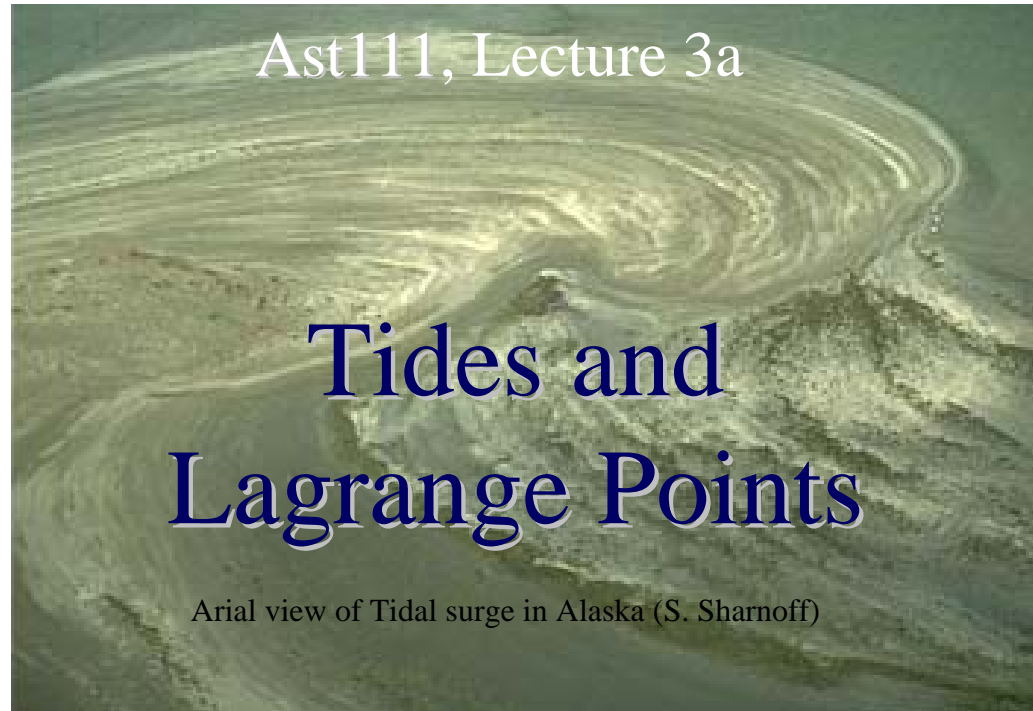
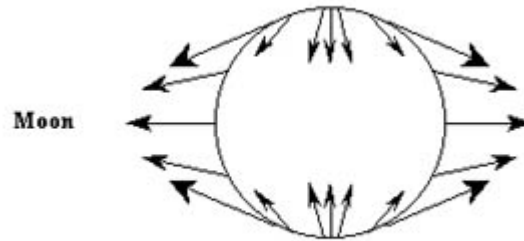


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- Tides
- Tidal disruption
- Lagrange Points
- Tadpole Orbits and Trojans
- Tidal Bulges

Tidal Force



In the frame rotating with the Earth,
the force due to gravity from the sun

$$F = -\frac{GM_{\odot}}{R^2} \hat{\mathbf{R}} \text{ where } \mathbf{R} \text{ is the vector from the Sun}$$

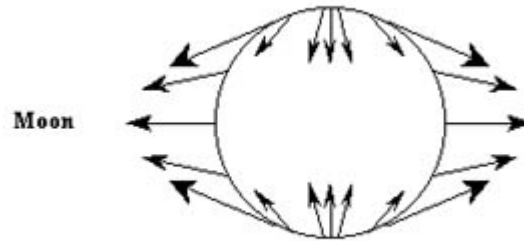
to the Earth.

We expand this force in a local coordinate system
which is centered on the Earth.

Remember how to do an expansion: It's a Taylor series

$$f(x + \delta) = f(x) + f'(x)\delta + f''(x)\frac{\delta^2}{2} + \dots$$

Tidal Force (continued)



$$F = -\frac{GM_{\odot}}{d^2} \hat{\mathbf{d}} + \frac{GM_{\odot}}{d^3} (3(\hat{\mathbf{d}} \cdot \vec{\mathbf{r}})\hat{\mathbf{d}} - \vec{\mathbf{r}}) \dots$$

Here d is the distance between the Sun and Earth,
and r is the distance from the center of the Earth.

$\hat{\mathbf{d}}$ represents the unit vector between Earth and Sun

$\vec{\mathbf{r}}$ represents a vector from the Center of the Earth

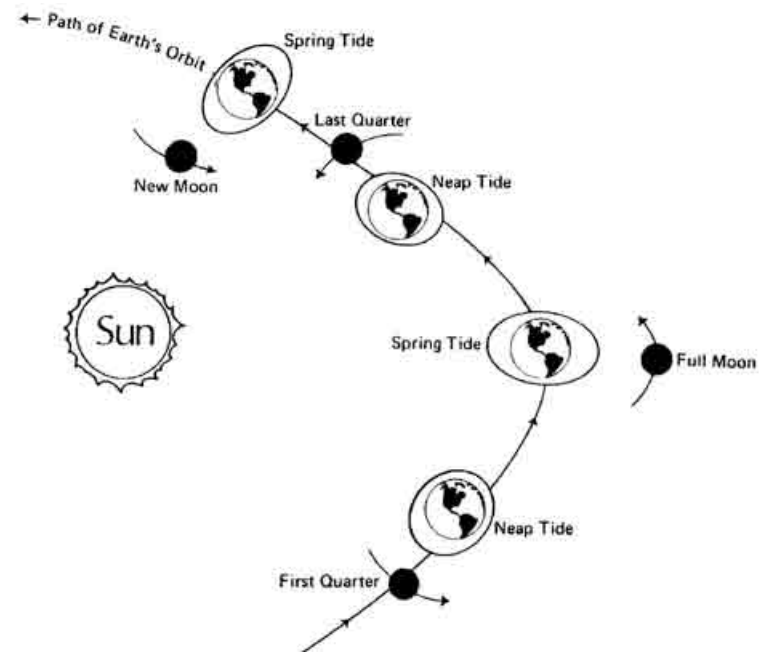
The direction of the tidal force depends on the direction of \mathbf{r}

$$F_t = \frac{2GM_{\odot}}{d^3} r$$

Tidal force is $2F_t$ outward for \mathbf{r} toward or away from the Sun,
and is $-F_t$ in the plane perpendicular to this line.

Tides on the Earth

- Both the Sun and Moon exert tides on the Earth.
- When the Sun and Moon align, the tide is largest.



The Hill Sphere

- Where is an object under the influence primarily of the Sun's gravitational field, and where is primarily under the influence of a planet's gravitational field?

Where the Force from self gravity from the planet exceeds the tidal force from the Sun, we say we are within the Hill Sphere.

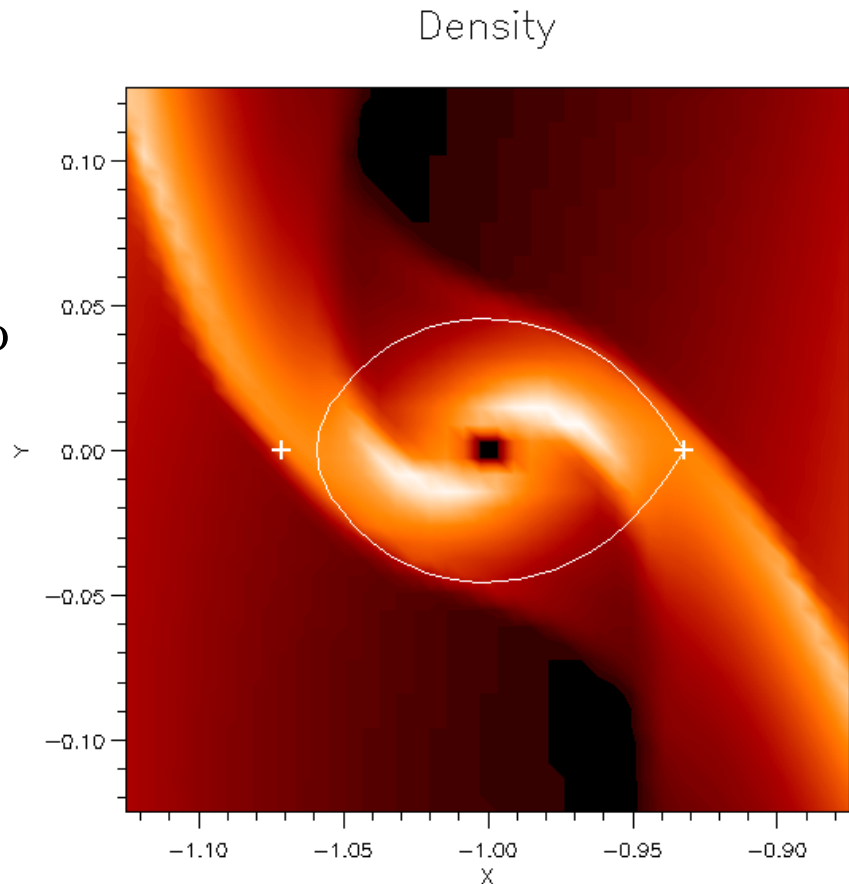
$$\frac{GM_p}{r^2} > F_t \quad \rightarrow \quad \frac{GM_p}{r^2} > \frac{2GM_\odot r}{d^3}$$

The Hill radius

$$r_H \equiv d \left(\frac{M_p}{3M_\odot} \right)^{1/3}$$

The Roche Lobe

- Gas or planetesimals which go inside the Hill Sphere, can hit the planet and so can **accrete** onto the planet.
- An object which gets so close to the Sun that its surface reaches outside the Hill sphere, is said to “*overflow its Roche Lobe*”.
- Material from a planet which gets outside the Roche Lobe can accrete onto the star.
- The Roche Lobe is a surface that quantifies the shape of the Hill Sphere.

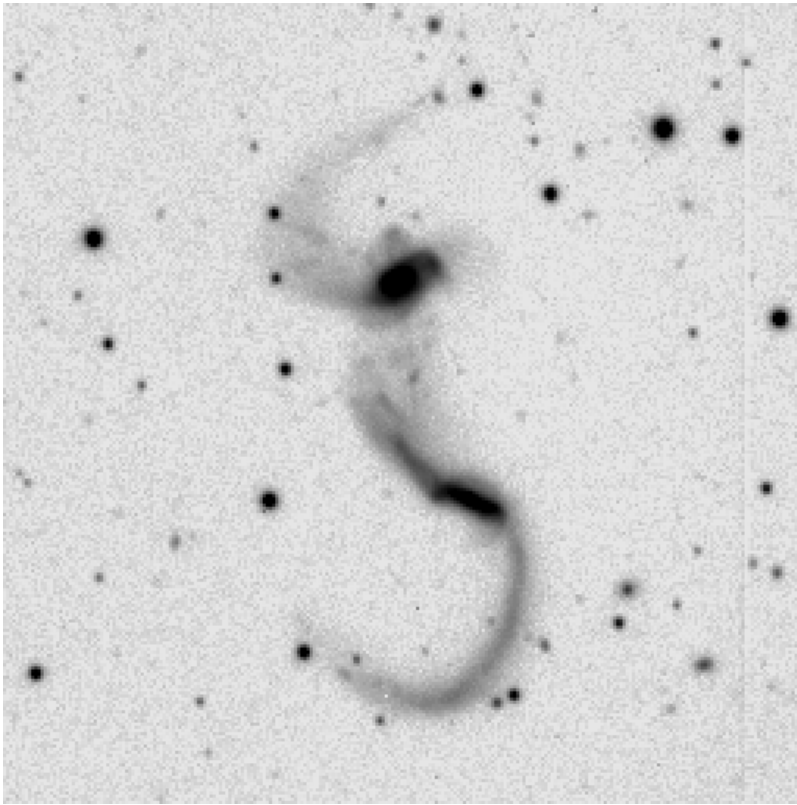


Simulation of planetary accretion by Siebert

Tidal Disruption

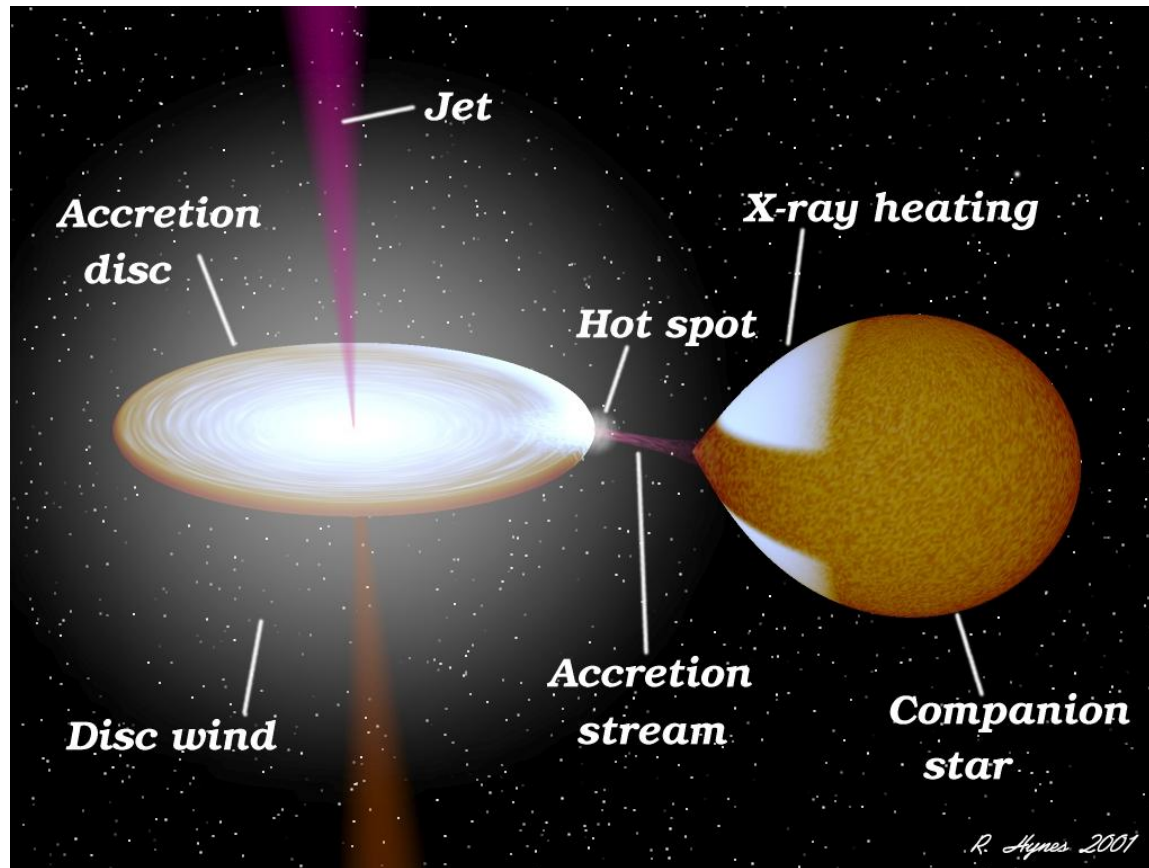
- When an object approaches a planet or a star and exceeds the size of its Hill Sphere or Roche Lobe, it can be tidally disrupted.
- An object that has a constant density is disrupted all at once. Tidal disruption.
- An object which is denser in the center will lose its surface first. Roche Lobe overflow.

Tidal disruption

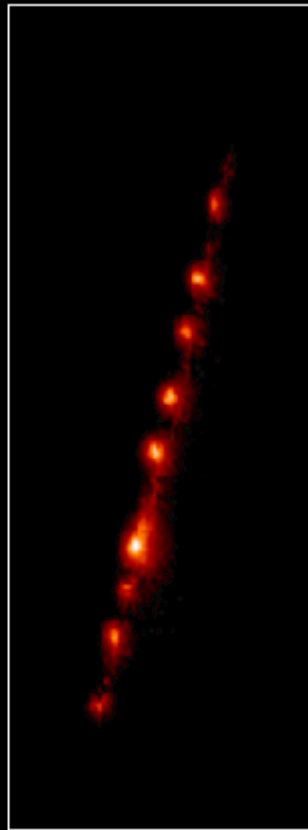
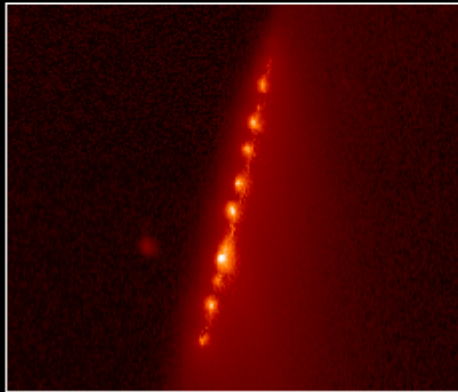
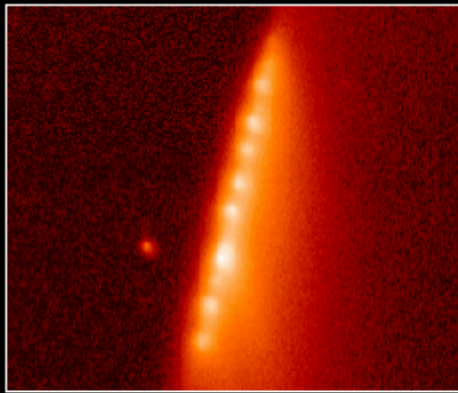


Examples of images of galaxies where the outer parts are being tidally disrupted.

Roche Lobe Overflow



Tidal disruption of Comet Shoemaker Levy 9



Prior to impact comet Shoemaker Levy9 made a close approach to Jupiter. It was tidally disrupted into a series of fragments which subsequently impacted Jupiter.

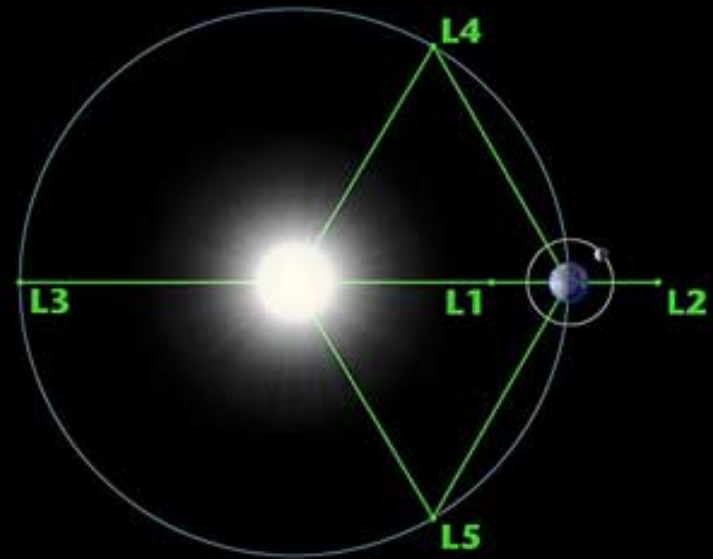
Observation from Kitt Peak by W. Winiewski

The **rubble pile** model for Comets

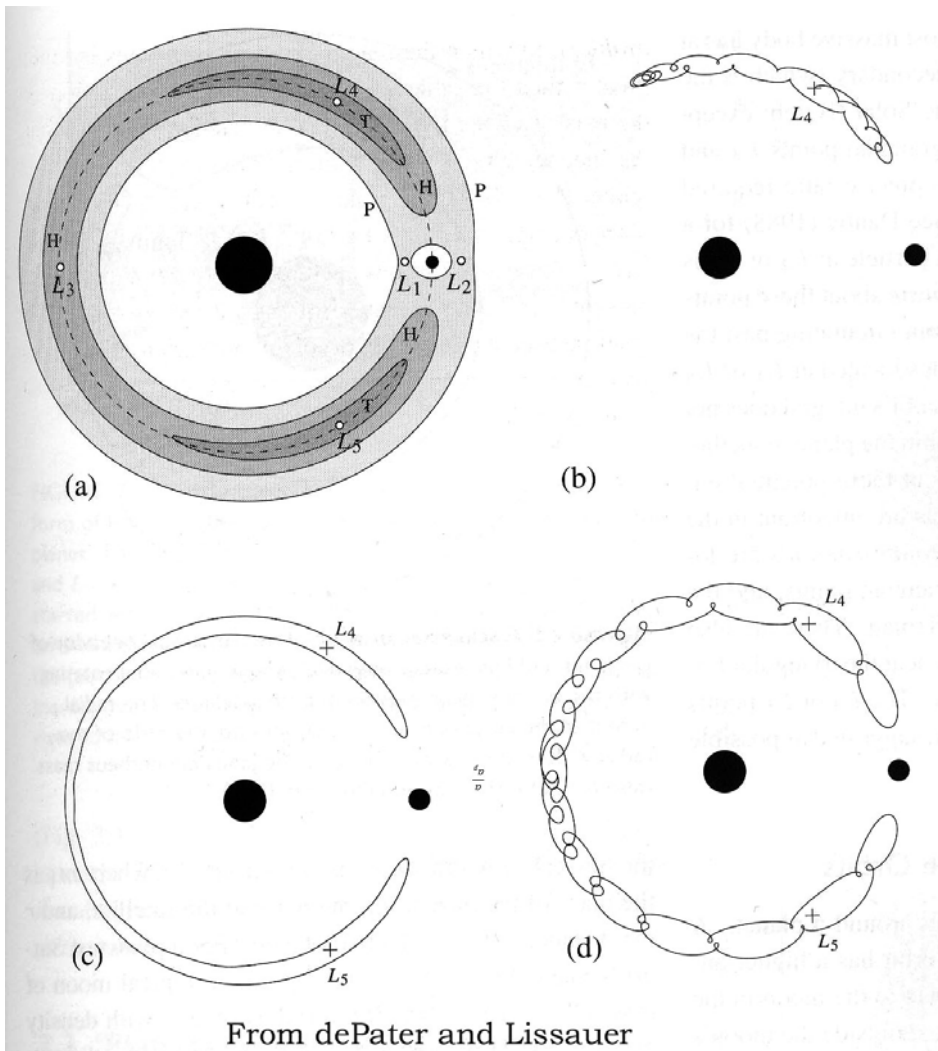
- Because comet Shoemaker Levy 9 disrupted, it could not have had a high density, and it could not have had a high material tensile strength.
- Models for the disruption of the Comet by E. Asphaug and collaborators suggested that the comet was a conglomeration of rock and ice, bound by self-gravity only.

Lagrange Points

- There are special points where a particle in a frame rotating with a planet feels no net force.
- These are known as Lagrange points.
- There are 5 of them.
- We can think of L1 and L2 as places where the tidal force from the Sun is balanced against gravity from the planet.



Horseshoe and Tadpole Orbits



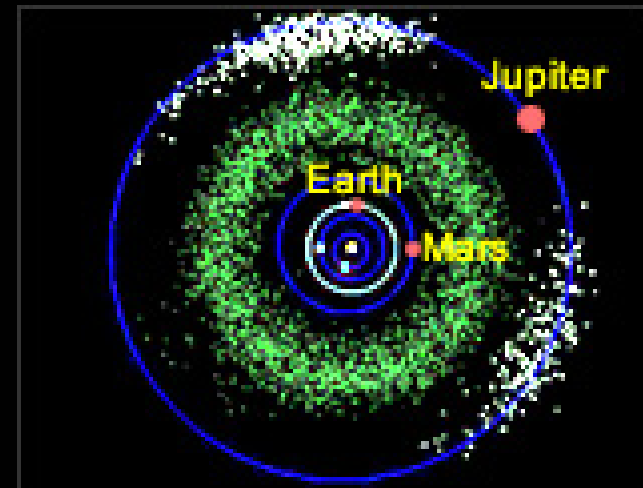
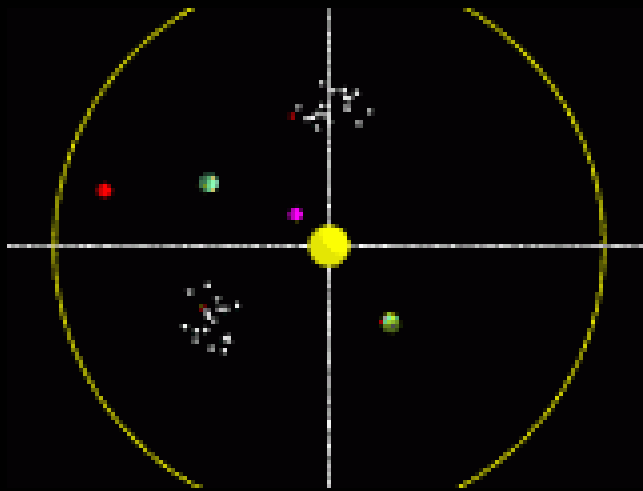
In the frame rotating with the Earth.

The L4 and L5 points are **stable**. Near these points there are small closed orbits.

The other Lagrange points are not stable. This means that a small nudge away from the point will cause the particle to move far away in its orbit.

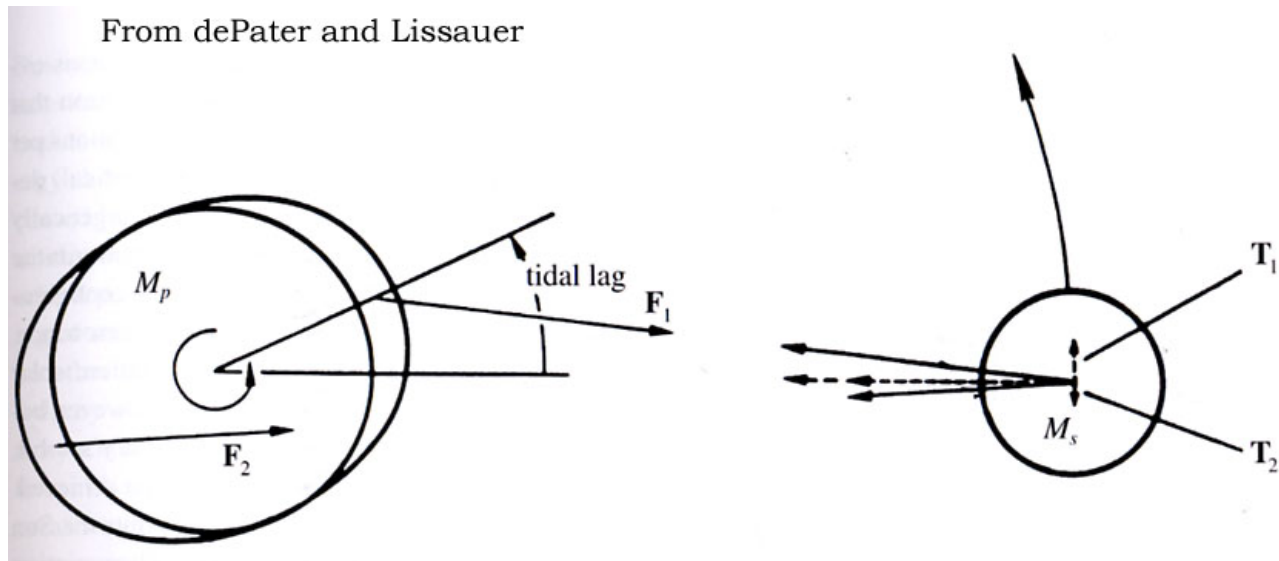
Space craft put in the L1 or L2 points must be maintained in these positions.

Trojan asteroids



A family of asteroids exists in tadpole orbits about L4, L5 with Jupiter.

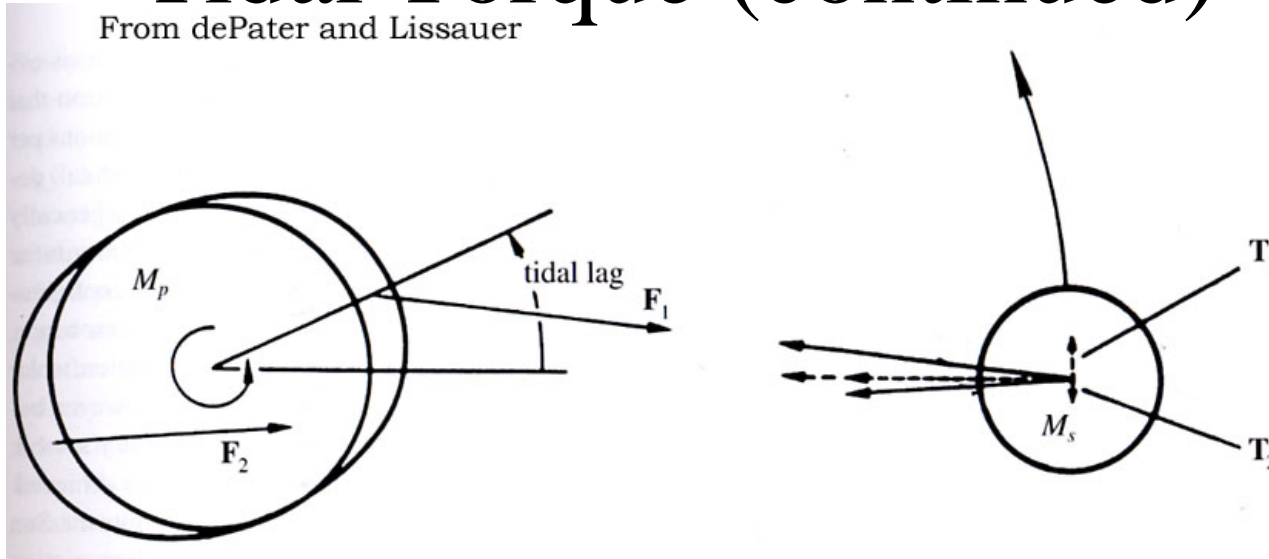
Tidal Torque



The moon causes the Earth to exhibit a tidal bulge. However, the Earth does not respond immediately. Energy is dissipated in making the tidal bulge. Consequently, the bulge lags the actual location of the moon.

Tidal Torque (continued)

From dePater and Lissauer



Because the earth is no longer a perfect sphere, and because of the misalignment, the earth exerts a torque on the moon.

The moon receives a positive net torque, and its orbit evolves outwards.

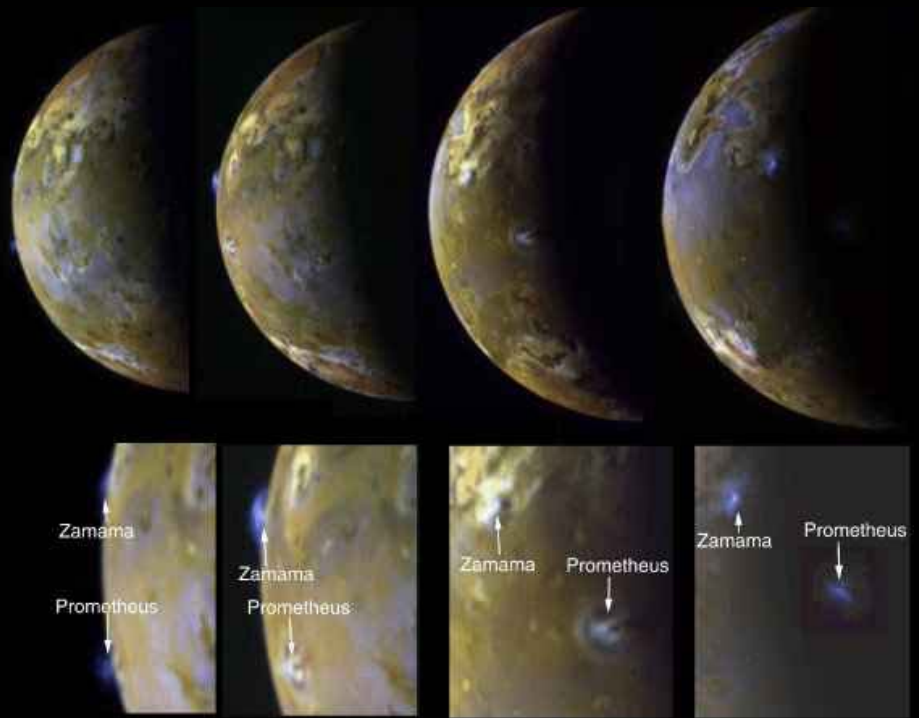
Angular momentum is extracted from the Earth's rotation. As the moon gets further away, the **day** lengthens.

Tidal Locking

- Raising a tidal bulge dissipates energy.
- This can causing heating.
- The energy typically comes out of the rotation, so the object is slowed down.
- For example, the moon is tidally locked with the Earth. We see the same face of the moon at all times.
- Pluto and Charon are tidally locked. The same hemisphere of each body always faces the other.

Tidal heating on Io

- Io is the innermost Galilean satellite.
- Because of its proximity to Jupiter, Io experiences strong tides.
- S. Peale in 1979 predicted that Io would have volcanoes because of tidal heating.
- Subsequent studies of Io have revealed active volcanoes, as well as extensive evidence for lava flows.
- Io has very very few craters, so the surface is reformed on short timescales.



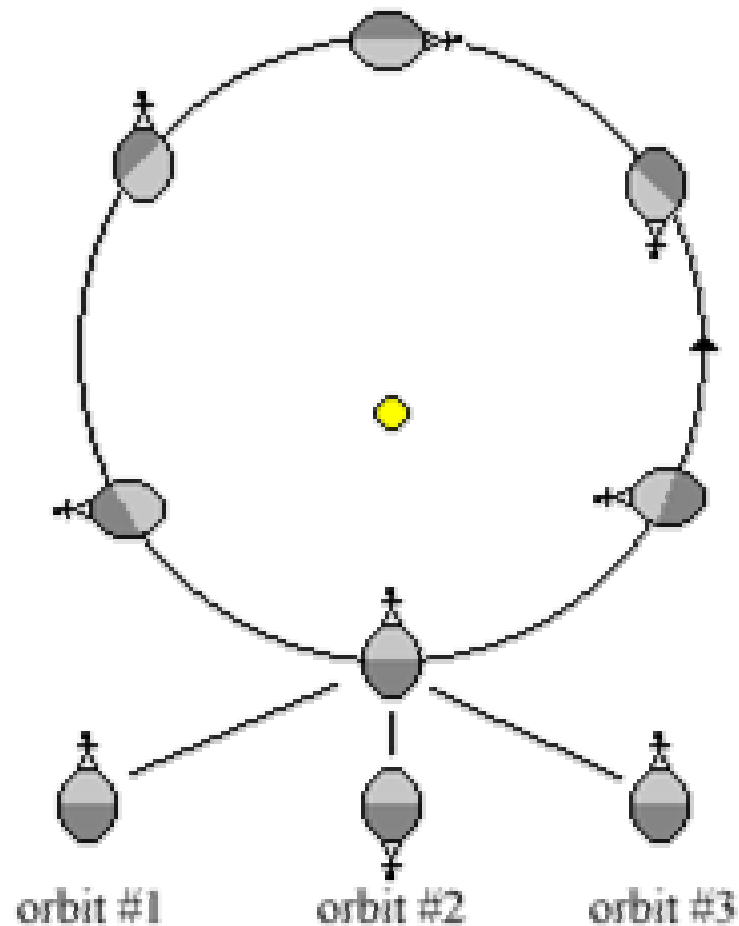
Credit NASA/JPL

Tidal Torque on Io

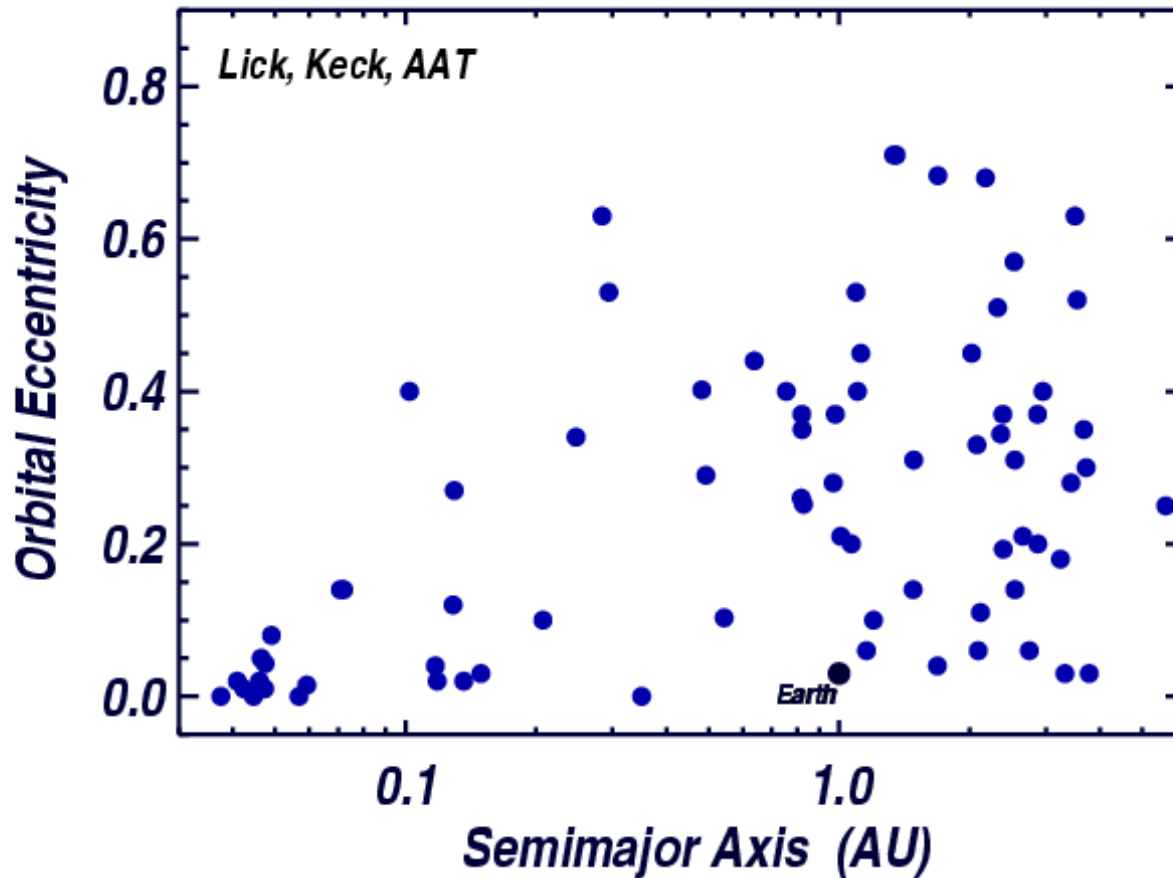
- Like the moon, Io is slowly gaining angular momentum, and so drifting away from Jupiter.
- As Io drifted outwards, it captured Europa into resonance. Likewise Europa captured Ganymede.
- The outwards motion of Io is thought to be responsible for the fact that the Galilean satellites are locked in each others mean motion resonances.

Mercury's Spin Orbit Resonance

- Mercury is locked in a 3:2 resonance such that it rotates 3 times for every 2 orbits.
- At every other perihelion, Mercury presents the same face to the Sun.
- Mercury probably became captured into this resonance because of tides excited by the Sun.



Low eccentricity Hot Exo-Planets



The low eccentricities of extrasolar planets with small major axes is thought to be a result of **tidal circularization**. This happens quickly when the planet is close to the parent star.

Review

$$\text{Tidal force} \sim \frac{2GMmr}{d^3}$$

$$\text{Hill sphere radius: } r_H \equiv d \left(\frac{M_p}{3M_\odot} \right)^{1/3}$$

Vocabulary and Concepts:

- Tidal heating, torque, bulges, disruption, and locking
- Lagrange points
- Roche Lobe overflow
- Stability of orbits
- Trojan asteroids