PROBLEM SET #1 **AST570**

Problem 1. On Hyperbolic orbits

Following Murray and Dermott problem 2.3.

A test particle of mass m approaches a planet of mass M and radius R from infinity with speed v_∞ and an impact parameter b.

a) Use the particle's energy and angular momentum with respect to the planet to derive expressions for the semi-major axis, a, the eccentricity, e (greater than 1 for a hyperbolic orbit), and for the pericenter distance q.

b) Show that the eccentricity may be written $e = 1 + 2v_\infty^2/v_0^2$ where v_0 is the escape velocity at q.

c) Use the expression for the true anomaly corresponding to the asymptote of the hyperbola to show that the overall deflection, ψ, of the test particle's orbit after it leaves the vicinity of the planet, is given by $\sin(\psi/2) = e^{-1}$.

d) Calculate the maximum deflection angle for a spacecraft skimming Jupiter with v_∞= 10 km/s.

Problem 2. Impulse approximation and velocity kick due to a passing moon

Consider a particle in a planetary ring with semi-major axis a that is in a circular orbit about a planet of mass M. There is a nearby moon, also in a circular orbit. The ratio of the moon to planet mass is μ. The difference between moon and particle semi-major axes (for the orbit around the planet) is da.

a) Using the impulse approximation, show that after a close approach by the moon that the particle has experienced a small change in velocity

$$dv \sim \mu \left(\frac{a}{da}\right)^2 na$$

where the mean motion $n = \sqrt{GM/a^3}$ is the mean motion of the moon.

b) Estimate the change in eccentricity of the particle, following the close approach by the moon. In the low eccentricity limit a particle has maximum radial velocity $v_r \sim ena$ where e is its eccentricity.

c) Why would you expect the impulse approximation to be invalid?

Murray and Dermott estimate (eqn 10.57)

$$e \sim \mu \left(\frac{a}{da} \right)^2$$

d) Why can the limit for $v_\perp$ that gives the impulse approximation be used here even though the difference in velocity between the two objects is not necessarily large? (Consider the formula for $\Delta v_\perp$ and $\Delta v_\parallel$ given in class and the criterion for deciding when the impulse approximation can be used or when a limit of these can be taken).

Problem 3. Tidal Forces

a) Show that the gravitational potential due to an object m and from tides from a distant object M can be written

$$\Phi(\mathbf{r}) = -\frac{Gm}{r} \left[1 + \left(\frac{M}{m} \right) \left(\frac{r}{D} \right)^3 P_2(\hat{\mathbf{r}} \cdot \hat{\mathbf{z}}) \right]$$

to lowest order in r/D, where D is the distance between the two objects and $\hat{\mathbf{z}}$ is a unit vector in the direction between m and M. Here $P_2(\mu) = \frac{1}{2}(3\mu^2 - 1)$ is the second Legendre polynomial, $r = |\mathbf{r}|$ and $\hat{\mathbf{r}} = \mathbf{r}/r$ is a unit vector.

b) Consider a comet that is grazing the Sun's surface. What is the maximum density object that can be tidally disrupted by the Sun? The Sun has a mean density of approximately 1.4g/cm^3. Consider a comet that is grazing Mercury's surface. What is the maximum density object that can be tidally disrupted? Mercury has a mean density of 5.4 g/cm^3.

Problem 4. Estimates of Gravitational Heating Rates

Consider a newly formed but unbound stellar cluster in the Galaxy in the solar neighborhood (approximately 8 kpc from the Galactic center) that is at low inclination in the Galactic plane. The disk of the galaxy contains molecular clouds with mass of about a million solar masses. The surface density of molecular clouds is about $1 M_\odot/\mathrm{pc}^2$. The velocity dispersion of the molecular clouds is about 10 km/s. The circular velocity in the Galaxy is about 200 km/s.

a) Estimate the velocity diffusion coefficient due to heating from encounters from molecular clouds.

b) Consider two stars in the unbound cluster that are initially separated by a distance $d = 1$pc. How fast do you expect that they diverge from one another because of encounters with molecular clouds?

c) Assume that the two stars that initially differ by 1km/s in their velocity? How fast would they diverge from one another in a smooth Galaxy?

Problem 5. Disruption of Binary Planetesimals by a close approach to a planet

Many massive Kuiper Belt objects are nearly equal mass binaries. For example, Pluto and Charon are separated by about 2×10^9 cm. Their masses are 13 and 2 $\times 10^{24}$g. Their densities are about 1.8 g/cm^3.

a) What is the Hill radius of the binary system? How far apart could the separation between the two objects be before the self gravity of the two objects is not enough to hold them together in the gravitational field of the Sun?

b) How close an approach to Neptune would disrupt this binary? The mass of Neptune is $102 \times 10^2 4$ kg.

c) Which mass is most likely to be left bound to Jupiter following tidal disruption with this planet?

For a comparison of binary objects see for example, *The Extreme Kuiper Belt Binary 2001 QW322*, Petit et al. 2008, Science 322, 432.

Problem 6. Orbital timescales in the outer parts of a Hill sphere

Consider an irregular satellite in the outer part of a Hill sphere of a planet. Let a_p be the semi-major axis of the planet (and for the planet's motion around the Sun) and a_s be the semi-major axis of the moon about the planet. Let r_H be the Hill radius of the planet.

For $a_s/r_H \to 1$ what is the orbital period of the moon around the planet in planetary years?