Today in Astronomy 142

- The extragalactic distance scale II: Hubble’s Law.
- Active galaxies I: quasars and their rather large luminosities. Eddington Luminosity

Jet and disk associated with the active nucleus of the large E0 galaxy M87 (HST; NASA/STScI).
Hubble’s Law

After his Cepheid-variable distance determination to M 31 that settled the Shapley-Curtis debate, Edwin Hubble continued to search for Cepheids in galaxies for which Slipher, Pease and Humason were spectroscopically determining radial velocities. By 1929 he had detected Cepheids in ten galaxies with measured radial velocities.

- He used these galaxies to calibrate yet another standard candle: the luminosity of the brightest individual star in a spiral galaxy. This could in principle be used for galaxies too distant in which to detect Cepheids.

- From observations of galaxies in clusters he noticed that galaxies of the same shape (“Hubble type”) were all about the same size. With Cepheid distances he determined that size for nearby examples, and could thereafter use galaxies of those types as standard rulers.
Hubble’s Law (continued)

Now having more than two dozen galaxies with measured radial velocity and distance, he plotted the two quantities and revealed a linear relationship between them:

\[ v_r = H_0 d \]

Hubble’s and Humason’s original (1929) data and result:

\[ v_r = H_0 d, \]

\[ H_0 = 464 \text{ km sec}^{-1} \text{ Mpc}^{-1} \]
The extragalactic distance scale, extended

Hubble realized that this relation would be a distance indicator, since the radial velocity of a galaxy could be determined completely independently of brightness or shape.

- Though the form he determined for the law was correct, the value Hubble inferred for $H_0$ was large, enough to cause concern even then. (It made the Milky Way look like the largest galaxy in the Universe, by far.) This was because...

- the Cepheid calibration was still corrupted by effects of extinction and multiple populations of Cepheids.

- Hubble’s Law also, implies that the Universe expands.
# Modern values of the Hubble constant

\[
H_0 = 57 \pm 4 \text{ km sec}^{-1} \text{ Mpc}^{-1} \quad \text{Sandage, Tammann and Saha (1996), using HST Cepheid galaxy distances to calibrate supernovae of type Ia as standard candles.}
\]

\[
H_0 = 64 \pm 3 \text{ km sec}^{-1} \text{ Mpc}^{-1} \quad \text{Riess, Press and Kirschner (1996), using similar methods.}
\]

\[
H_0 = 73 \pm 5 \text{ km sec}^{-1} \text{ Mpc}^{-1} \quad \text{The HST Hubble Constant Key Project team, using Cepheid distances for galaxies (e.g. Freedman 1996).}
\]
Recent Hubble-constant measurements

**SH0ES** and **CHP** teams:

\[ H_0 = 74.2 \text{ km sec}^{-1} \text{ Mpc}^{-1}. \]

- The SH0ES team’s method is thoroughly modern.
  - Only standard candles used are Cepheids and SNe Ia, which now overlap considerably in distance.
Recent Hubble-constant measurements (continued)

- The **WMAP** and **Planck** missions derive $H_0$ indirectly from model fits to cosmic-background anisotropies, and they get smaller values.

- Despite their adoption of SNe Ia, **Sandage & Tammann et al.** use 8 (!) standard candles, and multiple telescopes/instruments, including photographic emulsion. Thus they risk systematic error.
Redshift and radial velocity

In analogy with the form of the nonrelativistic of the Doppler shift expressed in terms of wavelengths,

\[ \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v_r}{c} \rightarrow \lambda = \lambda_0 \left( 1 + \frac{v_r}{c} \right) \]

astronomers define the redshift, \( z \):

\[ \lambda = \lambda_0 \left( 1 + z \right) \rightarrow z = \frac{\lambda - \lambda_0}{\lambda_0} . \]

This form is used for all radial velocities, even if they’re close to the speed of light; remember when you use it that \( c z \) is only if \( v_r \approx v_r \ll c \).

The largest redshift measured for a galaxy to date is \( z = 6.1 \).
The largest in Hubble’s original sample was \( z = 0.004 \).
SN Ia apparent magnitude and distance

For convenience, apparent magnitudes of SNe Ia are often plotted, or referred to, instead of distance. The translation:

\[ m_V^0 = M_V^0 + 5 \log \frac{d}{10 \text{ pc}} = M_V^0 + 5 \log \left( \frac{d}{10 \text{ pc}} \cdot \frac{10^6 \text{ pc}}{\text{Mpc}} \right) \]

\[ = M_V^0 + 5 \log \frac{d}{\text{Mpc}} + 25 \]

\[ \log \frac{d}{\text{Mpc}} = 0.2m_V^0 - 0.2M_V^0 - 5 \]

The absolute \(V\) magnitude of a SN Ia is (Riess et al. 2011):

\[ M_V^0 = -19.14 \]
SN Ia Hubble diagram

\[ v_r = cz = H_0 d \]
\[ \log cz = \log H_0 + \log d \]

The intercept of the log-log plot gives \( H_0 \).

Usually \( cz \) is in km/sec and \( d \) in Mpc.

\[ 0.2m_v^0 \text{ (mag)} = \log \left( \frac{d}{\text{Mpc}} \right) + 0.2M_V^0 + 5 \]
Active galaxies: the discovery of quasars

- Identified by visible-light astronomers as stars with extremely peculiar spectra (1950s).
- Actually reminiscent of the bright, blue, star-like galaxy nuclei of some spiral galaxies discovered in the 1940s by Carl Seyfert (and earlier by Milton Humason), but this went unnoticed at the time because no “nebulosity” was ever photographed in the surroundings of quasars.
- Maarten Schmidt (1963) was the first to realize that the spectrum of one quasar, 3C 273, is fairly normal, but seen with a radial velocity of about 48,000 km/sec (that is, $z = 0.16$).
Discovery of quasars (continued)

- Hubble’s Law implies that the quasars are very distant. 3C 273 lies at \( d = \frac{v}{H_0} = 740 \text{ Mpc} = 2.4 \text{ Gly} \).

- Yet they are bright: the quasars are extremely powerful. 3C 273 has an average luminosity of \(10^{12} L_\odot\), about 100 times that of the entire Milky Way and comparable to that of the very brightest galaxies known.

- Observations show that quasars consist of a bright core and a long, thin jet, and that the cores are very small.
  - Radio observations show directly that most of the brightness in 3C 273 is concentrated in a space smaller than 3 pc in diameter.
The brightness of quasars is highly, and randomly, variable. 3C 273 can change in brightness by a factor of 3 in only a month. This means that its power is actually concentrated in a region with **diameter no larger than one light-month** (= $7.9 \times 10^{16}$ cm = 5300 AU). Some of the more violent quasars vary substantially in an hour (1 light-hour = 7.2 AU).

In the 1980s, the suspicions of most astronomers were confirmed when imaging observations with CCD cameras revealed that quasars are the nuclei of galaxies. Until good CCDs were available, the “fuzz” comprising the galaxy surrounding each quasar was lost in the glare of the quasar.
A quasar host galaxy at $z = 0.33$

A New Technology Telescope (NTT) image of the host galaxy of a quasar at a redshift of 0.33. The quasar’s image is heavily overexposed, in order to show the galaxy (Roennback, van Groningen, Wanders and Orndahl 1996).
Quasars and host galaxies

HST observations: Bahcall and Disney 1997.
How are quasars powered?

Requirements: need to make $10^{12} L_\odot$ in a sphere with diameter $2.5 \times 10^{17}$ cm or smaller.

Here are a few ways one can produce that large a luminosity in that small a space.

- **Stellar power**: $10^7$ stars of maximum brightness, $10^5 L_\odot$.
  **Problem**: such stars only live $10^6$ years or so, and galaxies (and the Universe) must be more like $10^{10}$ years old. We see so many quasars in the sky that longer lifetimes than that are required.
How are quasars powered (continued)?

- **Stellar power II**: $10^{12}$ solar-type stars. Their lives would be long enough to explain the numbers of quasars we see.

**Problems:**

- stars would typically be only about $6 \times 10^{12}$ cm apart, less than half the distance between Earth and Sun. They would collide frequently, and soon you wouldn’t have your $10^{12}$ solar-type stars.
- they would weigh $10^{12} M_\odot$. The Schwarzschild radius for that mass is about 2 ly, larger than that of the space in which they’re confined. Thus if you assembled that collection of stars, they would form a black hole.

So let’s consider a black hole, gravitationally accreting matter.
How are quasars powered? (continued)

- **Accretion power**: accretion of mass by a black hole. Let a black hole accrete a mass \( m \); the energy released in the form of radiation is

\[
E = \varepsilon mc^2,
\]

where the efficiency \( \varepsilon \leq 1 \) (0.1 is considered reasonable).

\[
L = \frac{dE}{dt} = \varepsilon c^2 \frac{dm}{dt}
\]

\[
\frac{dm}{dt} = \frac{L}{\varepsilon c^2} = 0.7 M_\odot \text{year}^{-1} \quad \text{for } \varepsilon=0.1, \quad L = 10^{12} L_\odot.
\]

This is an infinitesimal drain on the total mass of a galaxy, so accretion power seems feasible.
Will any black hole do?

Not really. The large luminosity itself can stop the accretion by the outward pressure the light exerts on infalling material.

- So accretion will be able to take place steadily only if the force of gravity the black hole exerts on the infalling material exceeds the force from radiation pressure.

- Thus the more massive the black hole, the larger the luminosity it’s capable of emitting by accretion.

- The maximum luminosity via accretion, called the Eddington luminosity, is that for which the forces of gravity and radiation pressure balance.

We shall now derive a formula for the Eddington luminosity to see what it has to say about quasar black holes.
The Eddington luminosity: derivation

Photons per unit time incident on electron:

\[ \dot{n} = \frac{P_{\text{intercepted}}}{h \nu} = \frac{L}{4\pi R^2} \sigma_e \]

Cross-sectional area of electron, for light scattering

Distance from BH

"Classical radius of electron:" assume its rest energy comes from electrostatic potential energy.

\[ m_e c^2 = \frac{e^2}{r_e} \quad \Rightarrow \quad r_e = \frac{e^2}{m_e c^2} = 2.8 \times 10^{-13} \text{ cm} \]

Thomson cross section

\[ \sigma_e = \frac{8\pi r_e^2}{3} \]

(8/3, not 1: quantum mechanics)
The Eddington luminosity: derivation (continued)

- Momentum per incident photon: $p_{\text{photon}} = \frac{h}{\lambda} = \frac{h \nu}{c}$
- Since the electron radius is so much smaller than the wavelength of light, light will scatter from the electron isotropically (uniformly in all directions). The scattered light therefore has zero momentum on the average.
- Thus all the incident photon momentum is transferred to the electron, resulting in an outward force of

$$F_{\text{rad, } e} = \frac{dp}{dt} = p_{\text{photon}} \dot{n} = \frac{h \nu}{c} \frac{L \sigma_e}{4 \pi R^2 \nu} = \frac{2 r_e^2 L}{3 c R^2} = \frac{2 e^4 L}{3 m_e c^5 R^2}$$

- Similarly, there is a force on the proton, but since the proton radius is so small, $r_p = \frac{e^2}{m_p c^2} = 1.5 \times 10^{-16}$ cm, this force is negligible compared to that on the electron.
The Eddington luminosity: derivation (continued)

- Each electron will drag a proton with it, whether these particles are bound in an atom or reside in ionized gas, because matter on macroscopic scales has equal numbers of positive and negative charges and the electrostatic force between them is strong.

- Similarly, each proton will drag an electron with it. The gravitational force exerted by the black hole on each proton is of course much larger than that on an electron.

- Accretion takes place if $F_{\text{grav}, p} + F_{\text{grav}, e} > F_{\text{rad}, p} + F_{\text{rad}, e}$

  or, to good approximation, $F_{\text{grav}, p} > F_{\text{rad}, e}$
The Eddington luminosity: derivation (continued)

Thus in order to accrete at luminosity \( L \), the black hole mass \( M \) must be such that

\[
\frac{GMm_p}{R^2} > \frac{2e^4 L}{3m_e^2 c^5 R^2}
\]

\[
\frac{M}{L} > \frac{2e^4}{3Gm_p m_e^2 c^5} = 1.6 \times 10^{-5} \text{ gm sec erg}^{-1} = 3 \times 10^{-5} M_\odot L_\odot^{-1}
\]

Given \( M \):

\[
L < L_E = \frac{3Gm_p m_e^2 c^5}{2e^4}
\]

**Eddington luminosity**

Given \( L \):

\[
M > \frac{2e^4 L}{3Gm_p m_e^2 c^5}
\]
Quasar black holes have to be supermassive

The Eddington luminosity is the maximum luminosity that a body with mass $M$ can produce by accretion. Now consider a typical quasar like 3C 273, with $L = 10^{12} L_\odot$.

$$M > \frac{2e^4 L}{3Gm_p m_e^2 c^5} = 3 \times 10^7 M_\odot \quad \text{Supermassive black hole required!}$$

- There are quasars with luminosities as large as $L = 10^{14} L_\odot$; thus we should expect to find central black holes well in excess of $10^9 M_\odot$: equivalent to the mass of a good-size galaxy.

- The event-horizon radius of the minimum-mass black hole that would power 3C273: $R_{Sch} = \frac{2GM}{c^2} = 0.6\text{AU}$. 
Quasar black holes have to be supermassive (concluded)

Clearly such a black hole is quite different in origin from those we considered earlier this semester: since stars don’t come any larger than \( 100M_\odot \), these couldn’t have formed from stellar collapse.

- The origin of supermassive black holes is in fact not yet understood very well. Leading models involve the interaction of galaxies and the transfer of interstellar matter between them during the interaction, as we shall discuss next week.

- Observational consequences of supermassive BHs in galaxy nuclei: material within a galaxy that passes close to a black hole like this should exhibit very large (even relativistic) speeds, that should show up in Doppler shifts and proper motions.
Summary

- The Eddington Luminosity
- The Hubble constant and how to use it
- Why quasars are thought to be powered by massive black holes