians have a great sense of accuracy in pitch, however there is no fixed or absolute pitch for notes.
- Large microtonal variations... and these are carefully played by the musician.

What is a raga?

Associated with a particular theme scale and tune. Tonal framework for composition.

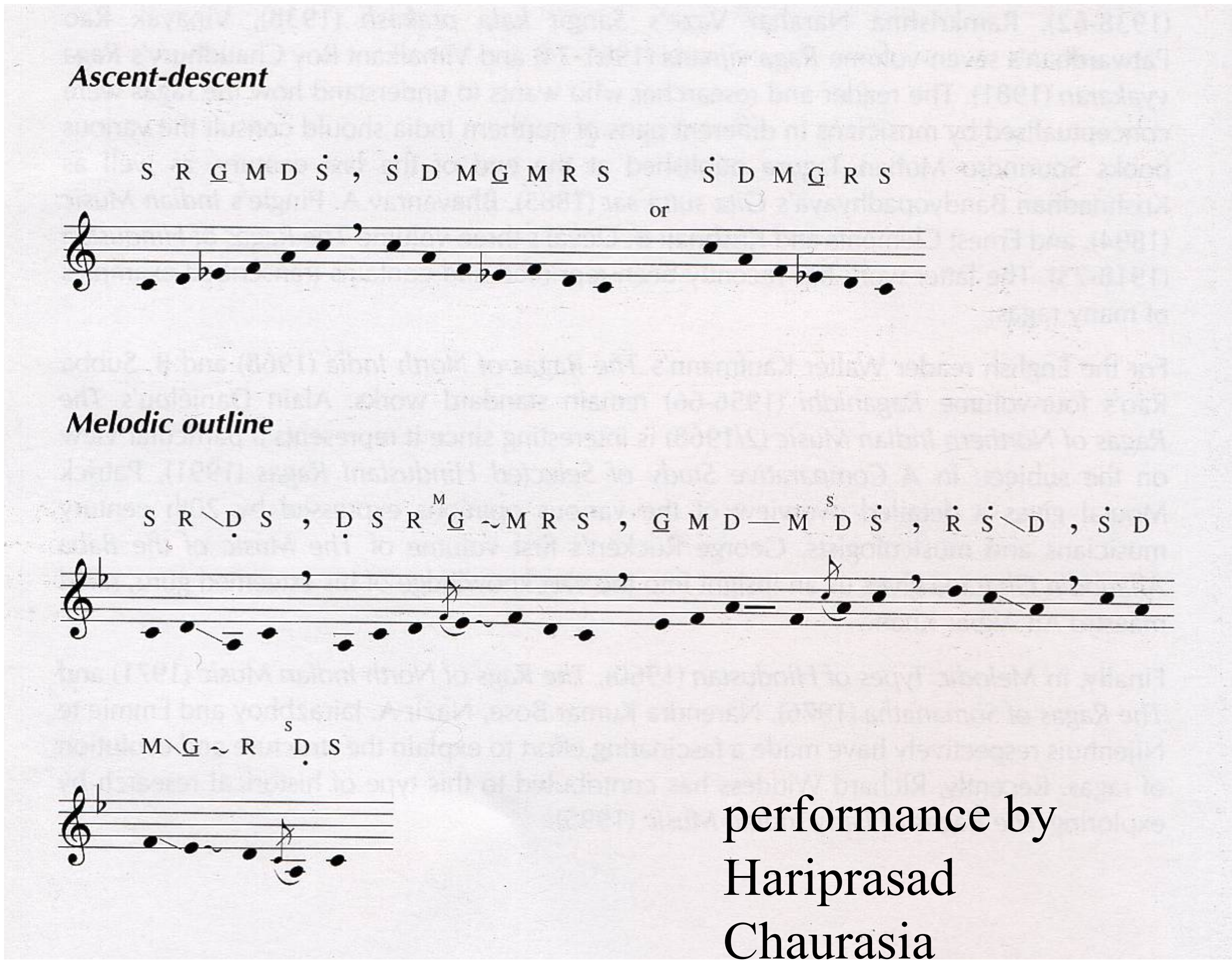
Ragas from CDs with the Raga Guide "A survey of 74 Hindustani Ragas"

Alhaiya bilaval

time: late morning – major scale

performed by Shruti Sadolikar-Katkar

Abhogi - Early night



performance by
Hariprasad
Chaurasia

Recommended Reading

- Berg and Stork Chap 9
- Hopkin Chap 3 and appendix 2
- Loy, Chapter 3