Today in Astronomy 142

• Stellar mass black holes
• The massive black hole at the Galactic Center

Figure: artist's conception of a blue supergiant - black hole binary system. (Dana Berry, Honeywell/NASA.)
Escape speeds from stars, white dwarfs and neutron stars: relativistic stars

Neglecting relativity,

\[ E = \frac{1}{2} m v^2_{\text{esc}} - \frac{G M m}{R} = 0 \]

\[ v_{\text{esc}} = \sqrt{\frac{2 G M}{R}} \]

= 619 km s\(^{-1}\) from the Sun

=6970 km s\(^{-1}\) from a 1\(M_\odot\) white dwarf

=154000 km s\(^{-1}\) from a 1\(M_\odot\) neutron star

\[ v_{\text{esc}} = c = 299792 \text{ km s}^{-1} \text{ when } R = \frac{2 G M}{c^2} \]

Schwarzschild radius
Beyond the neutron-star maximum mass: black holes

- The maximum mass of a neutron star is about $2 \, M_\odot$. There is no physical process that can support a heavier object without internal energy generation. (no source of pressure that will give hydrostatic equilibrium solution)

- A heavier object will collapse past neutron-star dimensions, and soon thereafter becomes a **black hole**: an object from which even light cannot escape if it lies within

$$R_{\text{Sch}} = \frac{2GM}{c^2}$$

of the object, as measured by a distant observer; this spherical surface is called the **event horizon**.

- With strong gravity, one **cannot** ignore general relativity. Curvature of space time due to mass
Near black holes

Near a black hole, according to general relativity:

- Clock-tick time intervals appear slow compared to those of identical clocks at large distance $r$, an effect known as gravitational time dilation or the gravitational redshift:

$$\Delta t_{\infty} = \Delta t \sqrt{1 - \frac{2GM}{rc^2}} < \Delta t_{\infty}$$

- Thus time appears (to a distant observer) to stop at the event horizon.

- Length $\Delta L$ measured simultaneously between two points on a radial line is greater than the coordinate distance $\Delta r$ (that measured by a distant observer):

$$\Delta L = \frac{\Delta r}{\sqrt{1 - \frac{2GM}{rc^2}}} > \Delta r$$
Observation consequences of time dilation
gravitation redshift

Consider a photon coming from the surface of a neutron star. It is emitted in the rest frame of the neutron star, then travels out of the gravitational potential well. A distant observer, stationary w.r.t the neutron star, will detect a redshifted photon. An emission line spectrum would be redshifted

$$\frac{\Delta \lambda}{\lambda} = \left(1 - \frac{2GM}{rc^2}\right)^{-1/2} - 1 \sim \frac{GM}{rc^2}$$
Gravitational redshift (continued)

\[ \frac{\Delta \lambda}{\lambda} = \left( 1 - \frac{2GM}{rc^2} \right)^{-1/2} - 1 \]

\[ \frac{\Delta \lambda}{\lambda} = \left( 1 - \frac{r_{Sch}}{r} \right)^{-1/2} - 1 \sim \frac{R_{sch}}{2r} \]

What sizescale for redshift from a WD? A neutron star?
Embedding diagrams

Circles in flat spacetime: \( C = 2\pi r = \pi d \). That is the very definition of\( \pi \).
Circular orbits in flat space (all in the same plane)

Distances between the orbits:

Local and distant observers would report the same distances.
Circular orbits in space warped by a black hole, same circumferences as before (still all in same plane)

Distances between the orbits, measured by a local observer:

1.057 1.074 1.106 1.135 1.185

The distances would still all be 1, in the viewpoint of a distant observer. Circumferences are the same in both viewpoints.

Horizon (circumference = 2\pi)
One way to visualize warped space: “hyperspace”

To connect these circles with segments of these “too long” lengths, one can consider them to be offset from one another along some imaginary dimension that is perpendicular to $x$ and $y$ but is not $z$. (If it were $z$, the circles wouldn’t appear to lie in a plane!). Such additional dimensions comprise **hyperspace**.
This is why you see the equatorial plane of a black hole represented as a funnel-shaped surface, as if made from a stretched rubber sheet. It’s important to note that the direction of the stretch is in hyperspace, though, so the scene would not look like a funnel to your eyes, which see just the three usual spatial dimensions.
Equivalence principle

A uniform gravitational field in one direction is indistinguishable from a uniform acceleration in the opposite direction.

You can’t tell the difference between being accelerated and being in a gravitational field.

If you are in an elevator you can’t tell the difference between a strong gravitational field and being accelerated upward.

All physics is the same otherwise.

Acceleration is independent of mass.

Gravitational acceleration does not depend on mass.
Differential Geometry

At every spot on the “manifold” we associate a flat Minkowski vector space (consistent with special relativity).
The manifold is “differentiable” so we can move smoothly from one spot to another.
We define distances between points with a “metric.”
For Minkowski space \( ds^2 = -c^2 \, dt^2 + dx^2 + dy^2 + dz^2 \)

Schwarzschild metric, near a black hole of mass \( M \)

\[
\begin{align*}
    ds^2 &= -c^2 \, dt^2 \left( 1 - \frac{2GM}{c^2 r} \right) + \frac{dr^2}{\left( 1 - \frac{2GM}{c^2 r} \right)} \\
    &\quad + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right)
\end{align*}
\]
Differential geometry

The metric allows distances to be calculated and is away to describe curving of space time.

Trajectories of particles under gravity alone (no other forces or accelerations) are then “geodesics” or lines that minimize the distance between two points.

Light travels on null geodesics  (note the negative sign which follows from special relativity)

\[
ds^2 = -c^2 dt^2 \left(1 - \frac{2GM}{(c^2 r)}\right) + \frac{dr^2}{\left(1 - \frac{2GM}{(c^2 r)}\right)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)
\]
The ISCO

\[ ds^2 = -c^2 dt^2 \left(1 - \frac{2GM}{c^2 r}\right) + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

Schwarzschild radius

\[ R_{\text{Sch}} = \frac{2GM}{c^2} = 2M \text{ (units } G=c=1) \]

apparent singularity in Schwarzschild metric

Radius of the Innermost Stable Circular Orbit (ISCO)

\[ R_{\text{ISCO}} = 6M = 3R_{\text{Sch}} \]
Light is bent near heavy objects

To order of magnitude we can estimate the angle of deflection. It depends on $r, G, M, c$

The only unitless relation would be

$$\frac{GM}{rc^2}$$

The angle of deflection turns out to be

$$\theta = \frac{4GM}{rc^2}$$
about black holes (continued)

- Orbits outside the BH, further away than 1.5 $R_{\text{Sch}}$ (in the coordinate system of a distant observer) still turn out to be ellipses.
- The resulting **coordinate speed** in orbit (for the coordinate system of a distant observer) is the same as is obtained for Newtonian gravity:

$$v \equiv r \frac{d\phi}{dt} = \sqrt{\frac{GM}{r}}$$

- At the horizon, the coordinate speed of light is zero: **light cannot escape**. Thus no information can reach a distant observer from, or within, the horizon.
about black holes (continued)

- Orbits with coordinate radius $< 3 \, R_{\text{Sch}}$ are unstable to small perturbations, if the black hole doesn’t spin.
- There are no orbits with coordinate radius $< 1.5 \, R_{\text{Sch}}$ for a non-spinning black hole; at this radius the local orbital speed is the speed of light, and smaller orbits would require (impossibly) higher speeds.
  - Thus you can’t orbit there, because your rest mass isn’t zero, but if you could, you could train your binoculars straight ahead (in the $\phi$ direction) and see the back of your head.
  - To get closer to the horizon, one would have to descend vertically and balance gravity with thrust, as in a rocket launch.
  - If the black hole spins, the innermost stable orbit and the photon orbit are smaller than 3 and 1.5 $R_{\text{Sch}}$ if the particle orbits in the same direction as the spin, and larger if it orbits in the opposite direction.
Interesting facts about black holes (continued)

Within $r = 1.5 \ R_{\text{Sch}}$, all geodesics (paths of light) terminate at the horizon.

- Thus: from near the horizon, the sky appears to be compressed into a small range of angles directly overhead; the range of angles is smaller the closer one is to the horizon, and vanishes at the horizon. (The objects in the sky appear bluer than their natural colors as well, because of the gravitational Doppler shift).

- Space itself is stuck to the horizon, since one end of all the geodesics are there. If the horizon began to rotate, the ends of the geodesics would rotate with it. (This harmonizes with time stopping there.)
about black holes (continued)

- Gravitational acceleration

\[
a = \frac{GM}{r^2} \frac{1}{\sqrt{1 - 2GM/rc^2}}
\]

which has its familiar Newtonian form at large distances but blows up at \( r = R_{\text{Sch}} \). Thus, in a vertical descent to a hovering position just above the horizon, very large gravitational accelerations would be encountered.
Tidal forces near a black hole are as you’d expect from Newtonian gravity. For an object of length $\Delta r$ in the radial direction and $\Delta x$ in the crosswise directions,

$$\Delta a_r = \frac{2GM}{r^3} \Delta r = \frac{c^6}{4G^2M^2} \Delta r$$

close to the event horizon.

$$\Delta a_\theta = \Delta a_\phi = \frac{GM}{r^3} \Delta x = \frac{c^6}{2G^2M^2} \Delta x$$

For a 2 m person and a 10 $M_\odot$ BH, the radial tidal acceleration at the event horizon is $2 \times 10^{10}$ cm sec$^{-2}$ ($2 \times 10^7$g). It is 1g for a 4.6 $\times$ 10$^4$ $M_\odot$ BH. Thus if you want to fall freely past the horizon of a BH to see what happens, choose a large one, so as not to be torn apart before you get there.
Yet black holes emit light!

Details of the process, called Hawking radiation:

- Virtual particle-antiparticle pairs, produced briefly by vacuum fluctuations, can be split up by the strong gravity near a horizon. Both of the particles can fall in, but it is possible for one to fall in with the other escaping.

- The escaping particle is seen by a distant observer as emission by the black hole horizon: black holes emit light (and other particles)!

- The energy conservation debt involved in the un-recombined vacuum fluctuation is paid by the black hole itself: the black hole’s mass decreases by the energy of the escaping particle, divided by \( c^2 \). The emission of light (or any other particle) costs the black hole mass and energy.
Hawking radiation

Total Flux depends on Area of black hole event horizon

\[
A = 4\pi R_{Sch}^2 = 4\pi \left( \frac{2GM}{c^2} \right)^2 = 16\pi \frac{G^2 M^2}{c^2}
\]

\[
= 16\pi M^2 \quad \text{units } G=c=1
\]

Thermal, described with a temperature
Black hole evaporation

- Hawking radiation is emitted more efficiently if the gravity at the horizon is stronger. Recall: horizon gravity is stronger for smaller-mass black holes.
- Emission is the same as that of a blackbody at temperature
  \[ T = \frac{hc^3}{16\pi^2kGM} \]
- Thus an isolated black hole will eventually evaporate
  - Evaporation takes this long –
    - \(10^9 M_\odot\) black hole: \(10^{94}\) years.
    - \(2 M_\odot\) black hole: \(10^{67}\) years.
    - \(10^8\) gram black hole: 1 second (!)
“Black holes have no hair”

Meaning: after collapse is over with, the black hole horizon is smooth: nothing protrudes from it; and that almost everything about the star that gave rise to it has lost its identity during the black hole’s formation. No “hair” is left to “stick out.”

- Any protrusion, prominence or other departure from spherical smoothness gets turned into gravitational radiation; it is radiated away during the collapse.
- Any magnetic field lines emanating from the star close up and get radiated away (in the form of light) during the collapse.
- The identity of the matter that made up the star is lost. Nothing about its previous configuration can be reconstructed.
“Black holes have no hair” (continued)

- Even the distinction between matter and antimatter is lost: two stars of the same mass, but one made of matter and one made of antimatter, would produce identical black holes.

The black hole has only three quantities in common with the star that collapsed to create it: **mass, spin and electric charge**.

- That is, in common with the star as it was immediately before the formation of the horizon…

- Only very tiny black holes can have much electric charge; stars are electrically neutral, with equal numbers of positively- and negatively-charged elementary particles.

- Spin makes the black hole depart from spherical shape, but it’s still smooth.
Distinctive features that can indicate the presence of a black hole

Observe **two or more** of these features to find a black hole: 
**Gravitational deflection of light**, by an amount requiring black hole masses and sizes.

**X-ray and/or γ-ray emission** from ionized gas falling into the black hole.

Orbital motion of nearby stars or gas clouds that can be used to infer the mass of (perhaps invisible) companions: a **mass too large to be a white dwarf or a neutron star** might correspond to a black hole.

**Motion close to the speed of light**, or apparently greater than the speed of light (“superluminal motion”).

**Extremely large luminosity** that cannot be explained easily by normal stellar energy generation.

**Direct observation of a large, massive accretion disk.**
Paradigm stellar-mass black hole: GRO J1655-40

GRO J1655-40 is an X-ray transient source discovered by the Compton GRO in 1994. Appeared as a nova in visible light.

- Rapidly-variable X-ray emission: time scales of these variations show that the object is a few hundred km. Very small.

- Has a stellar companion, a star rather similar to the Sun (about 1.1 $M_{\odot}$). Is a single line spectroscopic binary. The period is 2.92 days, and the velocity amplitude 90 km s$^{-1}$. Thus the mass function is 3.2 $M_{\odot}$.

- Inclination estimated at $\sim 65^\circ$. Constraint on binary mass ratio via rotational broadening of the stellar companion.

- Thus we know the mass of the X-ray bright companion rather precisely: it must be between 5.5 and 7.9 $M_{\odot}$, with a most probable value of 7.0 $M_{\odot}$, way too much to be a neutron star (Shahbaz et al. 1999)

- Also has radio jets with motions close to the speed of light, tilted 85$^\circ$ from the line of sight.
GRO J1655-40
(continued)

Jets: speed 0.92c, with some of the ejecta on the left moving at (projected) superluminal velocities. (We’ll discuss superluminal motion later in the context of quasars.)

Thus: it’s too small and massive to be a white dwarf or a neutron star, is X-ray bright, and is associated with relativistic ejection speeds: sounds like a black hole.

(radio maps from Hjellming and Rupen, NRAO)
GRO J1655-40 black hole spin

A 7 $M_\odot$ nonspinning black hole has a horizon circumference 130 km, and an innermost stable orbit circumference of 390 km. Material in this orbit will circle the black hole 314 times per second.

- However, one often sees the X-ray brightness of GRO J1655-40 modulate at 450 times per second for long stretches of time (Tod Strohmayer 2001, *ApJL* 552, L49).
- Nothing besides very hot material in a stable orbit can do this so reproducibly at this frequency.
- Thus there are stable orbits closer to the black hole than they can be if the black hole doesn’t spin.
Most probably, the black hole in GRO J1655-40 is spinning at about 40% of its maximum rate. Within the uncertainties the spin rate lies in the range 12%-58% of maximum; zero spin is quite improbable.

In blue: innermost stable orbits per second for 7.0 $M_{\odot}$ black holes, with uncertainties.

In red: measured orbits per second, with uncertainties (by Tod Strohmayer, with the Rossi X-ray Timing Explorer).
GRO J1655-40 (continued)

The picture of GRO J1655-40 which emerges from these results.

- Like GRO J1655-40, all the other stellar-mass black holes we know (~40 to date) belong to X-ray emitting binary systems.
- Without X-ray emission they would be inconspicuous.

Animation by Dana Berry, Honeywell/ NASA
White dwarfs, neutron stars and black hole horizons

The diagram shows the relationship between circumference (cm) and mass (M☉) for white dwarfs (WD), neutron stars (NS), and black holes (BH). The Earth and Rochester are indicated on the circumference axis.
The Galactic Center region is filled with relativistic electrons and magnetic fields, producing strong radio emission. The nucleus is marked by a bright radio source, Sagittarius A, which looks like an exploding bubble.
The central radio emission

**Sagittarius A East (blue):** a hypernova remnant, which was produced by a violent explosion only several tens of thousands of years ago. The origin is unknown. Explanations range from a star disrupted by a black hole to a chain reaction of ordinary supernovae or even a gamma-ray burst.

**Sagittarius A West or Minispiral (red):** Gas and dust streamers ionized by stars and spiraling around the very center, possibly feeding the nucleus.

**Sagittarius A *:** A bright and very compact radio point at the intersection of the arms of the Minispiral (difficult to see in this image)
Rapid motions of stars near Sag A*

If one looks at this region with big telescopes and near-infrared cameras one can see lots of stars. If one takes pictures every year it seems that some stars are moving very fast (up to 1500 kilometers per second). The fastest stars are in the very center - the position marked by the radio nucleus Sagittarius A* (cross).

Figure: Near-infrared image of stars in the Galactic Center produced by Eckart & Genzel.
The dark mass

What stirs up the stars? The only answer we can find is that a huge dark mass in the center pulls them around with its gravitational force. Using the velocities one can calculate, with the help of Kepler's law, how much mass is required within a certain distance from the nucleus. If the mass distribution is determined by the stars in the Galaxy the "enclosed mass" should become smaller and smaller going inwards, because less and less stars are included.

However, when approaching Sagittarius A*, the mass remains constant indicating a large amount of mass in a very small region. The mass one finds is 2.6 million times the mass of our sun (or a trillion times the mass of the earth).
Massive Stars near the Galactic Center

How did a star with such a small pericenter get in such a close orbit?
We still do not know whether we are really dealing with a black hole or something even more exotic. The defining feature of a black hole is its event horizon - the final barrier between our world and the hole. Even light which comes too close to the event horizon will be attracted by the black hole and disappear into the event horizon forever. This leads to an unusual optical appearance of any emission in the very vicinity around a black hole. Here are some calculations obtained with a ray-tracing code for an optically thin emission in region which exhibit an interesting feature: literally a "black hole" in the middle. This is what we call the shadow cast by the black hole - the photons (light) we do not see have vanished into the event horizon.

From Falcke, Melia & Agol 2000
Fermi Bubble in the Galactic Center region

Credit NASA/DOE/Fermi LAT/D. Finkbeiner et al.
Summary

- Black holes as end states
- Some phenomenology of black holes
- The Schwartzschild radius
- The ISCO
- Gravitational redshift
- Bending of light rays near a massive object
- The Galactic Center