Today in Astronomy 142: observations of stars

What do we know about individual stars?

- Determination of stellar luminosity from measured flux and distance
  - Magnitudes
- Determination of stellar surface temperature from measurement of spectrum or color
  - Blackbody radiation
  - Bolometric correction

At right: the old open cluster M29 (2MASS/IPAC).
What do we want to know about stars?

As a function of mass and age, find...
- Structure of their interiors
- Structure of their atmospheres
- Chemical composition
- Power output and spectrum
- Spectrum of oscillations
- All the ways they can form
- All the ways they can die
- The details of their lives in between
- The nature of their end states
- Their role in the dynamics and evolution in their host galaxy

For the visible stars, measure...
- Flux at all wavelengths, at high resolution (probes the atmosphere)
- Magnetic fields, rotation rate
- Oscillations in their power output (probes the interior)
- Distance from us, position in galaxy, how they are moving
- Orbital parameters (for binary stars)

... and match the measurements up with theories.
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(We can do what’s in green, this semester)
Flux and luminosity

Primary observable quantity: flux, \( f \), (power per unit area) within some range of wavelengths. The flux at the surface of an emitting object is often called the surface brightness. Most stars, like many astronomical objects, emit light isotropically (same in all directions).

- Since the same total power \( L \) must pass through all spheres centered on a star, and is uniformly distributed on those spheres, the flux a distance \( r \) away is

\[
f = \frac{L}{4\pi r^2}
\]

To obtain luminosity (total power output), one must
- add up flux measurements over all wavelengths
- measure the distance

Best distance measurement: trigonometric parallax
Trigonometric parallax

Relatively nearby stars appear to move back and forth with respect to much more distant stars as the Earth travels in its orbit.

Since the size of the Earth’s orbit is known very accurately from radar measurements, measurement of the parallax $p$ determines the distance to the star, precisely and accurately.

$p$ is an angle
Since the Earth’s orbit is (nearly) circular, parallax can be measured no matter what the direction to the star.
Trigonometric parallax (continued)

Mean distance between centers of Earth and Sun:

\[ 1\text{AU} = 1.496 \times 10^{13} \text{ cm} \]

Thus

\[ r = \frac{1\text{AU}}{\sin p} \approx \frac{1\text{AU}}{p} \quad \text{small angle approximation} \]

where the parallax \( p \) is measured in radians.

- Parallax is usually expressed in fractions of an arcsecond (\( \pi \) radians = 180 degrees = 10800 arcminutes = 648000 arcseconds); the distance to an object with a parallax of 1 arcsecond, called a \textbf{parsec}, is \( 3.087 \times 10^{18} \text{ cm} = 3.2616 \text{ ly} \).
- Good parallax measurements: about 0.02 arcsec from the ground, \( \sim 0.001 \text{ arcsec} \) from the \textit{Hipparcos} satellite.
Parallax examples

What is the largest distance that can be measured by parallax with ground-based telescopes? With the *Hipparcos* satellite?

First note that we can write the distance-parallax relation as

\[ r = 1 \text{ parsec} \times \left( \frac{\text{arcsec}}{p} \right) \]

- If the smallest parallax we can measure is 0.01 arcsec, the largest distance measurable is \(1/0.01 = 100\) parsecs.
- Similarly, *Hipparcos* got out to \(1/0.001 = 1000\) parsecs.
- The ESA satellite *Gaia*, launched last year, will measure parallax with an accuracy of 0.00002 arcsec, thus perhaps stretching out to 50 000 parsecs = 50 kpc.
  - Compare: we live 8 kpc from the center of our Galaxy.
If we lived on Mars, what would be the numerical value of the parsec?

Mars’s orbital semimajor axis is 1.524 AU, so the “parsec” is a factor of about 1.524 larger:

\[
\begin{align*}
    r &= 1 \text{ "parsec"} = \frac{1.524 \text{ AU}}{1 \text{ arcsec}} \\
    &= \frac{1.524 \times 1.496 \times 10^{13} \text{ cm}}{1 \text{ arcsec}} \left( \frac{3600 \text{ arcsec}}{1^{\circ}} \right) \left( \frac{180^{\circ}}{\pi} \right) \\
    &= 4.703 \times 10^{18} \text{ cm} = 4.971 \text{ light years.}
\end{align*}
\]

But Mars’s orbit is twice as eccentric as Earth’s, so those last decimal places are affected by the orbit.
The thirty nearest stars

Figure: Chaisson and McMillan, *Astronomy Today*
Astronomer units: magnitudes

Ancient Greek system: first magnitude (and less) are the brightest stars, sixth magnitude are the faintest the eye can see.

The eye is approximately logarithmic in its response: perceived brightness is proportional to the logarithm of flux.

To match up with the Greek scale,

\[ 5 \text{ mag} \Leftrightarrow \text{ a factor of 100 in flux} \]

\[ 1 \text{ mag} \Leftrightarrow \text{ a factor of } (100)^{1/5} \approx 2.5 \text{ in flux} \]

or, for two stars with apparent magnitudes \( m_1 \) and \( m_2 \),

\[ m_2 - m_1 = 2.5 \log \left( \frac{f_1}{f_2} \right) \]
Magnitudes (continued)

magnitude = -2.5 \log_{10} (flux) + \text{constant}

Logarithmic scale often used for measurements based on human perception
• Decibels in audio (not inverted $10 \log_{10} \text{power}$)
• Richter scale for earthquakes (not inverted)
Magnitudes continued

- Brightness $\uparrow$ $\equiv$ Magnitude $\downarrow$

- Magnitudes are dimensionless: they are related to ratios of fluxes or distances.

- Another legacy from the Greeks: magnitudes run backwards from the intuitive sense of brightness.
  - Brighter objects have smaller magnitudes. Fainter objects have larger magnitudes. They’re like “rank.”

- Fluxes from a combination of objects measured all at once add up algebraically as usual. The magnitudes of combinations of objects do not.

- Calibration is done through comparisons
Magnitudes - the reference or zero point

A practical definition of zero apparent magnitude is Vega (α Lyrae), which has $m = 0$ at nearly all wavelengths (less than 20 microns).

**Absolute magnitude** is the apparent magnitude a star would have if were placed 10 parsecs away.

An A0 star at 10 pc has magnitude zero in all bands

**Bolometric magnitude** is magnitude calculated from the flux from all wavelengths, rather than from a small range of wavelengths.

In this week’s workshop you’ll show that the absolute and apparent bolometric magnitudes, $M$ and $m$, of a star are related by:

$$M = m - 5 \log \left( \frac{r}{10 \text{ pc}} \right) = 4.75 - 2.5 \log \left( \frac{L}{L_{\odot}} \right)$$
Magnitudes - zero point

magnitude = -2.5 \log_{10} (\text{flux}) + \text{constant}

magnitude = -2.5 \log_{10} (\text{flux}/\text{flux}\_\text{comp})

constant = 2.5 \log_{10} (\text{flux}\_\text{comp})

\text{flux}\_\text{comp} \text{ is the flux that gives you zero magnitude}

we call the constant the zero point and is related to the flux that gives you zero magnitude
Example (IAA problem 11.2). Two stars, with apparent magnitudes 3 and 4, are so close together that they appear through our telescope as a single star. What is the apparent magnitude of the combination?
Magnitudes (continued)

Example (IAA problem 11.2). Two stars, with apparent magnitudes 3 and 4, are so close together that they appear through our telescope as a single star. What is the apparent magnitude of the combination?

Call the stars $A$ and $B$, and the combination $C$:

First how much larger flux does $A$ have than $B$?

\[
m_B - m_A = 1 = 2.5 \log \left( \frac{f_A}{f_B} \right) \Rightarrow \frac{f_A}{f_B} = 10^{2.5} = 10^{0.4} = 2.512
\]

\[
\frac{f_A}{f_B} + 1 = \frac{f_A + f_B}{f_B} = \frac{f_C}{f_B} = 3.512
\]

Add 1 to both sides:

So \[
m_B - m_C = 2.5 \log \left( \frac{f_C}{f_B} \right)
\]

\[
m_C = m_B - 2.5 \log \left( \frac{f_C}{f_B} \right) = 4 - 2.5 \log (3.512) = 2.64
\]
Magnitudes and Photometric calibration

Observe an object at the same time as another object with known brightness (two stars in the same field of view).
Find correction factor for known object
Use that correction factor to measure apparent magnitude of your object.

Apparent magnitude – how bright the object is observed to be.
Absolute magnitude

If you know the distance to the object and you measure the apparent magnitude, then you can determine the brightness for the object as if were observed at 10pc away.
This is the absolute magnitude.
Directly related to the stars intrinsic luminosity

Ways to know distances?
Blackbody emission

Because stars are **opaque** at essentially all wavelengths of light, they emit light much like ideal **blackbodies** do.

Blackbody radiation from a perfect absorber/emitter is described by the Planck function:

\[
B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}
\]

*Power per unit area, bandwidth and solid angle.*

\(B_\lambda\) units: erg s\(^{-1}\) cm\(^{-2}\) wavelength\(^{-1}\) steradian\(^{-1}\)

For a small **bandwidth** (wavelength interval) \(\Delta \lambda (<< \lambda)\) and solid angle \(\Delta \Omega (<< 4\pi)\), the flux \(f\) emitted by a blackbody is

\[
f = B_\lambda(T) \Delta \lambda \Delta \Omega
\]
Although stars like the Sun are brightest at visible wavelengths, the Planck function for star-like temperatures is much wider than the visible spectrum. So most of a star’s luminosity comes out at ultraviolet and/or infrared wavelengths.
Planck function

\[ B_\nu(\nu) \quad \text{As a function of photon frequency} \]

\[ B_\lambda(\lambda) \quad \text{As a function of photon wavelength} \]
Blackbody emission (continued)

\[ B_\nu(\nu) \quad \text{flux integrated between } \nu \text{ and } \nu + d\nu \]

\[ B_\lambda(\lambda) \quad \text{flux integrated between } \lambda \text{ and } \lambda + d\lambda \]

\[ \nu = c/\lambda \]

\[ B_\nu(\nu) d\nu = B_\lambda(\lambda) d\lambda \]
Note that a given ratio of flux within the $B$ and $V$ wavelength ranges (see below) is characteristic of a certain temperature of blackbody, and that continuous stellar spectra are approximately blackbodies.

Figure: Chaisson and McMillan, *Astronomy Today*
Planck function

\[ B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT}} - 1 \]

- \( h \) Planck’s constant
- \( k \) Boltzmann’s constant
- \( c \) speed of light
- photon energy distribution
- Photons in a box derivation
- Energy levels set with Planck’s constant
- Energy levels populated proportional to \( \exp(E/kT) \)
Integrating the Planck function

\[ B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \]

\[ \int_0^\infty B_\lambda(T) d\lambda = \frac{2k^4 \pi^4}{15h^3 c^2} T^4 = \frac{\sigma_B T^4}{\pi} \]

using \[ x = \frac{\lambda kT}{hc} \]

\[ \int_0^\infty \frac{x^{-5}}{e^{1/x} - 1} dx = \frac{\pi^4}{15} \]

\[ \sigma_B = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4} \]

is the Stefan-Boltzmann constant.

\[ \sigma_B = \frac{2\pi^5 k^4}{15h^3 c^2} \]
Blackbody emission (continued)

The total flux emitted into all directions at all wavelengths from the surface of a blackbody is

\[
f = \int_{0}^{\infty} d\lambda \int d\Omega \cos \theta B_{\lambda}(T)
\]

\[
f = \int_{0}^{\infty} d\lambda B_{\lambda}(T) \ 2\pi \int_{0}^{\pi/2} \cos \theta \sin \theta d\theta
\]

\[
f = \frac{\sigma_B T^4}{\pi} \ 2\pi \int_{0}^{\pi/2} \cos \theta \sin \theta d\theta = \sigma_B T^4
\]

\[\sigma_B = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}\] is the Stefan-Boltzmann constant.

Luminosity of a spherical blackbody with radius \( R \):

\[
L = 4\pi R^2 \sigma_B T^4
\]
Blackbody emission (continued)

Wavelength of the Planck function’s maximum value changes with temperature according to

\[ \lambda_{\text{max}} T = 0.29 \text{ cm K} \ . \]

Reminders about solid angle:

- Solid angle for cone with small apex angle \( \theta \) is

\[ \Omega = \pi \theta^2 \ . \]

- Differential element of solid angle in spherical coordinates:

\[ d\Omega = \sin \theta d\theta d\phi \ . \quad \theta \in [0, \pi] \]
Solid angles

Cone, radius $\Delta \theta$:
\[
\Omega = \int_0^{2\pi} \left[ \int_0^{\Delta \theta} \sin \theta \, d\theta \right] \, d\phi = 2\pi \left( 1 - \cos \Delta \theta \right)
\]

Cone with $\Delta \theta \ll 1$:
\[
\Omega \approx 2\pi \left( 1 - \left[ 1 - \frac{\Delta \theta^2}{2} \right] \right) = \pi \Delta \theta^2
\]

Whole sky:
\[
\Omega = \int_0^{2\pi} \left[ \int_0^\pi \sin \theta \, d\theta \right] \, d\phi = 4\pi
\]

Sky above a plane:
\[
\Omega = \int_0^{2\pi} \left[ \int_0^{\pi/2} \sin \theta \, d\theta \right] \, d\phi = 2\pi
\]
Stars are blackbodies to (pretty good) approximation.

Solar flux per unit band-width as seen from Earth’s orbit, compared to the flux per unit bandwidth of a 5777 K blackbody with the same total flux.

(Wikimedia Commons)
Effective Temperature

\[ L = 4\pi R^2 \sigma_B T^4 \]  
for a black body

\( T_e \) is the temperature of the blackbody of the same size as a star (not exactly a black body), that gives the same luminosity:

\[ T_e = \left( \frac{L}{4\pi R^2 \sigma_B} \right)^{\frac{1}{4}} \]

The Sun’s luminosity and radius gives an effective temperature \( T_{\odot e} = 5800 \) K

Blackbody: surface brightness and spectrum described by 1 parameter, the temperature
Color and temperature (continued)

$p$ Ophiuchi. a molecular cloud at $T = 60$ K.

A young stellar object. The star's surroundings have $T = 600$ K.

The Sun's surface, $T = 6000$ K.

Evolved stars in $\omega$ Centauri, at $T = 60,000$ K.

Figure: Chaisson and McMillan, *Astronomy Today*.
Stellar photometry

To facilitate comparison of measurements by different workers, astronomers have defined standard bands, each defined by a center wavelength and a bandwidth, for observing stars.

- At visible wavelengths the bands in most common use (Johnson 1966) are as follows:

<table>
<thead>
<tr>
<th>Band</th>
<th>Wavelength (nm)</th>
<th>Bandwidth (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>360</td>
<td>70</td>
</tr>
<tr>
<td>B</td>
<td>430</td>
<td>100</td>
</tr>
<tr>
<td>V</td>
<td>540</td>
<td>90</td>
</tr>
<tr>
<td>R</td>
<td>700</td>
<td>220</td>
</tr>
</tbody>
</table>

- Measurement of starlight flux (or magnitudes) within such bands is called photometry.
Color and effective temperature

**Color** is the difference between apparent magnitude at different wavelengths, or 2.5 times the logarithm of the ratio of fluxes in the two bands.

**Color index**: the color between the $B$ and $V$ bands:

$$B - V = m_B - m_V = M_B - M_V$$

$$= 2.5 \log \left( \frac{f(V)}{f(B)} \right)$$

In the absence of extinction, color is an index of the effective temperature $T_e$ of stars.

For $T_e < 12000$ K, and taking $B = V = 0$ at $T_e = 10000$ K (like Vega),

$$B - V \approx -0.93 + 9000 \frac{K}{T_e}$$

relating color to temperature.
Example: for these spectra, the visible colors are

\[ B - V = 2.5 \log \left( \frac{f_V}{f_B} \right) \]

- 3000 K: \( B - V = -0.85 \)
- 10000 K: \( B - V = -0.46 \)
- 30000 K: \( B - V = +1.23 \)

top to bottom.
Color and effective temperature (continued)

At 30000 K most of the radiation is emitted bluer than visible wavelengths.

At 300K most of the radiation is emitted redder than visible wavelengths.
Color index and effective temperature (Johnson 1966)
Color, magnitude and bolometric correction

Once the shape of a star’s spectrum (i.e. its temperature) is determined from its colors, the total flux is also determined.

- In magnitude terms: the ratio of total flux to flux within one of the photometric bands is expressed as a bolometric correction to the star’s magnitude in that band, usually $V$:

$$m = m_V + BC$$

- In blackbody terms: once the temperature of the body is known, so is the total flux ($f = \sigma T^4$). If stellar spectra were really blackbodies, the bolometric correction would be

$$BC = m - m_V = 2.5 \log \left( \frac{f_V}{f} \right) = 2.5 \log \left( \frac{B_\lambda (\lambda_V, T) \Delta \lambda_V \Delta \Omega}{\sigma T^4} \right)$$
Bolometric correction
(continued)

Shape of spectrum, and this correction factor, can be determined if one knows the color of the star (difference between apparent magnitudes at two wavelengths).

Scaling factor for flux is the same as an offset in magnitude. This offset is called the bolometric correction, $BC$, and is determined from observed or theoretical stellar spectra.
Example. Two stars are observed to have the same apparent magnitude, 2, in the $V$ band. One of them has color index $B-V = 0$, and the other has $B-V = 1$. What are their apparent bolometric magnitudes?

From the graph, $BC = -0.4$ and $-0.38$ in these two cases, so their bolometric magnitudes are practically the same, too, 1.6 and 1.62.

That these magnitudes are both less than the apparent $V$ magnitude is a sign that these stars produce substantial power at wavelengths far outside the $V$ band. The bluer star (with $B-V = 0$) turns out to be brightest at ultraviolet wavelengths; the other one is brightest at red and infrared wavelengths.
Are perfect Black Body’s ever observed?

Not really: In stars there are absorption lines and emission lines, absorption edges, more than one temperature is seen from different places. Many objects emit thermally as well as non-thermally.

Cosmic Microwave Background spectrum is a counter example. It’s really very close to a Black Body.

The frequency spectrum of the cosmic microwave background records interactions between the evolving matter and radiation fields. Observations from ground-based, balloon-borne, and satellite instruments show the CMB to agree with a blackbody spectrum (dotted line) across 3 decades of frequency and 4 orders of magnitude in intensity. This agreement with a blackbody spectrum indicates that the early universe was once in thermodynamic equilibrium, and limits energetic events since the Big Bang: the cosmic microwave background accounts for more than 99.993% of the radiant energy in the universe.

Credit COBE team
Wavelength vs Temperature

For Black Body’s the wavelength of the emission peak is set by the temperature.
There is little emission at vastly different other wavelengths.
So you can often estimate the temperature of a thermal object from what wavelength (or frequency) you observe it at.

• $10^6$K stuff (plasma) emits in X-rays. Detected by CHANDRA
• $10^4$K stuff (plasma) emits in UV. Detected by HST, IUE, FUSE
• $10^3$K objects emit in optical. By eye.
• $10^2$K stuff (cold gas and dust) emits in mi-far infrared. Detected by SIRTF/SST (previously ISO and IRAS).
Note: there is also non-thermal emission -- then this rule doesn’t apply (e.g. synchrotron emission)
Near-Infrared Broad Band Filters

Atmospheric absorption along with filter transmission

From Simmons and Tokunaga et al 2001
Some practical Astronomy

- RA (right ascension) tells you when your object is up.
  - 12 hours is up highest Mar 21
  - 0 hours is up highest Sept 21
- DEC (declination) tells you how far away from the north pole.
  - Polaris is at +90. Anything below 0 degrees is hard to observe from the northern hemisphere.

- Spectral types and luminosity class: A5 V
  - A5 is the spectral type
  - V is the luminosity class.
  - V is called a dwarf, also is on the main sequence. Late type stars are fainter than the Sun, early type are much brighter than the Sun. Range in luminosities is about $10^5$, even though the range in mass is only 0.1 to $100 \, M_\odot$.
  - III’s are giants, typically $10^3$ times more luminous than the Sun
  - I’s are supergiants, typically $10^5$ times more luminous than the Sun.
Practical astronomy (continued)

• Using parallax to estimate distance:
The distance in parsecs is $1/(\text{parallax in arcsecs})$.

• Estimating the absolute magnitude. The distance modulus is

$$DM = 5\log(D/10\text{pc})$$

$$M_V = m_V - DM$$

• X-ray emission from stars is often not from the stellar photosphere, (of order 5000K) but from the corona (of order a million K)

• Far infrared emission from stars is often not from the stellar photosphere but from circum-planetary dust that absorbs light from the star and reradiates it at longer wavelengths (of order 100K).
Practical astronomy (continued)

Spectral types: O,B,A,F,G,K,M
Classified based on what their spectrum looks like.
Is also a temperature scale.
Is also a mass scale --- on the main sequence.
M-stars are cool and low mass \( \sim 0.1 \, M_\odot \) and faint \((0.01 \, L_\odot)\).
O stars are very very hot and more massive \( \sim 10 M_\odot \) and bright \((10^4 \, L_\odot)\).
Beta Pictorus (A5V) is more massive and luminous than the sun (a G star)
Epsilon Eridani (K2V) is less massive and less luminous than the sun.
Practical Astronomy (continued)

Figuring out how bright things are in solar luminosities.

- Figure out how bright the object is intrinsically.
- Divide this luminosity by the luminosity of the Sun.

Equivalently:

- Take the absolute magnitude of the object, subtract the absolute magnitude of the sun at the same band.
- Convert magnitudes into luminosity.

**Example:** A star has an absolute magnitude in V band of 7.0
The sun has an absolute magnitude in V band of 4.8

\[
\frac{7-4.8}{10^{-2.5}} = 0.13
\]

The star is about 1/10\textsuperscript{th} as bright as the sun. To do better than this use bolometric corrections.
Summary

- Magnitudes and how to calculate with them
- Black body function
- The relation between measured color and estimated temperature.
- Bolometric correction
- Broad band photometric measurements.