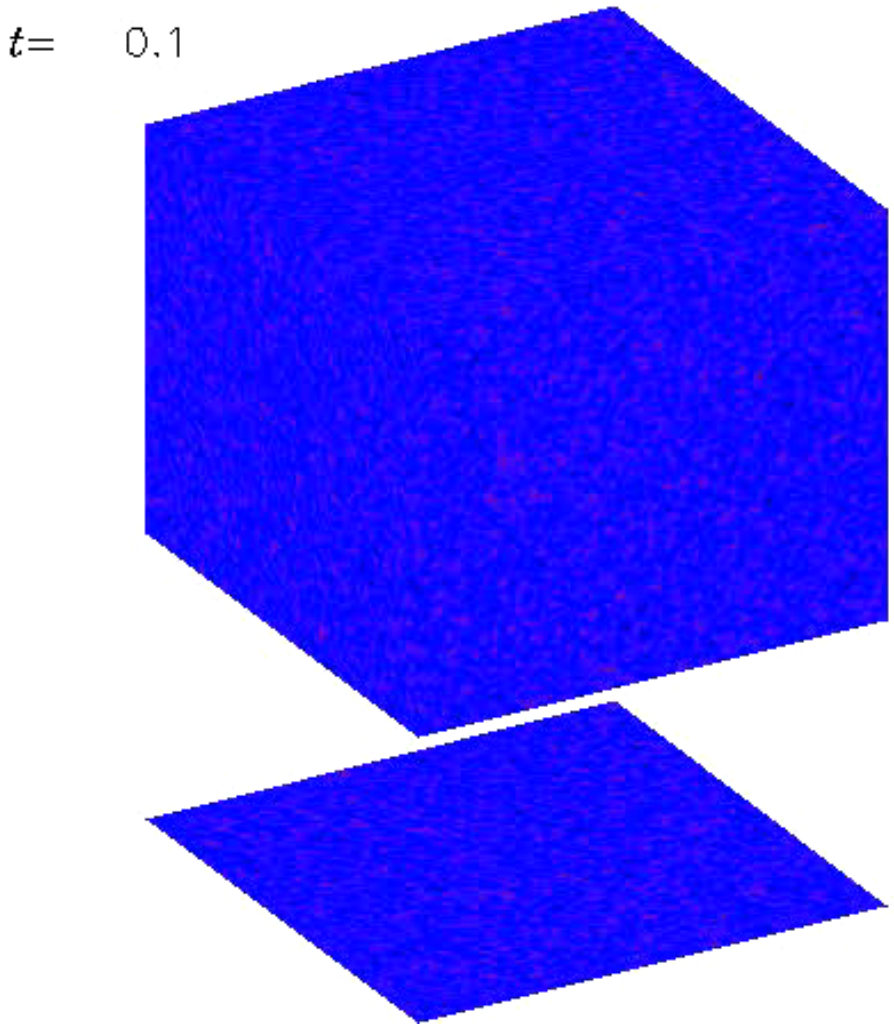


Radial drift of particles is unstable to streaming instability
Johansen & Youdin (2007); Youdin & Johansen (2007)

Planetesimal Formation

gas drag

settling of dust

turbulent diffusion

damping and excitation mechanisms for planetesimals embedded in disks

minimum mass solar nebula

particle growth

core accretion

Gas drag

Gas drag force $$F_D = C_D \rho_{gas} \pi s^2 v^2 / 2$$

where *s* is radius of body, *v* is velocity difference

- Stokes regime when Reynold's number is less than 10

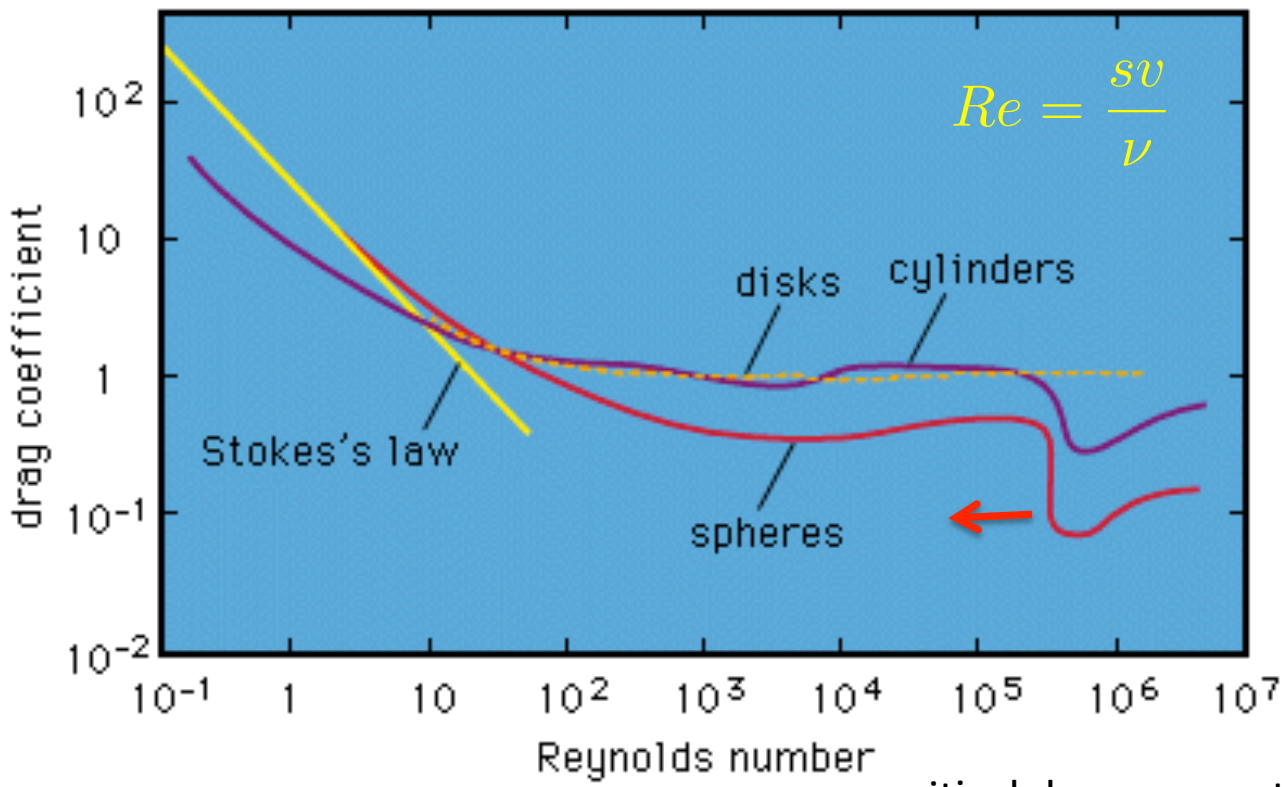
$$C_D \sim 24 Re^{-1}$$

- High Reynolds number regime C_D ~0.5 for a sphere
- If body is smaller than the mean free path in the gas → Epstein regime (note mean free path could be meter sized in a low density disk)

$$F_D = \frac{4\pi}{3} \rho_{gas} s^2 c_s v$$

Essentially ballistic except the cross section can be integrated over angle

Drag Coefficient



Note: we do not used turbulent viscosity to calculate drag coefficients

critical drop moves to the left in main stream turbulence or if the surface is rough

Stopping timescale

$$F = M dv/dt \qquad\qquad t_s = \frac{Mv}{F_D}$$

- Stopping timescale, t_s, is that for the particle to be coupled to gas motions
- Smaller particles have short stopping timescales
- Useful to consider a dimensionless number $t_s\Omega$ which is approximately the Stokes number

Settling timescale for dust particles

- Use gravitational force in vertical direction, equate to drag force for a terminal velocity

$$F_z = M\frac{dv_z}{dt} \sim M\frac{v_z}{t_s} \qquad F_z = -M\Omega^2 z$$

$$v_z = (t_s\Omega)z\Omega$$

- Timescale to fall to midplane

$$t_{settle} = z/\dot{z} = z/v_z = \Omega^{-1}(\Omega t_s)^{-1}$$

- Particles would settle unless something is stopping them
- Turbulent diffusion via coupling to gas

Turbulent diffusion

- Diffusion coefficient for gas $D_0 = \alpha c_s h$
- For a dust particle $D = \alpha c_s h / Sc$
- Schmidt number *Sc* $Sc = (1 + St)$
- Stokes, *St*, number is ratio of stopping time to eddy turn over time
- Eddy sizes and velocities $l \sim \sqrt{\alpha} h \qquad dv \sim \sqrt{\alpha} c_s$
  - Eddy turnover times are of order $t \sim \Omega^{-1}$
- In Epstein regime

$$St \sim \frac{3}{4\pi} \frac{M}{s^2} \frac{\Omega}{\rho_g c_s}$$

- When well coupled to gas, the Diffusion coefficient is the same as for the gas
- When less well coupled, the diffusion coefficient is smaller

Following Dullemond & Dominik 04

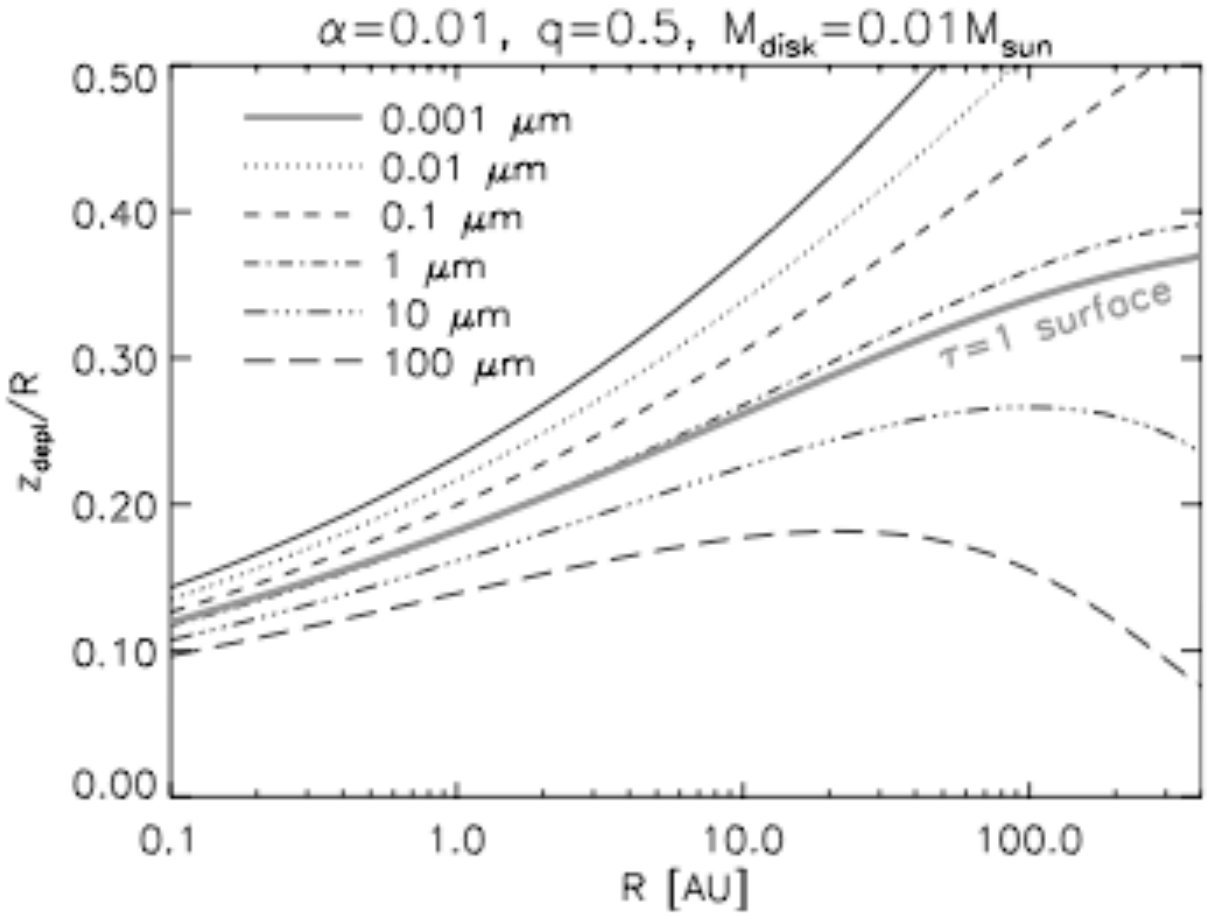
Mean height for different sized particles
Diffusion vs settling

- Diffusion processes act like a random walk $\bar{z} = \sqrt{< z^2 >}$
- In the absence of settling $\bar{z} = \sqrt{Dt}$
- Diffusion timescale $t_d \sim \bar{z}/\dot{\bar{z}}$ $\dot{\bar{z}}\bar{z} = \frac{D}{2}$

  $t_d = 2\bar{z}^2/D$
- To find mean z equate t_d to t_{settle} $t_{settle} = \Omega^{-1}(\Omega t_s)^{-1}$
- This gives $z^2 = \frac{D}{2\Omega}(\Omega t_s)^{-1}$

  and so a prediction for the height distribution as a function of particle size

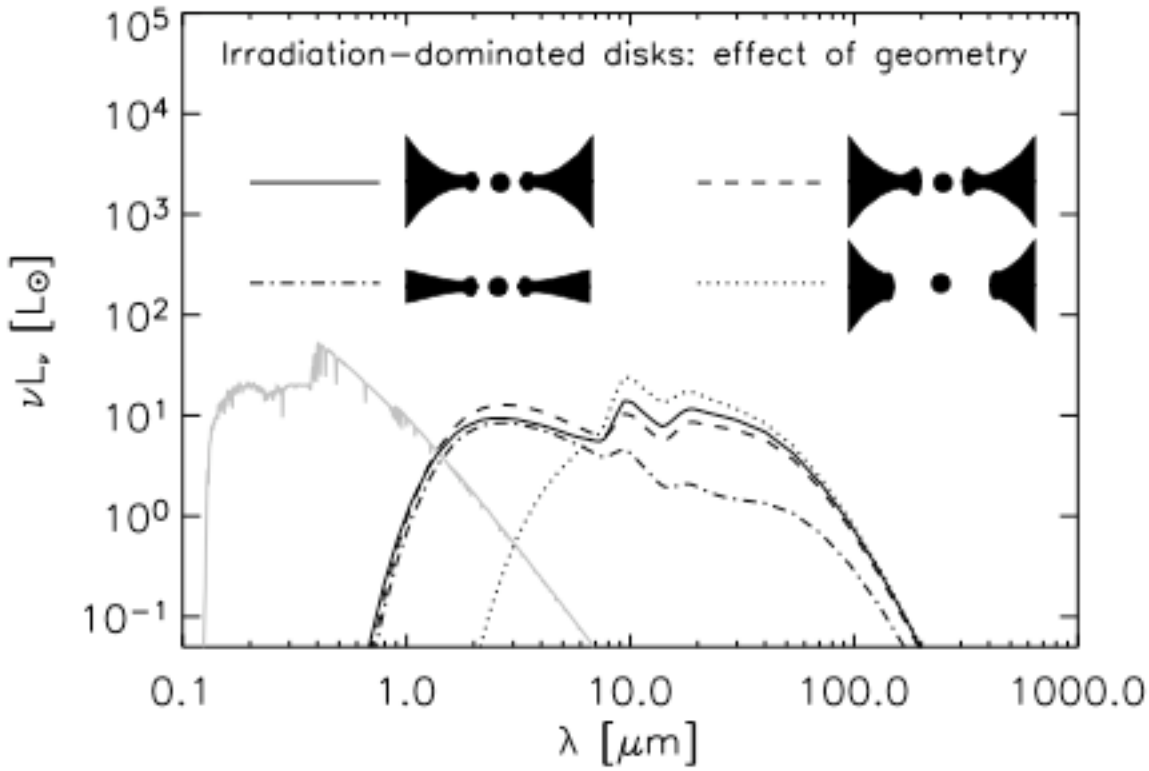
Equilibrium heights



Dullemond & Dominik 04

Effect of sedimentation on SED

Dullemond & Dominik 04



Minimum Mass Solar Nebula

- Many papers work with the MMSN, but what is it? Commonly used for references:

  Gas $\Sigma_g = 2400 \left(\frac{r}{1\mathrm{AU}}\right)^{-3/2} \mathrm{g\ cm^{-2}}$

  Dust $\Sigma_d = 10 \left(\frac{r}{1\mathrm{AU}}\right)^{-3/2} \mathrm{g\ cm^{-2}}$
- Solids (ices) 3-4 times dust density
- The above is 1.4 times minimum to make giant planets with current spacing
  - Hyashi, C. 1981, Prog. Theor. Physics Supp. 70, 35
- However could be modified to take into account closer spacing as proposed by Nice model and reversal of Uranus + Neptune (e.g. recent paper by Steve Desch)

Larger particles (~km and larger)

- Drag forces:
  - Gas drag, collisions,
  - excitation of spiral density waves, (Tanaka & Ward)
  - dynamical friction
  - All damp eccentricities and inclinations
- Excitation sources:
  - Gravitational stirring
  - Density fluctuations in disk caused by turbulence (recently Ogihara et al. 07)

Damping via waves

- In addition to migration both eccentricity and inclination on averaged damped for a planet embedded in a disk.

Tanaka & Ward 2004

$$\dot{e}/e = -0.78\ t_{wave}^{-1}$$
$$\dot{i}/i = -0.54\ t_{wave}^{-1}$$

$$t_{wave} = \Omega_p^{-1} \left(\frac{M_p}{M_*}\right)^{-1} \left(\frac{\Sigma_g(r_p) r_p^2}{M_*}\right)^{-1} \left(\frac{c_s}{V_K}\right)^4$$

- Damping timescale is short for earth mass objects but very long for km sized bodies
- Balance between wave damping and gravitational stirring considered by Papaloizou & Larwood 2000

Note more recent studies get much higher rates of eccentricity damping!

Excitation via turbulence
Stochastic Migration

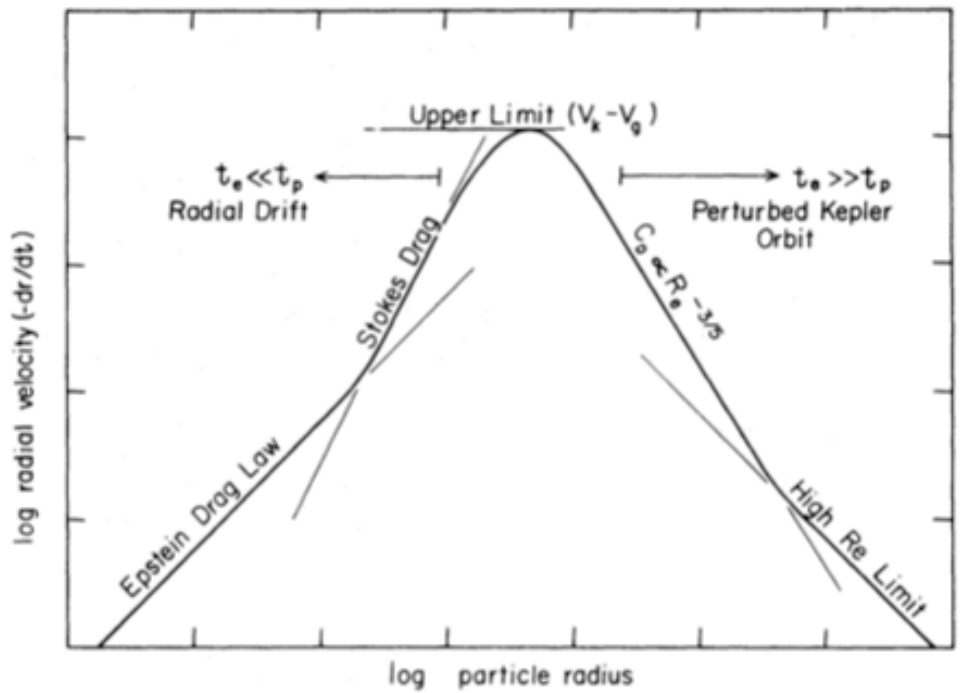
- Johnson et al. 2006, Ogihara et al. 07, Laughlin 04, Nelson et al. 2005
- Diffusion coefficient set by torque fluctuations divided by a timescale for these fluctuations $t_{fluc} \sim \Omega^{-1}$
- Gravitational force due to a density enhancement scales with $M_p 2\pi G \Sigma_g$
- Torque fluctuations $(\delta\Gamma)^2 \sim (\gamma M_p 2\pi r G \Sigma_g)^2$
- parameter γ depends on density fluctuations δΣ/Σ
- γ~α though could depend on the nature of incompressible turbulence
- See recent papers by Hanno Rein, Ketchum et al. 2011

Eccentricity diffusion because of turbulence

- We expect eccentricity evolution $< e^2 >= Dt$
- with
$$D \sim \frac{(\delta\Gamma)^2}{L^2}\Omega \sim \gamma^2 \left(\frac{\Sigma_g r^2}{M_*}\right)^2 \Omega$$
- or in the absence of damping
$$\bar{e} \sim 100\gamma \left(\frac{\Sigma_g r^2}{M_*}\right) (\Omega t)^{1/2}$$
- constant taken from estimate by Ida et al. 08 and based on numerical work by Ogihara et al.07
- Independent of mass of particle
- Ida et al 08 balanced this against gas drag to estimate when planetesimals would be below destructivity threshold

In-spiral via headwinds

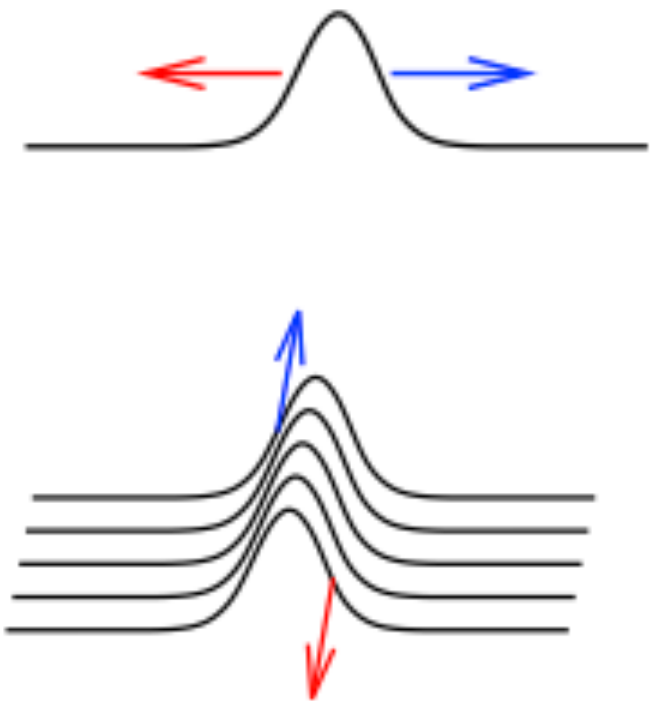
- For m sized particles headwinds can be large
  - possibly stopped by clumping instabilities (Johanse & Youdin) or spiral structure (Rice)
- For planetary embryos type I migration is problem
  - possibly reduced by turbulent scattering and planetesimal growth (e.g., Johnson et al. 06)



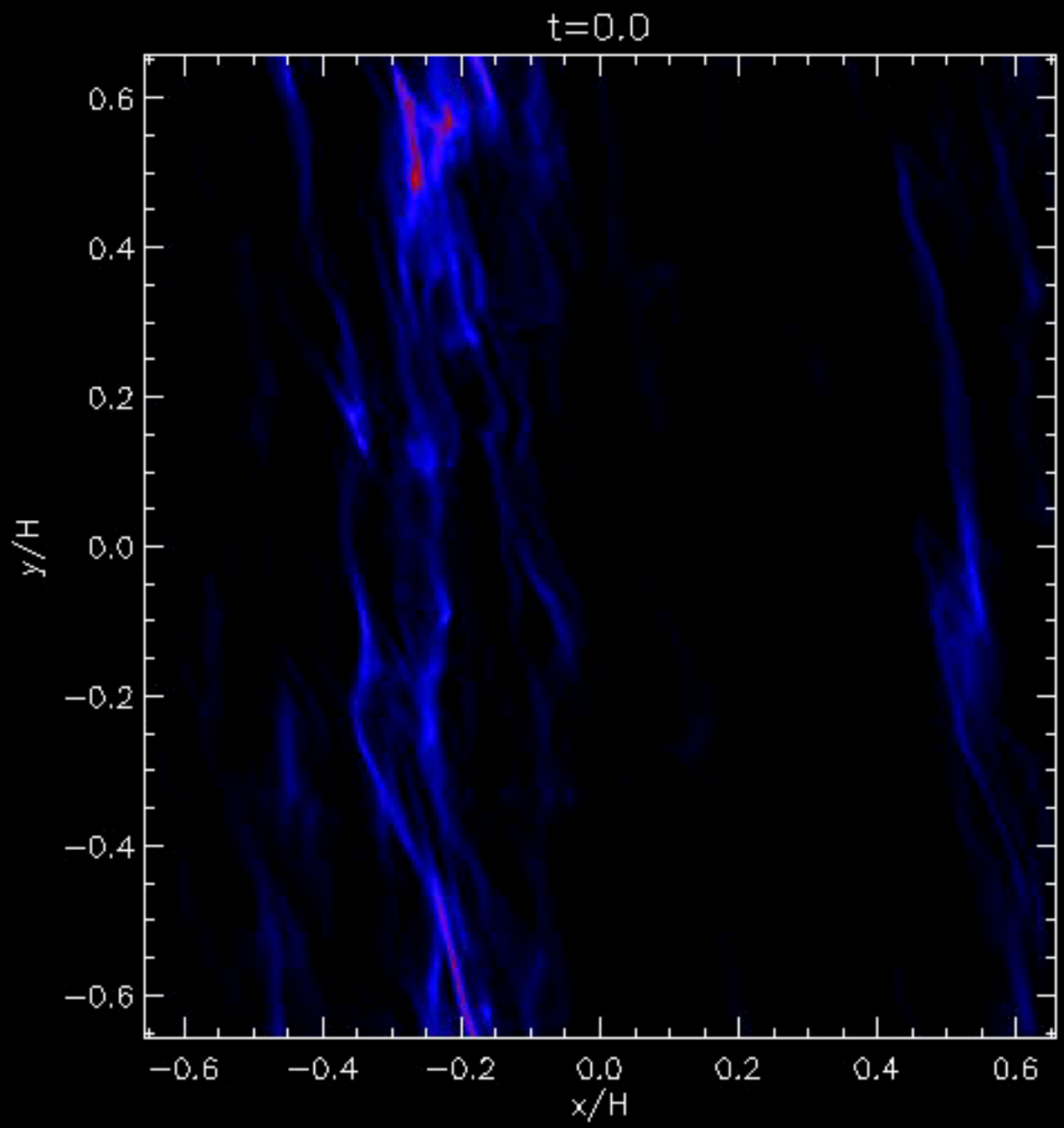
Heading is important for m sized bodies, above from Weidenschilling 1977

Density peaks
Pressure gradient trapping

- Pressure gradient caused by a density peak gives sub Keplerian velocities on outer side (leading to a headwind) and super Keplerian velocities on inner side (pushing particles outwards).
- Particles are pushed toward the density peak from both sides.



figures by Anders Johansen



Formation of gravitational bound clusters of boulders

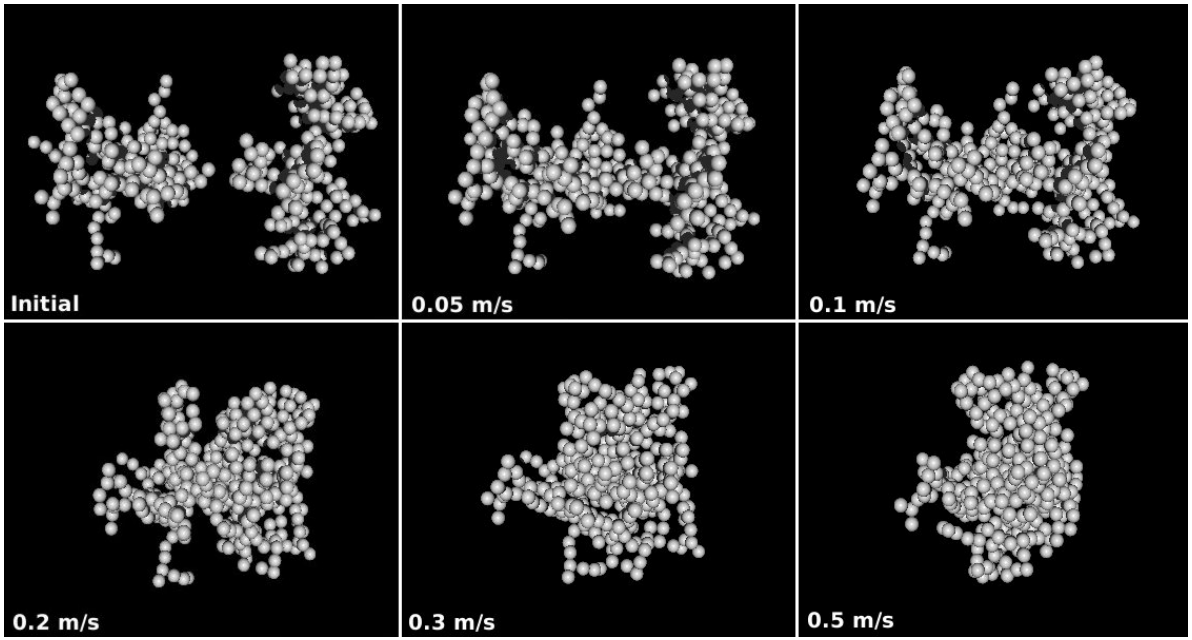
points of high pressure are stable and collect particles

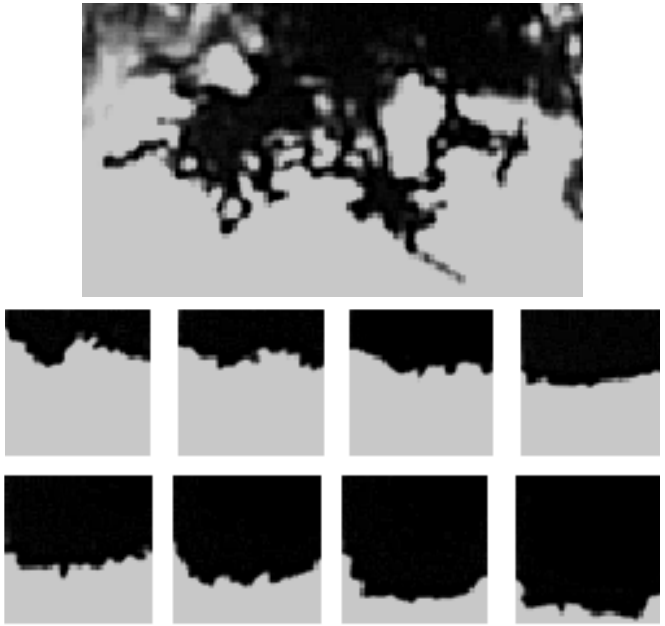
Johansen, Oishi, Mac Low, Klahr, Henning, & Youdin (2007)

The restructuring/compaction growth regime

($s_1 \approx s_2 \approx$ 1 mm…1 cm; $v \approx 10^{-2}$…10^{-1} m/s)

Low impact energy: hit-and-stick collisions





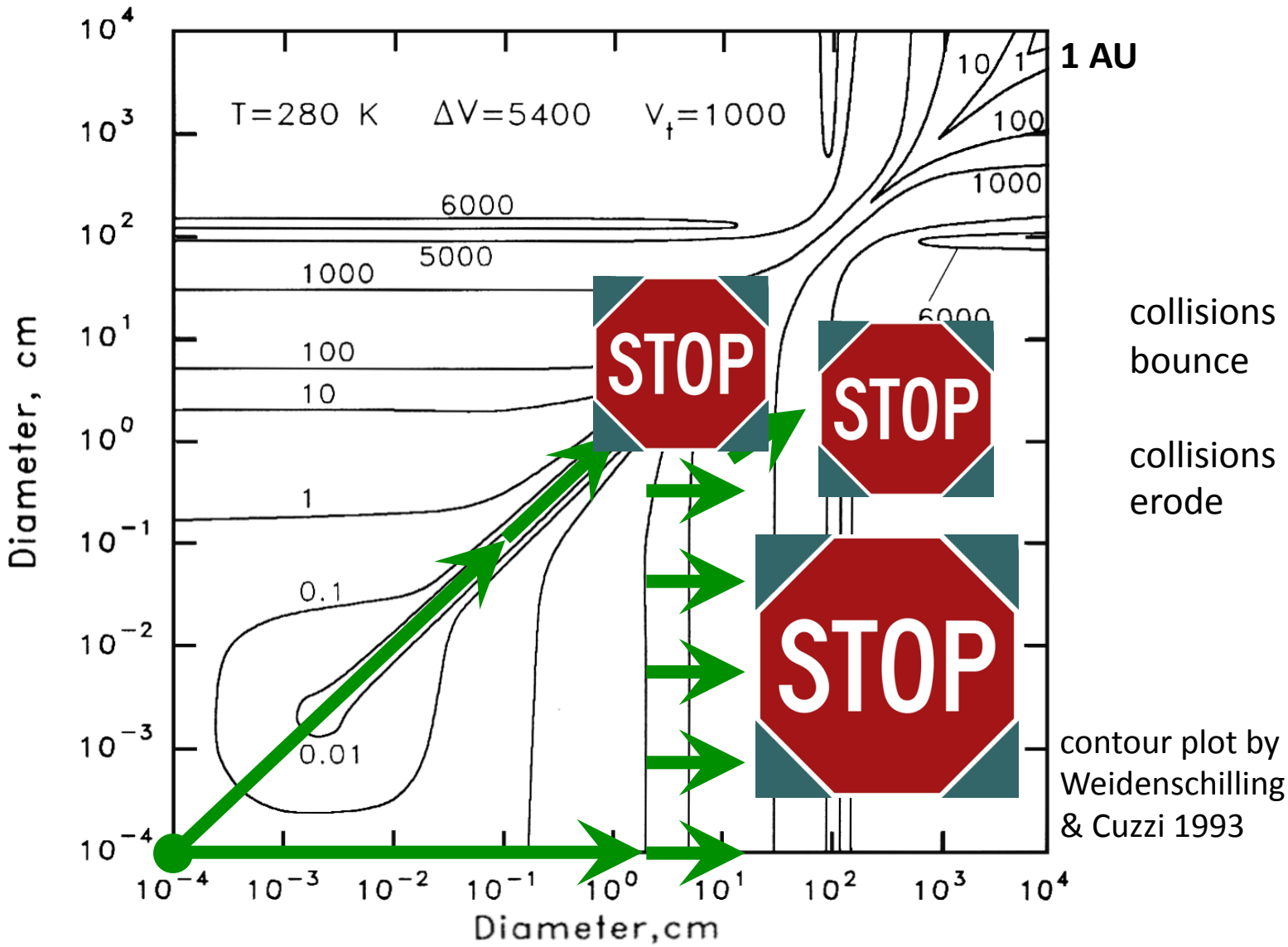
Paszun & Dominik, pers. comm.

Intermediate impact energy: compaction

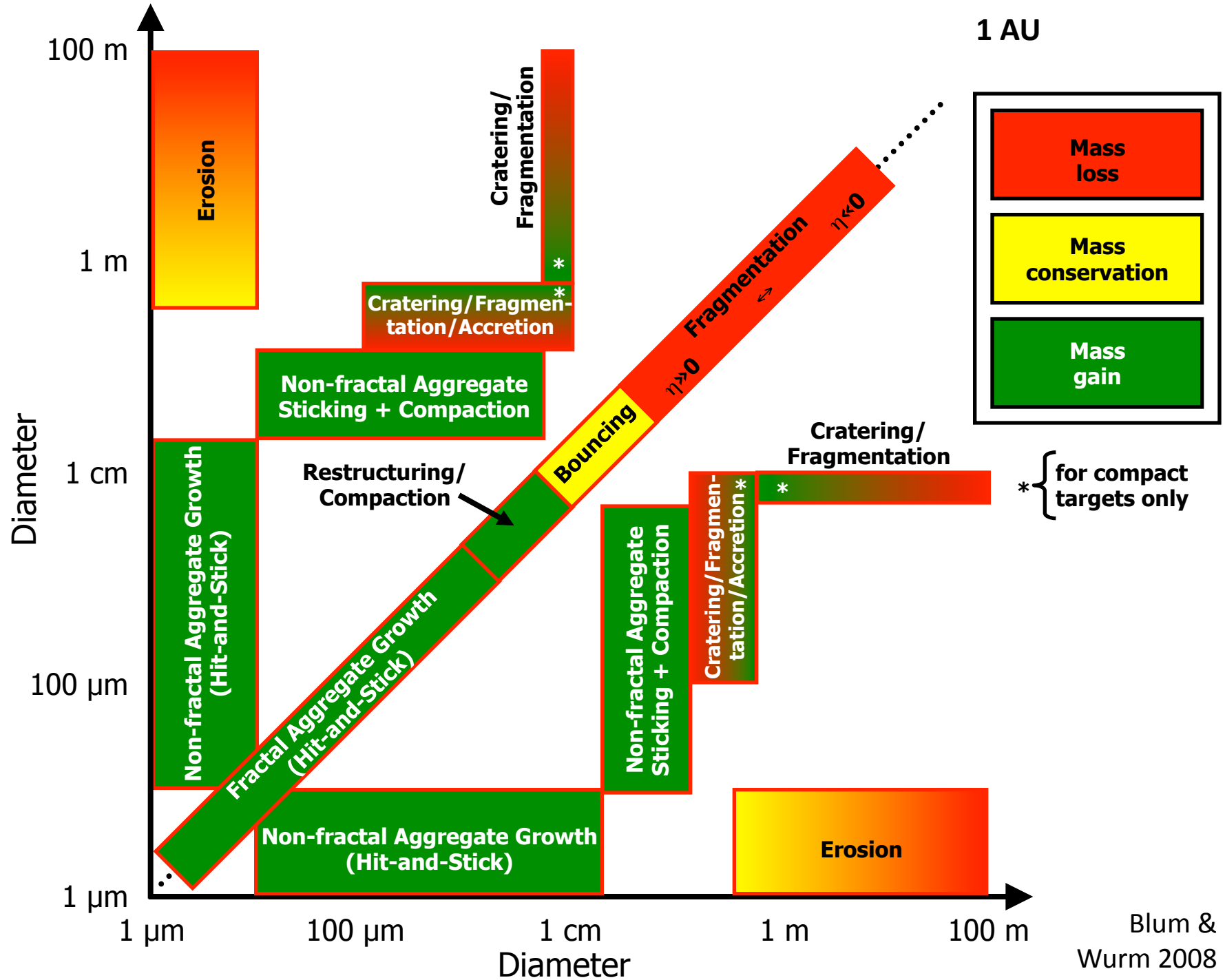
Blum & Wurm 2000

- ▷ **Collisions result in sticking**
- ▷ **Impact energy exceeds energy to overcome rolling friction**
  (Dominik and Tielens 1997; Wada et al. 2007)
- ▷ **Dust aggregates become non-fractal (?) but are still highly porous**

From a talk by Blum!



from a talk by Blum



Blum & Wurm 2008

Clumps

- In order to be self-gravitating clump must be inside its own Roche radius

$$s < r_H \qquad\qquad s < r\left(\frac{\rho_s s^3}{M_*}\right)^{1/3}$$

$$\rho_s > M_*/r^3 \gtrsim 6\times 10^{-7}\mathrm{g\ cm}^{-3}\left(\frac{M_*}{M_\odot}\right)\left(\frac{r}{1\mathrm{AU}}\right)^{-3}$$

- Concentrations above 1000 or so at AU for minimum solar mass nebula required in order for them to be self-gravitating

Clump Weber number

- Cuzzi et al. 08 suggested that a clump with gravitational Weber number We< 1 would not be shredded by ram pressure associated turbulence
- We = ratio of ram pressure to self-gravitational acceleration at surface of clump

$$We = \frac{\rho_g c^2}{G\rho_d^2 s^2} = C^{-2}\left(\frac{c}{v_K}\right)^2\left(\frac{r}{s}\right)^2\left(\frac{M_*/r^3}{\rho_g}\right)$$

- Analogy to surface tension maintained stability for falling droplets
- Introduces a size-scale into the problem for a given Concentration C and velocity difference c. Cuzzi et al. used a headwind velocity for c, but one could also consider a turbulent velocity

Growth rates of planetesimals by collisions

$$\frac{dM}{dt} = \pi s^2 \rho \sigma \left(1 + \frac{v_{esc}^2}{\sigma^2}\right)$$

- With gravitational focusing
- Density of planetsimal disk ρ,
- Dispersion of planetesimals σ
- Σ=ρh, σ=hΩ
- Ignoring gravitational focusing $\frac{dM}{dt}\Omega^{-1} \sim \Sigma(M/\rho_d)^{2/3}$
- With solution $s \propto t$

Growth rate including focusing

- If focusing is large

$$F_g = \left(1 + \frac{v_{esc}^2}{\sigma^2}\right) \propto \frac{M}{s}$$

$$\frac{dM}{dt}\Omega^{-1} \propto M^{4/3}$$

- with solution $s(t) \propto \frac{1}{C - \Omega t}$ quickly reaches infinity
- As largest bodies are ones where gravitational focusing is important, largest bodies tend to get most of the mass

Isolation mass

- Body can keep growing until it has swept out an annulus of width r_H (hill radius)

$$M = 2\pi a \Sigma r_H \qquad r_H = a(M/3M_*)^{1/3}$$

- Isolation mass of order

$$M_{iso} \sim a^3 \Sigma^{3/2} M_*^{-1/2}$$

- Of order 10 earth masses in Jovian region for solids left in a minimum mass solar nebula

Self-similar coagulation

- Coefficients dependent on sticking probability as a function of mass ratio
- Simple cases leading to a power- law form for the mass distribution but with cutoff on lower mass end and increasingly dominated by larger bodies
- dN(M)/dt =
  - rate smaller bodies combine to make mass M
  - subtracted by rate M mass bodies combine to make larger mass bodies

Core accretion (Earth mass cores)

- Planetesimals raining down on a core
- Energy gained leads to a hydrostatic envelope
- Energy loss via radiation through opaque envelope
- Maximum limit to core mass that is dependent on accretion rate setting atmosphere opacity
- Possibly attractive way to account for different core masses in Jovian planets

Giant planet formation
Core accretion vs Gravitational Instability

- Two competing models for giant planet formation championed by
  - Pollack (core accretion)
  - Alan Boss (gravitational instability of entire disk)
- Gravitational instability: Clumps will not form in a disk via gravitational instability if the cooling time is longer than the rotation period (Gammie 2001)

$$t_{cool} = \frac{U}{\sigma T^4} \gtrsim \Omega^{-1}$$

Where U is the thermal energy per unit area

  - Applied by Rafikov to argue that fragmentation in a gaseous circumstellar disk is impossible. Applied by Murray-Clay and collaborators to suggest that gravitational instability is likely in dense outer disks (HR8799A)
- Gas accretion on to a core. Either accretion limited by gap opening or accretion continues but inefficiently after gap opening

Connection to observations

- Chondrules
- Composition of different solar system bodies
- Disk depletion lifetimes
- Disk velocity dispersion as seen from edge on disks
- Disk structure, composition
- Binary statistics

Reading

- Weidenschilling, S. J. 1977, Areodynamics of Solid bodies in the solar nebula, MNRAS, 180, 57
- Dullemond, C.P. & Dominik, C. 2004, The effect of dust settling on the appearance of protoplanetary disks, A&A, 421, 1075
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- Johnson, E. T., Goodman, J, & Menou, K. 2006, Diffusive Migration Of Low-mass Protoplanets In Turbulent Disks, ApJ, 647, 1413
- Johansen, A., Oishi, J. S., Mac-Low, M. M., Klahr, H., Henning, T. & Youdin, A. 2007, Rapid Planetesimal formation in Turbulent circumstellar disks, Nature, 448, 1022
- Cuzzi, J. N., Hogan, R. C., & Shariff, K. 2008, Toward Planetesimals: Dense Chondrule Clumps In The Protoplanetary Nebula, ApJ, 687, 143
- Blum, J. & Wurm, G. 2008, ARA&A, 46, 21, The Growth Mechanisms of Macroscopic Bodies in Protoplanetary Disks
- Armitage, P. 2007 review
- Ketchum et al. 2011
- Rein, H. 2012