AST242 LECTURE NOTES PART 7

Contents

1. HII regions and Ionization Fronts

In star forming regions there are hot stars that have effective temperatures sufficiently high that they produce photons that are shorter wavelength than Lyα and so can ionized atomic hydrogen. The gas near these hot stars can then be ionized. HII regions consist of gas in equilibrium between photoionization and the inverse process which is known as recombination. Recombination is when ions capture free electrons. HII region is an astronomical term for singly ionized hydrogen. HI would be neutral atomic hydrogen.

First let us consider recombination. The recombination rate (number of recombinations per unit volume per unit time)

$$r = n_p n_e \langle \sigma v \rangle = n_p n_e \alpha(T) \tag{1}$$

where σ is a cross section and we have averaged the cross section over different velocities. Here n_p is the number density of free protons and n_e is the number density of free electrons. The number of free protons is approximately equal to the number of free electrons for a gas predominantly of hydrogen to be neutral; $n_p \sim n_e$.

Let x be the number of free electrons divided by the total number of hydrogen atoms (both ionized and neutral); $x = n_e/n_H$.

$$r = \alpha(T)n_p n_e = \alpha(T)n_e^2 = \alpha(T)x^2 n_H^2 \tag{2}$$

1.1. **The Strömgren Sphere.** Suppose we assume that inside some radius r_{strom} the gas is entirely ionized. In the center is an ionization source that emits ionizing photons at a rate Q_* (photons per second). We could estimate Q for a star

$$Q_* = \int_{h\nu=13.6 \text{ eV}}^{\infty} \frac{L_\nu}{h\nu} d\nu \tag{3}$$

Here 13.6 eV is the minimum energy needed to ionized neutral hydrogen. For an O5V star $Q_* \sim 3 \times 10^{49}$ s^{-1}. We assume that all ionizing photons are absorbed so the ionization rate integrated over the sphere is Q_*. The recombination rate integrated over the sphere

$$\int r dV = \frac{4\pi}{3} r_{strom}^3 \alpha(T) x^2 n_H^2 \tag{4}$$

If we balance the recombination rate against the ionization rate and assume that the medium is totally ionized ($x = 1$) then we find

$$r_{strom} = \left(\frac{3Q_*}{4\pi\alpha n_H^2} \right)^{\frac{1}{3}} \tag{5}$$

In reality we must not only balance ionization inside the HII region but also heating and cooling. The absorption of UV photons leads to heating. Recombination leads to emission of UV, optical and infrared photons and so leads to cooling. The balance between the two processes is achieved at an equilibrium temperature that is typically $\sim 10^4$K. At this temperature $\alpha \sim 2.6 \times 10^{-13}$ cm^3 s^{-1}. The radius of a Stromgren sphere at a density of $n_H \sim 10^4$ cm^{-3} is a fraction of a pc. This is much smaller than the mean free path due to Thompson scattering; ($\sigma_T = 6.8 \times 10^{-25}$cm^2). The optical and infrared photons emitted during recombination tend to escape easily (unless the system is young and very optically thick). An ion that has recently recombined will not necessarily be in its ground state. It will decay by emission lines (e.g., Balmer series transitions) eventually reaching the ground state. Its last transition is likely to be a Lyα photon. The lower energy emission lines will escape while the Lyα photons will be absorbed and reemitted. Consequently the energy of the ionizing photons will be reemitted primarily in photons that can escape the ionized gas. A significant fraction of the total energy absorbed is often emitted in Hα, for example, (about 1% for HII regions seen in extragalactic optical surveys).

1.2. **Early Evolution.** Consider the advance rate of the initial ionization front. The number density of electrons can be described with the following continuity equation

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}) = I - r \tag{6}$$

where I is the rate of ionizations per unit volume and r is the rate of recombinations per unit volume. We can describe the ionization rate

$$I = -\nabla \cdot J \tag{7}$$

where J is the flux of ionizing photons. Inside the front where the gas is nearly completely ionized the recombination rate $r = \alpha n_e^2$ as before. In the frame of the ionization front we can integrate the previous equation. Remember the relation for a conservation law the condition across a discontinuity. The jump condition becomes

$$n_{e,2}(s - v_2) + J = 0 \tag{8}$$

where s is the speed of the front, v_2 is the velocity inside the front, $n_{e,2}$ is the electron number density inside the front and J the flux of ionizing photons at the location of the front. The recombination rate drops out because it does not involve a derivative. The above equation has no term with the number density of electrons outside the front. This is because n_e is zero outside where the gas is not ionized. At early states we can assume that v_2 is zero and $n_{e,2}$ is the same as n_H outside the front. We assume that there is no ionizing flux outside the front. This lets us estimate the speed that the front advances

$$s \sim \frac{J}{n_H} \tag{9}$$

Early on when little of the ionizing flux is absorbed interior to the front this gives a speed

$$s \sim \frac{Q_*}{4\pi r^2 n_H} \tag{10}$$

We could have guessed at this speed just by considering how many atoms can be ionized per second and assume that there are no recombinations inside the front.

1.3. **Achieving Pressure equilibrium.** Consider what happens immediately after an O star begins to emit UV radiation. The photons are emitted and absorbed sending out an ionization front that travels very quickly (of order a fraction of the speed of light and so much faster than the sound speed). The gas does not have any time to respond. The front expands until the recombinations balance the ionizations. This happens when the ionization front is about the size of the Stromgren radius estimated above using the ambient density. At this time recombinations balance ionizations and the front stops expanding as all UV photons are absorbed. Inside

r_{strom} the gas is entirely ionized, but it is also now much hotter than it was when it was neutral. The balance between heating due to ionization and cooling due to emission of recombination lines (and other processes) heats the gas inside r_{strom} to about 10^4K. However the gas density inside is the same as it was outside. This means that interior to r_{strom} the pressure is much higher than outside. On a longer timescale the ionized region must expand until it is in pressure equilibrium with the ambient medium. We can estimate the side of a new sphere that achieves both ionization and pressure equilibrium by solving for a new density. The ration between the density inside and outside the sphere is

$$2n_{H,in}T_{in} \approx n_{H,out}T_{out} \tag{11}$$

where we can estimate T_{out} from that typical in a star formation region (100K or so) and $T_{in} \sim 10^4$K. The factor of 2 is because inside we assume the gas is totally ionized and so has both electrons and protons. Insert this into equation (5)

$$r_s = \left(\frac{3Q_*}{\pi\alpha n_{H,out}^2}\right)^{\frac{1}{3}} \left(\frac{T_{in}}{T_{out}}\right)^{\frac{2}{3}} \tag{12}$$

and we find a new radius.

How does the ionization front evolve between these two states? Between the two states we have an advancing ionization front and outward directed velocities caused by the pressure differential. The expanding bubble sweeps up ambient medium, so during expansion the bubble is surrounded by a dense cool shell of material.

1.4. **Jump conditions on an ionization front.** It turns out that complete ionization and temperature equilibrium at about 10^4K is a pretty good approximation inside the ionization front. We can assume temperature equilibrium outside the front in the ambient medium. The temperatures are set everywhere and we expect the largest pressure jump at the boundary of the front. We can consider the jump conditions at the ionization front itself.

$$\rho_1 u_1 = \rho_2 u_2 \tag{13}$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \tag{14}$$

Following our previous conventions the 1 subscript refers to pre-shock gas and we are working in the frame moving with the front. Here pre-shock means outside the ionization front where the gas is neutral. Let

$$\rho_1 = m_p n_{H,1} \qquad \rho_2 = m_p n_{H,2} \tag{15}$$

$$p_1 = n_{H,1} k_B T_1 \quad p_2 = 2n_{H,2} k_B T_2 \tag{16}$$

Note the factor of 2 in the pressure equation for the ionized gas inside the front. This implies that the gas interior to the front is entirely ionized (one electron for every hydrogen). We define two speeds

$$a_1^2 = k_B T_1/m_p \qquad a_2^2 = 2k_B T_2/m_p \tag{17}$$

with a_1 much smaller than a_2. Typically T_2 will be about 10^4K. Our jump conditions now look like

$$n_{H,1}u_1 = n_{H,2}u_2 \tag{18}$$

$$n_{H,1}(a_1^2 + u_1^2) = n_{H,2}(a_2^2 + u_2^2) \tag{19}$$

Solving for the density and velocity ratios

$$\frac{n_{H,2}}{n_{H,1}} = \frac{u_1}{u_2} = \frac{1}{2a_2^2}\left[(u_R u_D + u_1^2) \pm \sqrt{(u_1^2 - u_R^2)(u_1^2 - u_D^2)}\right] \tag{20}$$

with

$$u_R \equiv a_2 + \sqrt{a_2^2 - a_1^2}$$

$$u_D \equiv a_2 - \sqrt{a_2^2 - a_1^2} \tag{21}$$

Here u_R is for "rarefied' and u_D is for "dense." The velocity u_R is always larger than u_D. From equation (20) we see that physical solutions are found only if $u_1 > u_R$ (rarefied case) or $u_1 < u_D$ (dense case) and you never get solutions in between. The front is either called R-type or D-type. Fronts are further classified as "strong" or weak depending upon the sign in the middle of equation (20). If the plus sign is used then the density contrast is high and the front is considered "strong". Conversely if the negative sigh is used the density contrast is lower and the front is considered "weak."

When a_2 is much larger than a_1 then

$$u_R \sim 2a_2 \qquad u_D \sim \frac{a_1}{2}\left(\frac{a_1}{a_2}\right) \tag{22}$$

1.5. **Evolution of ionization fronts.** Early on in the evolution of an HII region, the initial velocity is low and so we expect an R-type front. However the velocity at the front boundary should increase with time. At some point the velocity will become high enough that the front can be called R-critical. When the front is R-critical there could be a discontinuity in the jump conditions because the ratio of densities or velocity in the frame moving with the cannot smoothly go from the R-type solutions to the D-type solutions. However, at this time the front itself will be moving close to the sound speed in the neutral ambient medium and so an acoustic shock can be generated by the expansion of the HII region. At this time the front may develop additional structure. During the second phase a D-type ionization front may lie inside an external shock front. Neutral gas is swept up into a thin shell by the expansion process.

To more carefully study ionization fronts, a number of additional factors can be taken into account. The clumpy structure of the ISM could be considered. For example, expansion could be faster in some directions than others. For old regions, some fraction of the UV radiation can escape the region rather than be absorbed. Young regions can be so optically thick that only radio emission can escape. For these "compact" HII regions estimates of their lifetime apparently are difficult to reconcile with estimates of their density and confinement by ambient material. The presence of circumstellar disks or motion in the parent molecular cloud could also affect the HII region evolution.

2. Cooling Flows

Early type galaxies and galaxy clusters can be luminous X-ray emitters. They are full of diffuse hot $T \sim 10^6 - 10^7$K gas. It has been difficult to reconcile cooling rates based on the X-ray emission with heating mechanisms and the quantity of detected cooler gas. While clusters are often discussed in terms of 'cooling flows' it is possible that there are sufficient sources of heating that they don't efficiently cool. Many models assume steady state solutions or mean properties or a single phase of the interstellar medium. These assumptions may be insufficient to capture the thermal dynamics.

2.1. **Hydrostatic equilibrium and Beta models.**

Consider a galaxy cluster filled with hot gas. We can consider a distribution in hydrostatic equilibrium.

$$\frac{\partial p}{\partial r} = -\rho \frac{\partial \Phi}{\partial r} \tag{23}$$

We adopt $p \approx 2 n_H k_B T$ (assuming that the gas is primarily hydrogen) and $\rho = n_H \bar{m}$. We can also relate the potential to the velocity of a particle in a circular orbit, $\frac{\partial \Phi}{\partial r} = \frac{v_c^2}{r}$. Neglecting temperature gradients this gives

$$\frac{2 k_B T}{\bar{m}} \frac{r}{\rho} \frac{\partial \rho}{\partial r} = -v_c^2 \tag{24}$$

Writing the derivative in a logarithmic form

$$\frac{\partial \ln \rho}{\partial \ln r} = -v_c^2 \frac{\bar{m}}{2 k_B T} \tag{25}$$

If the circular velocity is nearly constant as a function of radius then the solution is

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-\beta} \tag{26}$$

where $\beta \equiv v_c^2/a^2$ with $a^2 = \frac{2k_BT}{\bar{m}}$. These are known as beta models (though I should check the exponent and see if appropriately defined here). Typically a core radius is included into the potential and the density profile turns over at this radius.

2.2. **Cooling Rate.** The cooling rate per unit volume

$$\dot{q} = n_e n_p \Lambda(T, Z) \text{ erg s}^{-1} \text{ cm}^{-3} \tag{27}$$

For pure Brehmsstrahlung emission the cooling function

$$\Lambda_{rad} = 2.1 \times 10^{-27}\sqrt{T} \text{ erg cm}^{-3} \tag{28}$$

for T in Kelvin. The cooling time can be estimated as

$$t_{cool} \sim \frac{e}{\dot{q}} \sim \frac{n_e k_B T}{n_e^2 \Lambda_{rad}} \tag{29}$$

We have assumed $n_p = n_e$ and we have not bothered with factors of order 1 for the internal energy. From the above estimate we find that the cooling time scales with $t_{cool} \propto T^{1/2}\rho^{-1}$. The problem arises from the estimates of the density based on the beta models. The density increases with decreasing radius. The cooling timescale at smaller radii can be short. By short we mean on galactic timescales or under a few Gyrs.

Estimated cooling rates can be as high as a few hundred $\dot{M}$/yr over regions of radius of order 50 kpc and with X-ray luminosities as large as $10^{11} - 10^{12} L_\odot$. Cooling timescales in the center can be as short as order 1 Gyr. Note these luminosities are as large as entire galaxy luminosities. To stop the cooling something able to dump power of order a quasar is needed. This is why only something as energetic as kinetic energy associated with black hole outflows (and from a massive black hole) is capable of heating the medium sufficiently to stop a cooling flow.

2.3. **Cooling flows.** If there is gas cooling then one expects a slow radial inflow and a slow temperature gradient where cooler gas is found in the inner regions.

We can go back to Euler's equation and again assume a steady state solution but do note assume a zero radial velocity. Euler's equation can be written

$$\frac{\partial}{\partial \ln r}\left(\frac{u^2}{2}\right) \approx -\frac{2k_BT}{\bar{m}}\frac{\partial}{\partial \ln r}\left(\ln\rho + \ln T\right) + v_c^2 \tag{30}$$

where I have kept both temperature and density gradient. Depending on the radial temperature and density gradients the right hand side will not be zero and there can be a radial velocity which we expect would be small compared to the circular velocity.

Above we only consider a gas pressure but there could be other pressure terms. Sometimes a turbulent velocity is incorporated into the pressure. Cosmic ray or magnetic pressures may also contribute. Note here we have assumed that the gas is single phase and can be described by a bulk temperature, pressure and density as a function of radius. However cooler clouds or filaments may condense out of the hot gas and the medium could be multi-phase. There is no reason to expect that cooling is a smooth or stable process, but it may be particularly unstable at the critical temperature regime near the entropy floor. For example if small clouds serve as nucleation cites then cooling may be particularly patchy. Alternatively, an instability may be required and this may happen only in particular localized regions.

Some systems do show streamers in ionized gas, large amounts of molecular material and have star formation, though galaxies with large amounts of cooler gas remain a small subset of the wider class of cluster galaxies with luminous X-ray emission.

2.4. **Role of conductivity.** The hot outer regions might heat the inner regions via heat conduction. The energy equation

$$n\frac{\partial}{\partial t}\left(\frac{5k_BT}{2\bar{m}}\right) = -n^2\Lambda + \kappa\nabla^2 T \tag{31}$$

with thermal conductivity coefficient $\kappa \propto T^{5/2}$ depending on temperature as it depends on the velocities of particles and Coulomb interactions between them (the form of the coefficient often used is known as the Spitzer conductivity coefficient). It is difficult to imagine balancing the terms on the right hand side of the equation, and simulations that include conductivity can catastrophically cool via development of instabilities.

2.5. **Entropy Floor.** For an ideal gas the pressure

$$p = nk_BT = K\rho^{5/3} \tag{32}$$

where the right hand side has $\gamma = 5/3$ for a monotonic adiabatic gas. Adiabatic variations don't change K so the quantity K can be used to quantify entropy and is sometimes used in discussions of the "entropy floor" for the cluster medium

$$K = \frac{k_BT}{\bar{m}\rho^{2/3}} = k_BTn_e^{-2/3} \tag{33}$$

The standard thermodynamic entropy per particle $s = k_B \ln K^{3/2}+$ constant. A similar quantity is $k_BTn_e^{-2/3}$ but one can convert between K and this quantity using typical abundances.

It is desirable to normalize K. One choice for normalization is to use the mass inside the radius r_{200}. The radius r_{200} is that for which the mass interior to this radius is 200 times the critical cosmological density, ρ_{cr}. The mass M_{200} is then the mass interior to r_{200}. The characteristic temperature T_{200} is that such that $\frac{2k_BT_{200}}{\bar{m}} = \frac{GM_{200}}{r_{200}}$.

With a global baryon fraction $f_b = \frac{\Omega_b}{\Omega_M}$ and the other virial quantities we can define a characteristic entropy scale

$$K_{200} = \frac{k_B T_{200}}{\bar{m}(200 f_b \rho_{cr})^{2/3}} \tag{34}$$

Cooling reduces entropy. It is possible to solve for the critical entropy as a function of temperature for which the gas radiates an energy equivalent to its thermal energy in a time t_0. This threshold entropy is close to the observed entropy floor renormalized value of the entropy for a timescale similar to the age of the universe. The ratio depends on redshift presenting an additional problem as the threshold would have been even higher in the past. One solution is to have a feedback mechanism dependent on the entropy threshold (as proposed by Mark Voit and collaborators). Entropy gradients tend to increase with increasing radius presenting a challenge for feedback models as they must be constrained to put heat into the gas evenly and not reverse the entropy gradient. One mechanisms is driving sound waves that dissipate through conduction (Fabian et al. 2003).

Suppose some cooling does happen. While star formation is unlikely to provide enough energy to significantly heat the cluster gas it can serve as a sink for cold gas.

2.6. **Chandra observations.** Chandra observations have revealed the following:

Chandra spectroscopy failed to find evidence for lowish temperature gas (emission line emitting rather than dominated by Brehmstrahlung and of order upper 10^5 K). This and previous results are sometimes described in terms of an "entropy" floor (Peterson et al?).

Even quiescent elliptical galaxies can show complex X-ray structure. This structure may be telling us about episodes of past radio galaxy activity. (e.g. NGC 4636?)

Galaxies like M87 exhibit cavities in the X-ray emitting gas that are filled by radio lobes. Lots of detail recently found at low radio frequencies. These lobes are filled with cosmic rays. They could be buoyant. They could be exciting sound waves into the X-ray emitting gas which when damped by conduction heat the X-ray gas. The kinetic energy estimated from these lobes (and associated PdV evacuation work) is large enough to heat the X-ray gas sufficiently to stop cooling flows.

The energy estimated to push a cavity is of order pdV. The volume is estimated from the area on the sky, the pressure in the ambient material from the X-ray emission. Pressure is of the same size as the internal energy density. If the volume filled by lobes is similar to the volume of X-ray emitting gas then we would estimate that $pdV \sim E$ the total energy. If radio lobes are continually displacing significant fractions of the hot gas in a Gyr timescale then a heating rate can be similar to the cooling rate.

See Blanton's review talk and the 2003 on line conference book. Some statements are contradicted by Cen A's southern lobe which does have a high Mach number and hot shock edge.

Note this section is severely lacking in detail and citations. The PdV and entropy floor work should be properly cited. M87 and Perseus A and Cen A and NGC 4636 studies could be cited (and other beautiful examples). I would like to add more on role of possible role of cosmic rays and conduction and the turbulent α type of models.

For more information:
http://www.astro.virginia.edu/coolflow/proc.php

3. Acknowledgements

Following Shu's book on ionization fronts.

On cooling flows I at first followed the 1994 Fabian ARAA review, and the related article on ned. Then I was inspired by a 2003 conference proceeding article by Mark Voit and an on-line talk by Elizabeth Blanton.