

# AST242 LECTURE NOTES PART 10 – COSMIC MICROWAVE BACKGROUND

## CONTENTS

### 1. COSMIC MICROWAVE POWER SPECTRUM

The temperature anisotropy on the sky can be decomposed into spherical harmonics

$$(1) \quad \Theta(\theta, \phi) = \frac{\Delta T}{T}(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$$

Note that because  $\Theta$  is real

$$(2) \quad a_{lm}^* = (-1)^m a_{l-m}$$

The coefficients  $a_{lm}$  are assumed to be statistical random variables with a zero mean and a variance described by the power spectrum. The expectation value

$$(3) \quad \langle a_{lm} a_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}$$

The coefficients  $C_l$  can be computed from the correlation function of  $\Theta$  or

$$(4) \quad C(\alpha) = \langle \Theta(\mathbf{n}) \Theta(\mathbf{m}) \rangle$$

$$(5) \quad = \sum_{lm} \sum_{l'm'} \langle a_{lm} a_{l'm'}^* \rangle Y_{lm} Y_{l'm'}^*$$

$$(6) \quad = \sum_l C_l \frac{2l+1}{4\pi} P_l(\cos \alpha)$$

where  $\cos \alpha = \mathbf{n} \cdot \mathbf{m}$  and  $\mathbf{n}, \mathbf{m}$  are directions on the sky. The third step above requires an assumption of statistical isotropy. The mean-square anisotropy

$$(7) \quad \langle (\Delta T)^2 \rangle = T^2 C(0) = T^2 \sum_l C_l \frac{2l+1}{4\pi} \approx T^2 \int \frac{l(l+1)C_l}{2\pi} d \ln l$$

where the last approximation is valid for large  $l$ . We can relate the  $C_l$  coefficients to plane wave wavelengths and directions. We work with conformal position and time

$\mathbf{x}, \eta$ . Consider a plane wave with wavevector  $\mathbf{k}$  and  $\hat{\mathbf{k}} = \frac{\mathbf{k}}{|\mathbf{k}|}$ .

$$(8) \quad \Theta = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}_0} \sum_l (-i)^l (2l+1) a_l(\underline{\mathbf{k}}, \eta) P_l(\hat{\mathbf{k}} \cdot \mathbf{n})$$

and that finally

$$(9) \quad C_l = \frac{2}{\pi} \int \frac{dk}{k} k^3 \langle |a_l(k, \eta)|^2 \rangle$$

1.1. **xxxx.**