Normal modes of a column of air

Physics of music PHY103 Lecture #3

Detail of a feast for Nebamun, fragment of a scene from the tomb-chapel of Nebamun. Thebes, Egypt, Late 18th dynasty, around 1350BC

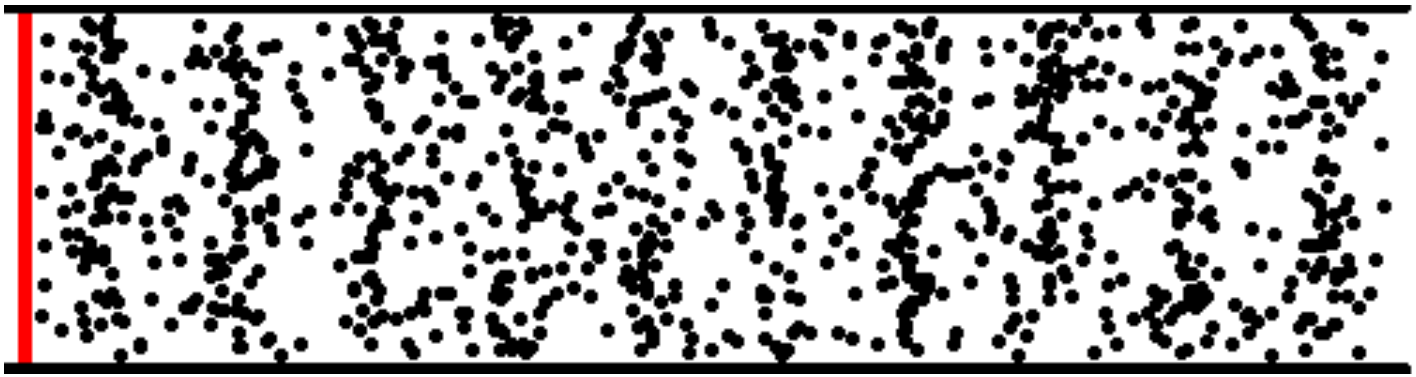
Wind Instruments

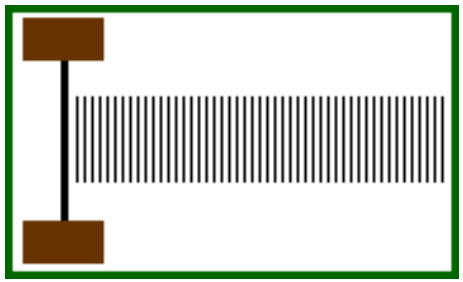
Tubes of air excited by blowing vibrations

- organ pipes, flutes, whistles, recorders
- brass family, reed instruments
- didgeridu
- ocarina

from DAVID O'BRIEN WHISTLES review
by Jessie Driscoll

Pressure waves in air
Longitudinal waves





Animation from
Dan Russel

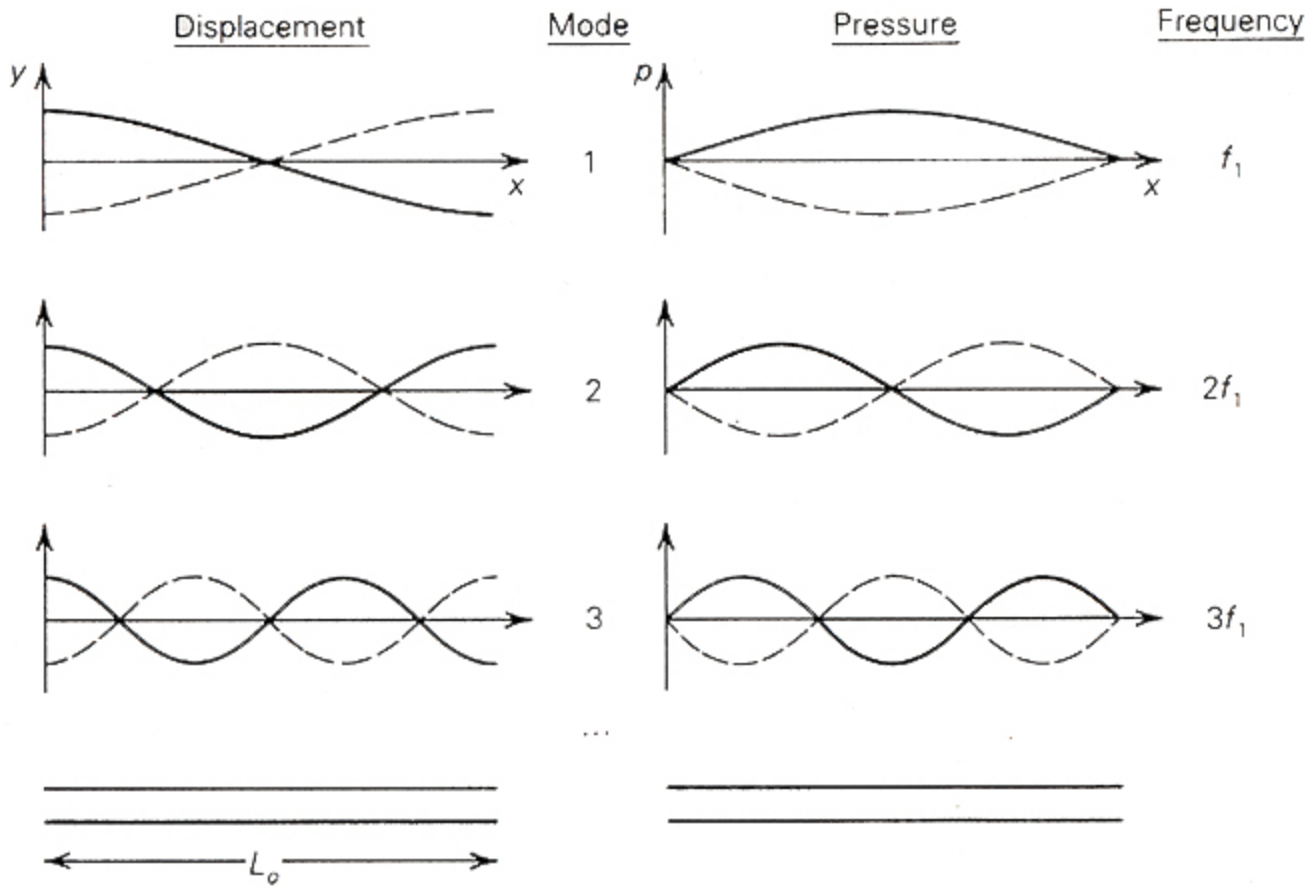
Standing waves or modes in a column of air

The motions shown are air velocity.

The shorter wavelength motions should be faster.

One of these is a pipe that is closed on one end and the other is open on both ends.
Which one is which?

Open/open tube



GURE 12.2 The first few natural modes of air confined in an open tube. At left, the curves are graphs maximum displacements at two times a half-cycle apart; they must have antinodes at the ends of the be. At right are corresponding graphs of acoustic pressure, which must have nodes at the ends. ındamental frequency is $f_1 = v/2L_o$.

Open/closed tube

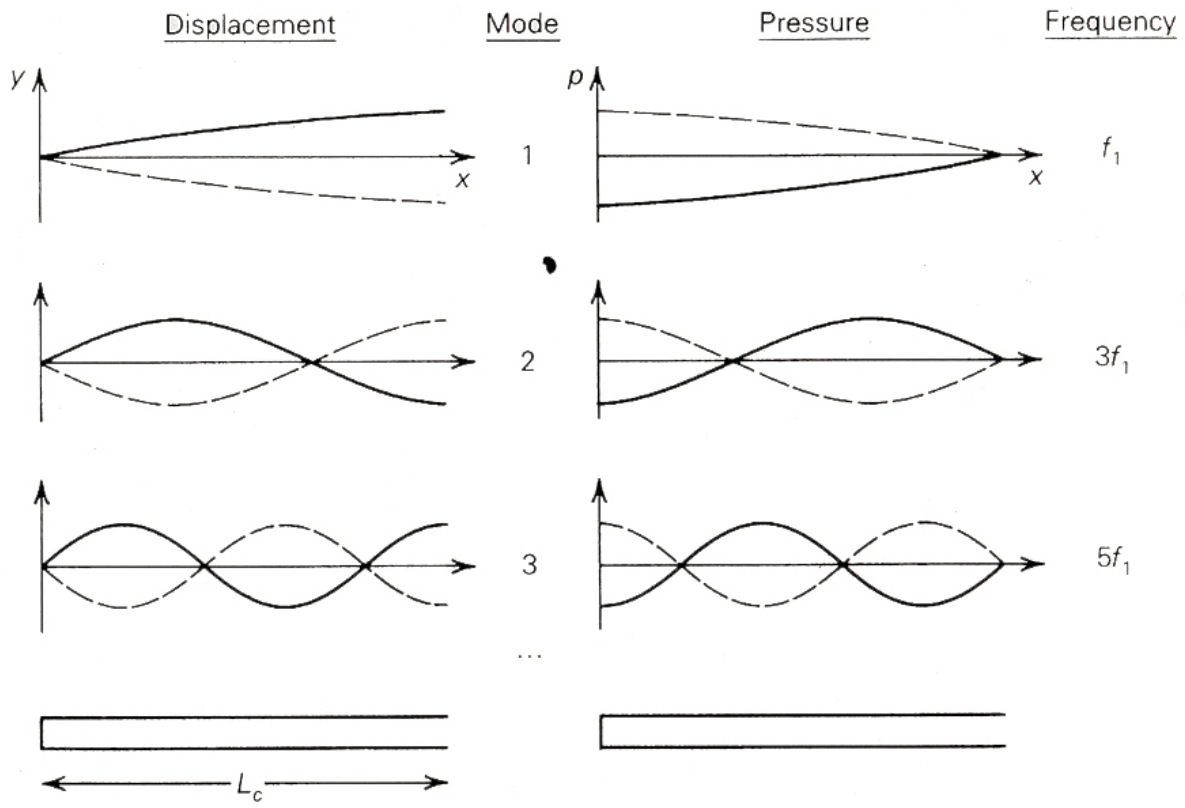


FIGURE 12.3 The first few natural modes of air confined in a closed tube (closed on one end). At left are displacement graphs; at right are pressure graphs for two times a half-cycle apart. Fundamental frequency is $f_1 = v/4L_c$.

Harmonics or overtones

- Closed/open tube only has odd harmonics (e.g., clarinet) *f, 3f, 5f, 7f*
- Open/open tube has all integer multiples *f,2f,3f,4f, 5f (*e.g., organ pipe*)*

In this case the tubes are the same but the **boundary conditions** are different.

The boundary: A closed end allows large pressures but no motions. An open ends allows motions but no pressure changes.

Music from overtones of pipes

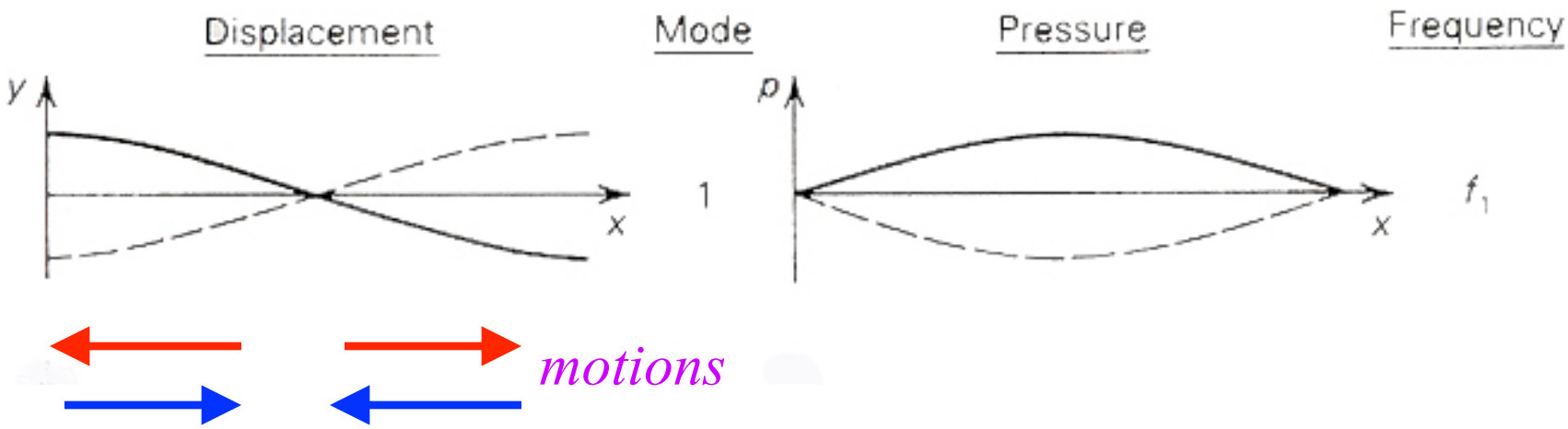
Sarah Hopkins

- Kindred Spirits
- from Gravikords, Whirlies and Pyrophones

note Doppler shift!

A wave reflecting off of the boundary

At an open boundary: the air bounces moving in and out of the boundary.

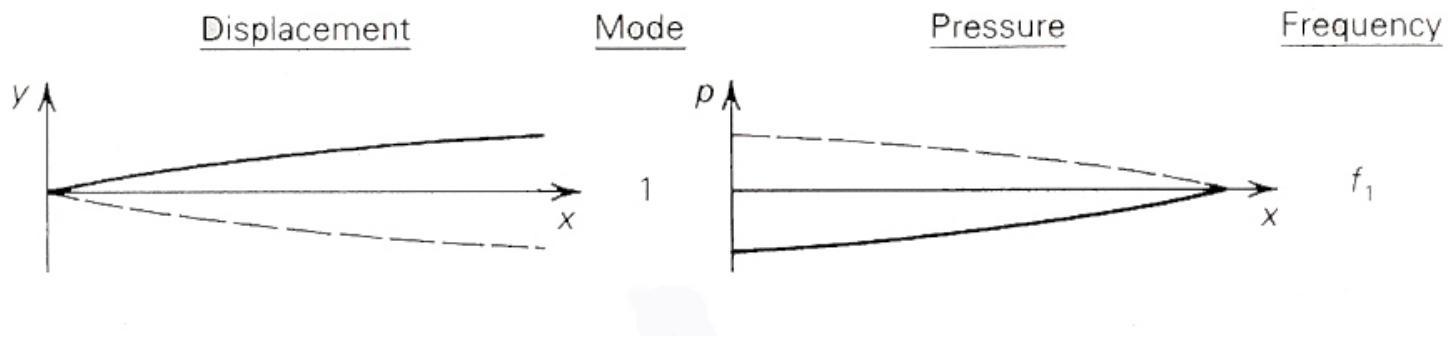


When the air moves out the pressure in the middle is low and the air is sucked back in.

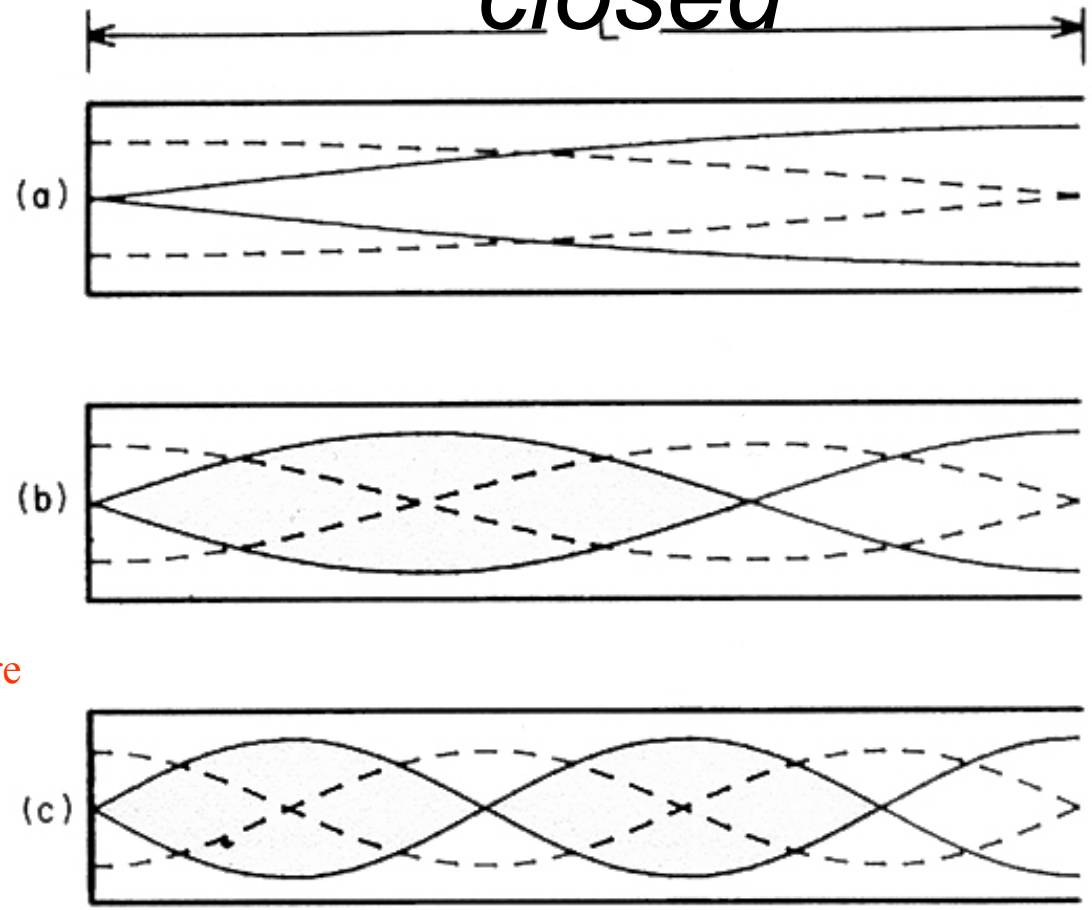
When the air moves in, the pressure is high in the middle and the air is pushed back out.

Wave reflecting off of the closed boundary

At a closed boundary: the wave reflects if it has a high pressure at the wall. The air compresses at the wall and then bounces back.



Normal modes of a column open/ closed



No pressure variation,

large motions

No motions,

large pressure variations

FIG. 9. First three vibration modes of an air column closed at one end and open at the other. Solid lines give displacement amplitudes; dashed lines, pressure amplitudes.

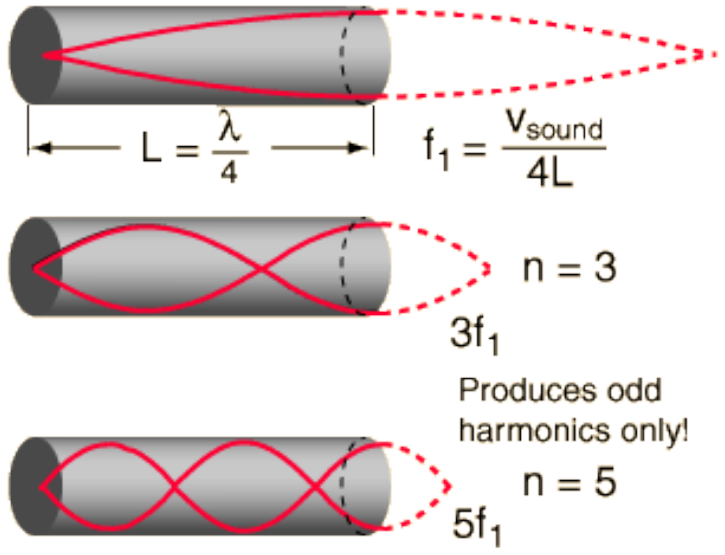
Which one has a lower fundamental tone? Open/open or open/closed?

Open/open

open/closed

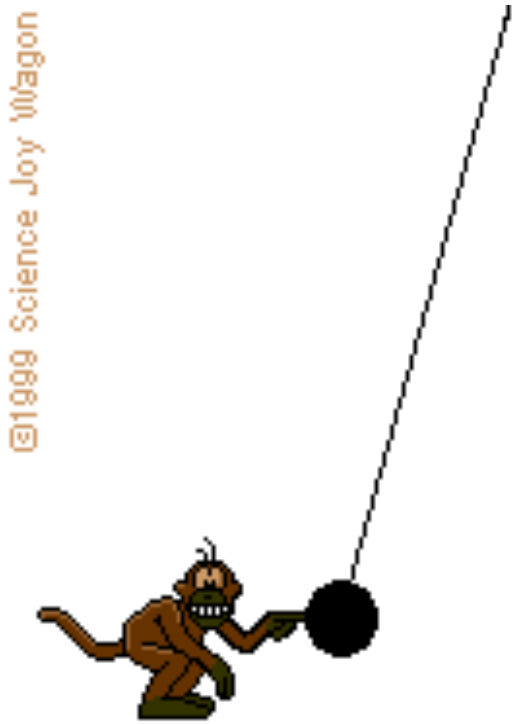
Length, fundamental and harmonics

- The open-closed pipe has a fundamental wavelength equal to four times its length and higher resonances at odd integer multiples of the fundamental frequency.



- The open-open pipe has a fundamental wavelength equal to two times its length and higher resonances which occur at all integer multiples of the fundamental frequency.

Resonant excitation



Small pushes correctly timed will add up and excite large amplitude motion.

Small pushes incorrectly timed will **tend to cancel out.**

Resonant excitation of a column of air

- How long does it take a disturbance to travel down the length of the tube and come back?

- Correctly timed excitations allow the mode to grow.

Boundaries

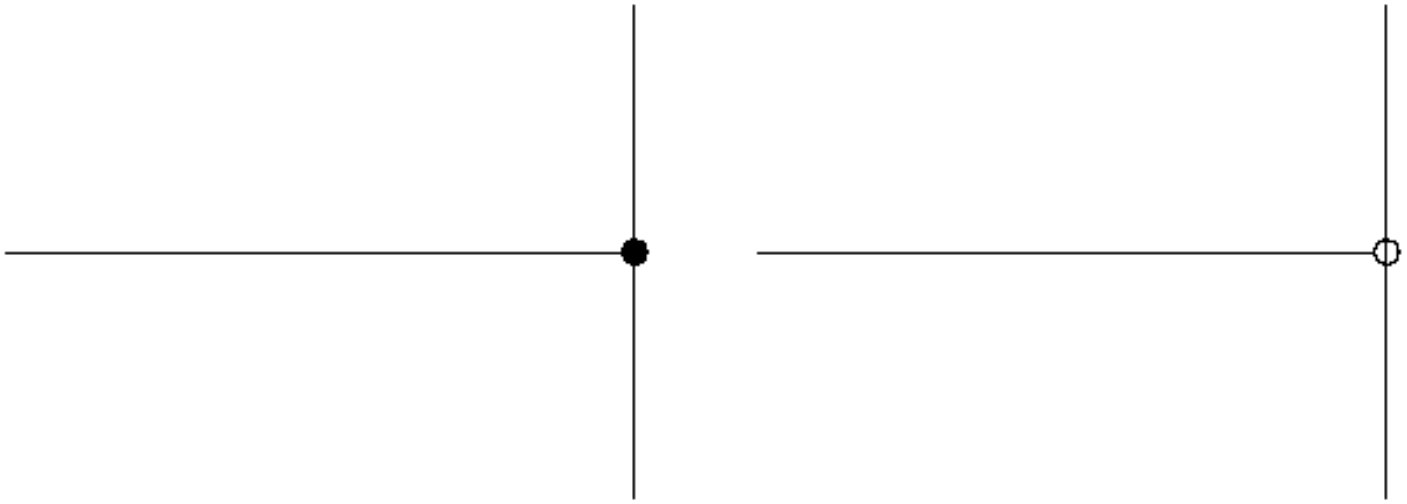
Open end

A travelling positive pressure pulse pushes the air out converting the pressure pulse into motion. The motion outwards excites a vacuum pulse inside moving the other way and flipping the sign of the pulse --- analogous to the fixed end of a string.

Closed end

At a closed boundary a high pressure pulse bounces against the boundary sending back a positive pressure pulse. Analogous to the free end of a string.

Waves reflecting off of boundaries



Animations courtesy of Dr. Dan Russell, Grad. Prog. Acoustics, Penn State

Reflection at boundary

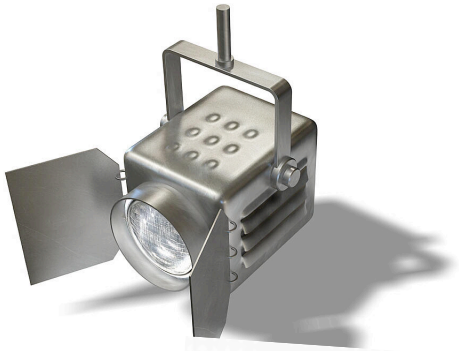
- Sign of wave reflected depends on nature of boundary
- Show on string, vs cable
- If the sign is opposite or same then 2 reflections needed to get back to original
- If sign is opposite on one side and same on other then 4 reflections needed

Open/Closed

- reeds
- horns
- digi
- panflutes

Open/Open

- flutes
- organ pipes
- recorders, whistles



Excitation of digi

movie by me+Raz Rivlis

Which modes will grow?

- If I put random pressure fluctuations into the pipe, some will grow and others will not.
- How do I describe the way the pipe reacts to an input sound?
- *Impedance* is a way to measure this.
- Relates input pressure to actual air velocity.
- Is a function of frequency

Speed of sound and excitation of a mode in an open column of air

The speed of sound is 330 m/s

If the column is 1.7m long then it takes

$$2 \times 1.7\text{m} \times \frac{\text{s}}{330\text{m}} = 0.010\text{s}$$ to travel back and forth.

This corresponds to a frequency

$$f = \frac{1}{0.01\text{s}} = 100\text{Hz}$$

If the column is excited at this frequency then a resonance is likely to be excited.

Water trumpet analogy for a trumpet

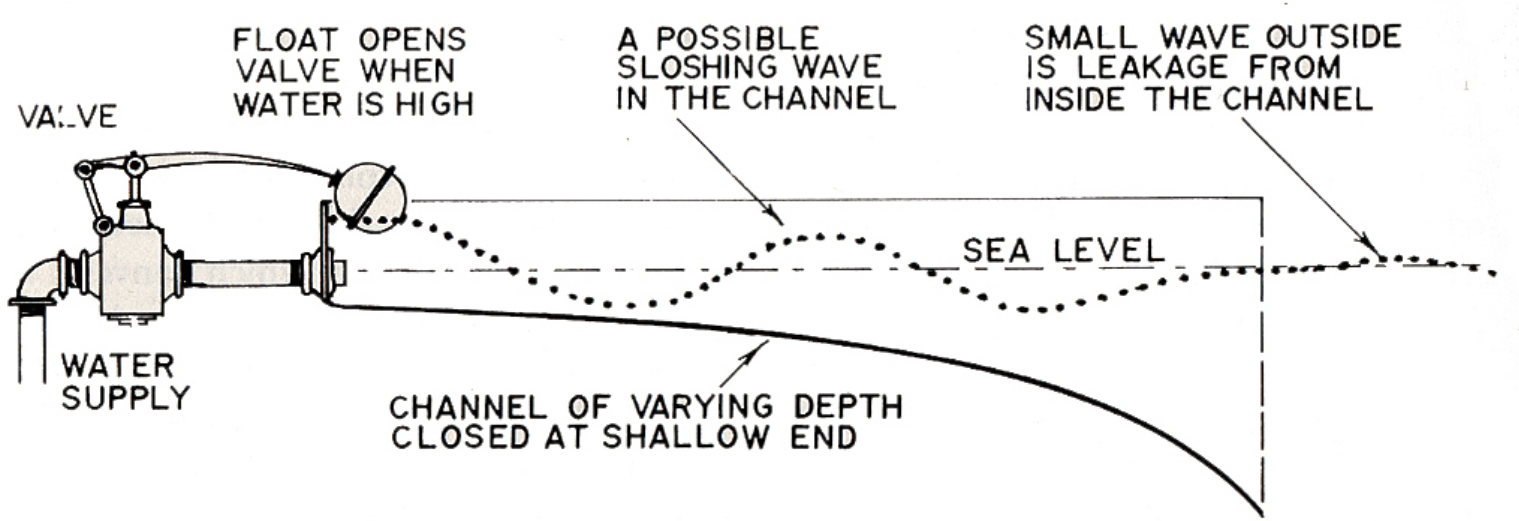
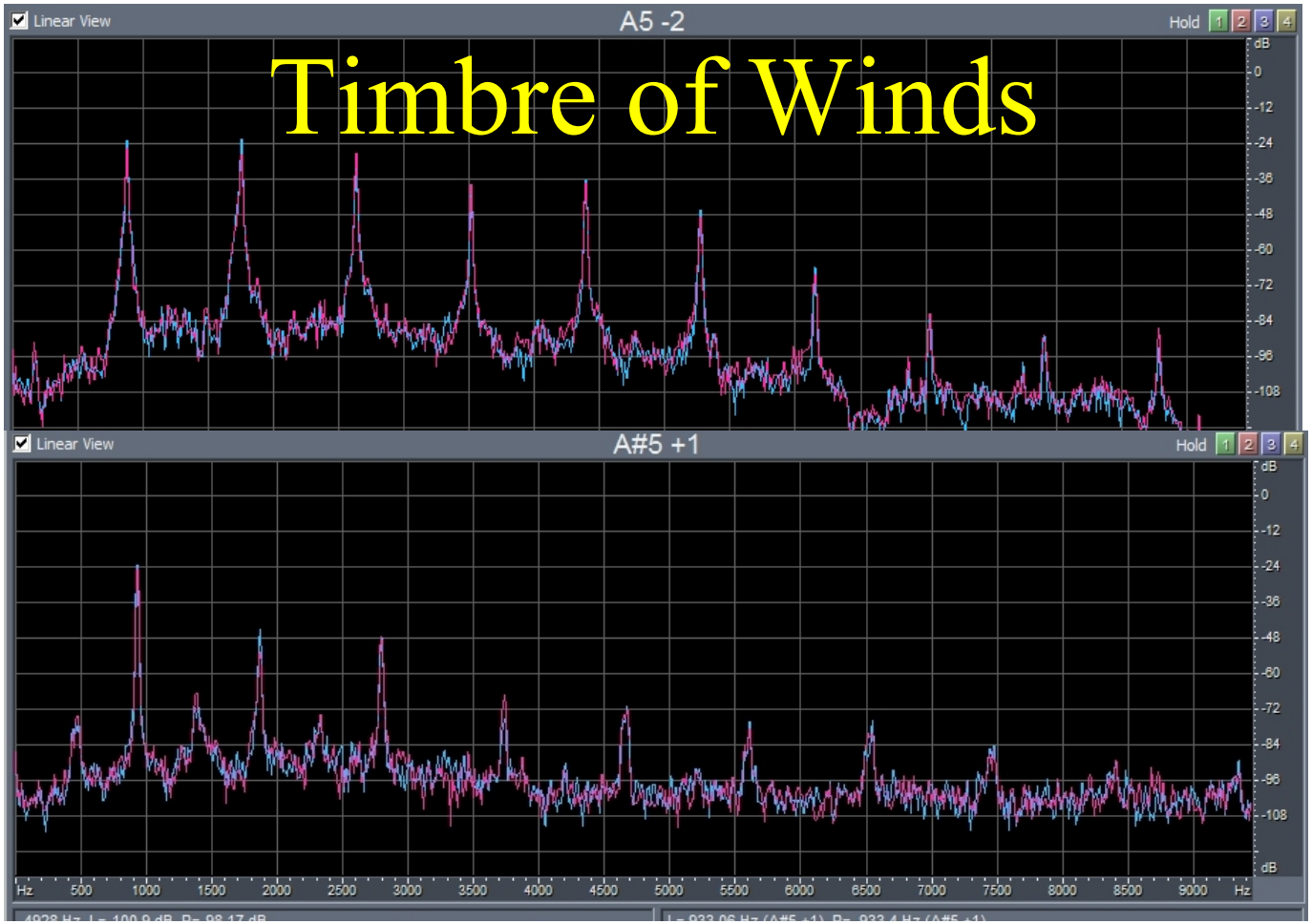


Fig. 20.2. A Water Trumpet

From Benade

Timbre of Winds



clarinet

flute

Despite closed end clarinet has all integer harmonics

Flute has stronger lower harmonic compared to clarinet -- Flute also seems to have some intermediate frequencies from the lower octave.

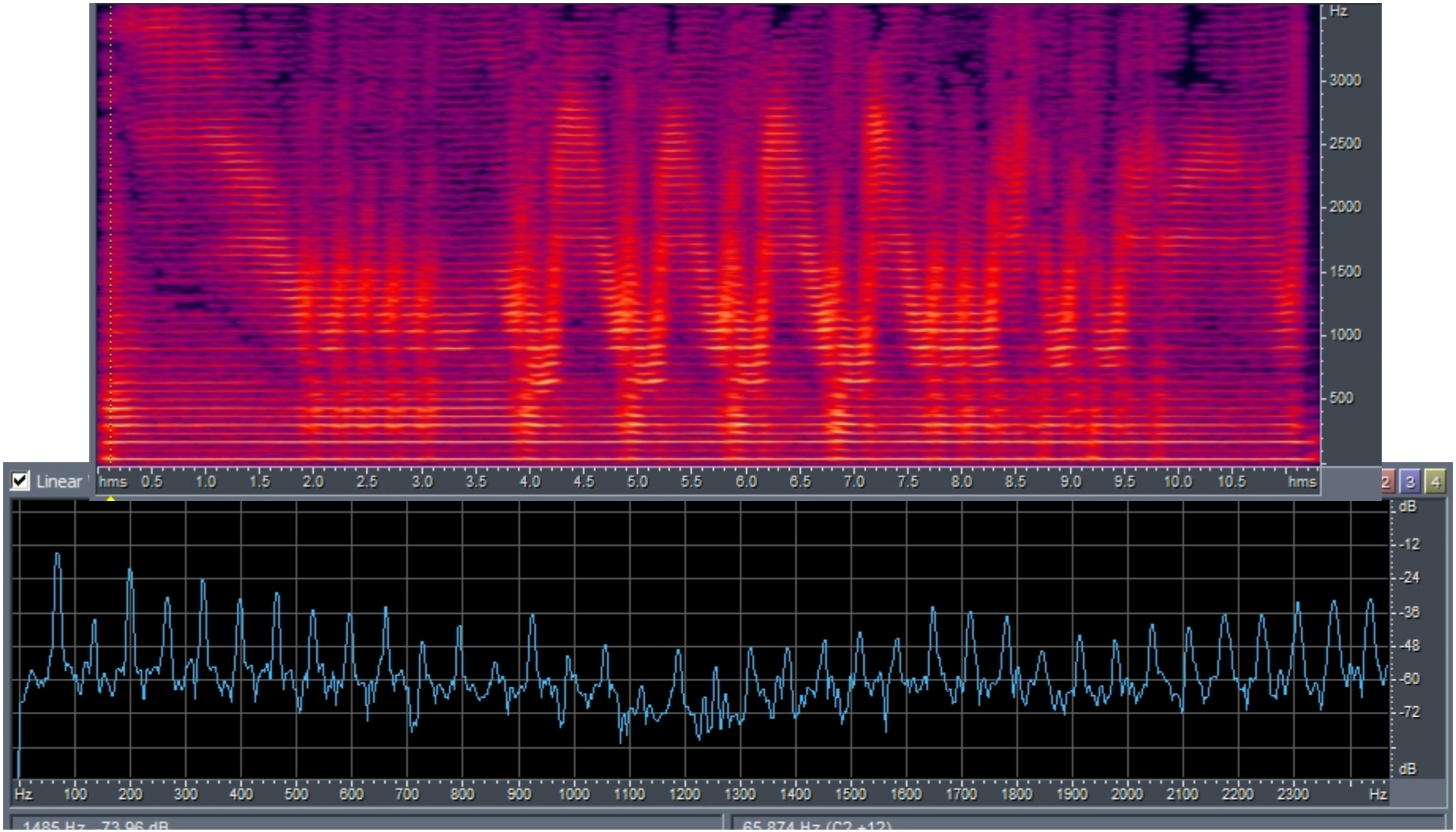
Clarinet and Brass

Clarinet

Horn

Horn has more broad band noise particularly at lower frequencies and at start of note. As was true for the flute there are tones in between but they are not the octave below.

Digderidu


Hz
3000
2500
2000
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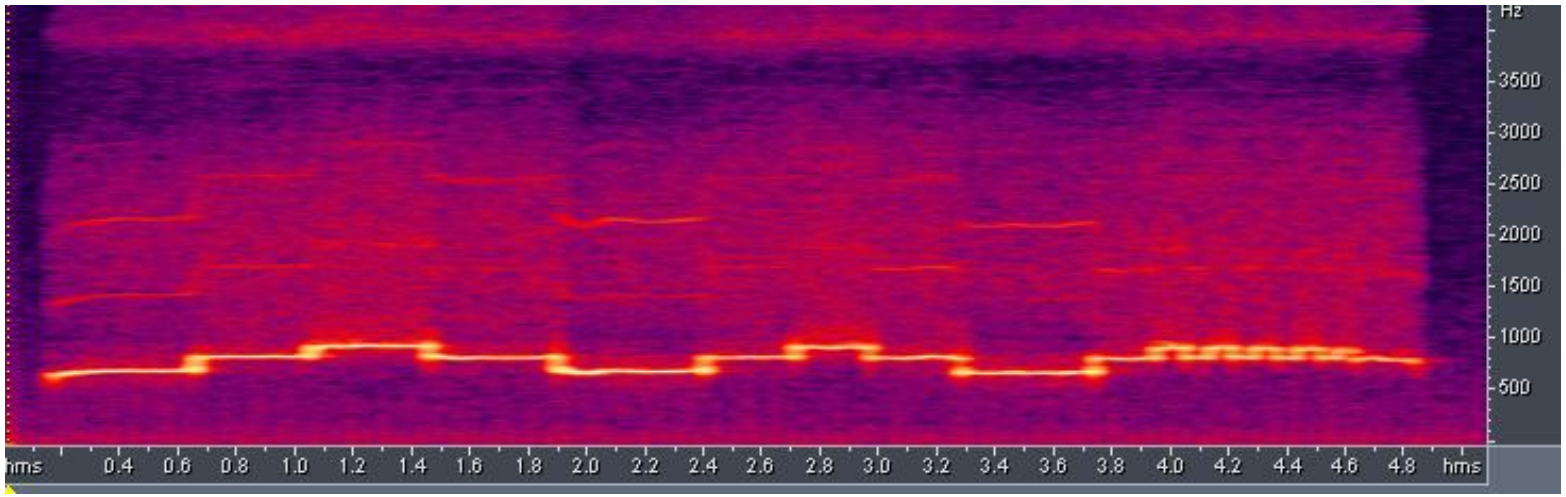
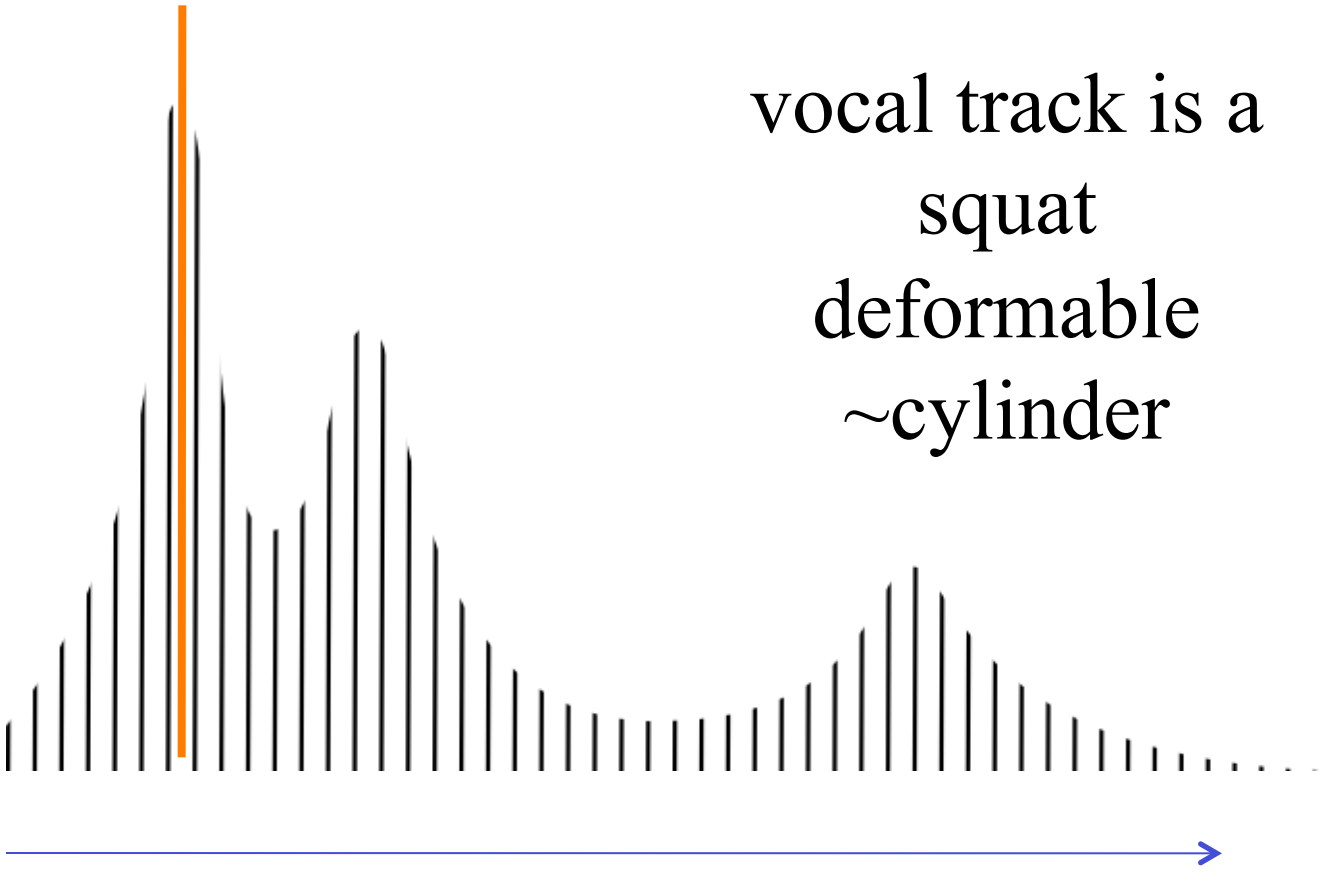
hms 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0 10.5 hms
Linear
2 3 4
dB
-12
-24
-36
-48
-60
-72
dB
Hz 100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000 2100 2200 2300 Hz

Ocarina

- Pitch adjusted by number of open holes rather than position of hole – not a tube!
- Almost pure tones



Whistle



Sliding whistle

- When all the way in, no longer acts like a tube of air. Acts more like an ocarina. Lost of higher harmonics. Noted by many in the lab!
- Even integer harmonics often weak because of inner closed end.

Spectrum and resonances

- On the string, each overtone frequency corresponds to a mode of oscillation
- In percussion instruments that is also often the case
- With wind instruments, integer multiple overtones are often seen but these are not always resonances of the instrument

Terms introduced

- Resonances
- Resonant excitation
- Pressure waves
- Boundary conditions
- Longitudinal waves
- Open/open and open/closed tubes and modes of oscillation in them
- Impedance

Recommended reading

- Chap 6 of Hopkin on Aerophones pages 73-75