Tides and Lagrange Points

- Tides
- Tidal disruption
- Lagrange Points
- Tadpole Orbits and Trojans
- Tidal Bulges
Tides

Moon

Moon's force strongest on near side.

Moon's force on the far side is less than at the center.

Forces relative to the Sun (or primary body)

Forces relative to the center of the Earth
Tidal Force

The force from gravity from the Sun

\[ F = -\frac{GM_\odot}{R^2} \hat{R} \]

in direction of vector between Earth and Sun

R is radius from Sun

We expand this force in a local moving coordinate system that is centered on the center of the Earth

To do the expansion use a Taylor series

\[ f(y + x) = f(y) + f'(y)x + f''(y)x^2 / 2 \ldots \]
Expanding Gravitational force in a Taylor series

\[ f(x + \delta) = f(x) + f'(x)\delta + f''(x)\frac{\delta^2}{2} + \ldots \]

- Force is a vector and we need to know how the force depends for three directions of varying the position. Expand each component separately. (Or expand the potential and then take the gradient of it.)

\[ f(x + \delta_x, y + \delta_y, z + \delta_z) = f(x, y, z) \]

\[ + \frac{\partial f(x, y, z)}{\partial x} \delta_x + \frac{\partial f(x, y, z)}{\partial y} \delta_y + \frac{\partial f(x, y, z)}{\partial z} \delta_z \]

\[ + \frac{\partial^2 f(x, y, z)}{\partial x^2} \frac{\delta_x^2}{2} + \frac{\partial^2 f(x, y, z)}{\partial x \partial y} \delta_x \delta_y + \ldots \]
Tidal force

- Expand the Force from the Sun about a distant point.

Force is stronger nearer the sun, so pulling out on this side. Force is weaker on the distant side, if we consider the strength at the center, we have overestimated, so the tidal part pulls away.
Tidal Force (continued)

\[ F = -\frac{GM_*}{d^2} \hat{d} + \frac{GM_*}{d^3} (3(\hat{d} \cdot r)\hat{d} - r) + \ldots \]

- \( d \) distance between Earth and Sun
- \( r \) is distance from center of Earth
- \( \hat{d} \) unit vector between Earth and Sun
- \( r \) vector from the center of the Earth

Magnitude of force depends on direction of \( r \)

Tidal force is

- \(+2F_t\) toward or away from Sun
- \(-F_t\) in plane perpendicular

gravitational force normally \( d^{-2} \)
tidal force here \( d^{-3} \)
Tides on the Earth

- Both the Sun and Moon exert tides on the Earth.
- When the Sun and Moon align, the tide is largest.
The Hill Sphere

Where is an object under the influence primarily of the Sun’s gravitational field, and where is primarily under the influence of a planet’s gravitational field?

Where the force from self gravity from the planet exceeds the tidal force from the Star, we are within the Hill Sphere

\[ \frac{GM_p}{r^2} > F_{\text{tidal}} \quad \Rightarrow \quad \frac{GM_p}{r^2} > \frac{2GM_*r}{d^3} \]

The Hill radius is the boundary

\[ r_H = d \left( \frac{M_p}{3M_*} \right)^{\frac{1}{3}} \]

Factors: If you use the factor of two you get the Roche radius. If you use the factor of 3 you get the Hill Sphere radius, that is the same as the location of the L1 Lagrange point.
The Roche Lobe

- Gas or planetesimals which go inside the Hill Sphere, can hit the planet and so can **accrete** onto the planet.
- An object which gets so close to the Sun that its surface reaches outside the Hill sphere, is said to "overflow its Roche Lobe".
- Material from a planet which gets outside the Roche Lobe can accrete onto the star.
- The Roche Lobe is a surface that quantifies the shape of the Hill Sphere.

Simulation of planetary accretion by Siebert
Tidal Disruption

• When an object approaches a planet or a star and exceeds the size of its Hill Sphere or Roche Lobe, it can be tidally disrupted.

• An object that has a constant density is disrupted all at once. Tidal disruption.

• An object which is denser in the center will loose its surface first. Roche Lobe overflow.
Tidal disruption

Examples of images of galaxies where the outer parts are being tidally disrupted.
Roche Lobe Overflow
Tidal disruption of Comet Shoemaker Levy 9

Prior to impact, comet Shoemaker Levy 9 made a close approach to Jupiter. It was tidally disrupted into a series of fragments which subsequently impacted Jupiter.

Observation from Kitt Peak by W. Winiewski
The **rubble pile** model for Comets

- Because comet Shoemaker Levy 9 disrupted, it could not have had a high density, and it could not have had a high material tensile strength.

- Models for the disruption of the Comet by E. Asphaug and collaborators suggested that the comet was a conglomeration of rock and ice, bound by self-gravity only.
Lagrange Points

• There are special points where a particle in a frame rotating with a planet feels no net force.
• These are known as Lagrange points.
• There are 5 of them.
• We can think of L1 and L2 as places where the tidal force from the Sun is balanced against gravity from the planet.
Lagrange points

- **L1** is a nice location for solar observatories
  - The Solar and Heliospheric Observatory (*SOHO*)
  - Advanced Composition Explorer (*ACE*)

- **L2** missions include:
  - WMAP, Planck, Herschel, JWST, GAIA
Lagrange points

- Balance the force from the planet with that of the Sun
- At L1 the Earth’s force exactly cancels the larger force from the Sun so that an object feels slightly less force, allowing it to remain in a orbit with the same orbital period as the Earth which is slightly further out
- At L2 the Earth’s force adds to the Sun’s allowing an object to orbit with a orbital period equivalent to that of the Earth even though the object is further away from the Sun

\[
F = \frac{GM_{\oplus}}{r^2}
\]

\[
F = \frac{GM_{\odot}}{R^2} \left(1 - \frac{2r}{R}\right)
\]

\[
F = \frac{GM_{\oplus}}{r^2}
\]

\[
F = \frac{GM_{\odot}}{R^2} \left(1 + \frac{2r}{R}\right)
\]
Inertial Frame  Rotating frame
Horseshoe and Tadpole Orbits

The L4 and L5 points are **stable**. Near these points there are small closed orbits.

The other Lagrange points are not stable. This means that a small nudge away from the point will cause the particle to move far away in its orbit.

Space craft put in the L1 or L2 points must be maintained in these positions.

In the frame rotating with the Earth.
Trojan asteroids

A family of asteroids exists in tadpole orbits about L4, L5 with Jupiter.
Restricted Three-body Problem

- Two massive bodies, in a circular orbit. Like Jupiter+Sun. Orbit is Keplerian. $M^*$ star, $M_p$ planet
- Consider the dynamics of a third massless particle.

\[
\frac{dv}{dt} = -GM_\star \frac{(\mathbf{r} - \mathbf{r}_\star)}{|\mathbf{r} - \mathbf{r}_\star|^2} - GM_p \frac{(\mathbf{r} - \mathbf{r}_p)}{|\mathbf{r} - \mathbf{r}_p|^2} - \text{Coriolis Centripetal}
\]

\[
\frac{dv}{dt} - 2\Omega \times v - \Omega \times \Omega \times \mathbf{r} = -\nabla \Phi
\]

In the rotating frame with gravitational potential from each interaction

\[
\frac{dv}{dt} - 2\Omega \times v = -\nabla \Phi_{eff}
\]

\[
\Phi_{eff} = -\frac{GM_\star}{|\mathbf{r} - \mathbf{r}_\star|} - \frac{GM_p}{|\mathbf{r} - \mathbf{r}_p|} + \frac{\Omega^2 r^2}{2}
\]
	pseudo potential
Effective potential contours

\[ \Phi = -G \left( \frac{M_1}{s_1} + \frac{M_2}{s_2} \right) - \frac{G(M_1 + M_2)}{2a^3} r^2 \]
Effective potential contours

L4, L5 are potential minima.
Stable minima.

L1, L2 are saddle points.
Unstable minima.

$M_1 = 5M_\odot$
$M_2 = 1M_\odot$
$a = 1 \text{AU}$
Quasi satellites

Same semi-major axis as a planet.

Both the planet and the quasi-satellite go around the Sun in one year.

The quasi-satellite appears to make an oblong loop when viewed from the planet.
Quasi satellites

- Earth, Venus, Neptune and (recently discovered) Pluto have quasi-satellites
- Goes outside the Hill sphere of the planet (this is different than regular satellites)
- Stays in the vicinity of the planet (different than tadpole or horseshoe orbits)
- Perturbations from the planet are important, lifetimes thousands of orbits but not necessarily the age of the solar system
Tidal Torque

The moon causes the Earth to exhibit a tidal bulge. However, the Earth does not respond immediately. Energy is dissipated in making the tidal bulge (notoriously hard to estimate). Consequently, the bulge lags the actual location of the moon.
Because the earth is no longer a perfect sphere, and because of the misalignment, the earth exerts a torque on the moon.

The moon receives a positive net torque, and its orbit evolves outwards.

Angular momentum is extracted from the Earth’s rotation. As the moon gets further away, the day lengthens.
Tidal Locking

- Raising a tidal bulge dissipates energy.
- This can causing heating.
- The energy typically comes out of the rotation, so the object is slowed down.
- For example, the moon is tidally locked with the Earth. We see the same face of the moon at all times.
- Pluto and Charon are tidally locked. The same hemisphere of each body always faces the other.
Tidal heating on Io

- Io is the innermost Galilean satellite.
- Because of its proximity to Jupiter, Io experiences strong tides.
- S. Peale in 1979 predicted that Io would have volcanoes because of tidal heating.
- Subsequent studies of Io have revealed active volcanoes, as well as extensive evidence for lava flows.
- Io has very very few craters, so the surface is reformed on short timescales.

Credit NASA/JPL
Tidal Torque on Io

• Like the moon, Io is slowly gaining angular momentum, and so drifting away from Jupiter.
• As Io drifted outwards, it captured Europa into resonance. Likewise Europa captured Ganymede.
• The outwards motion of Io is thought to be responsible for the fact that the Galilean satellites are locked in each others mean motion resonances.
Mercury’s Spin Orbit Resonance

- Mercury is locked in a 3:2 resonance such that it rotates 3 times for every 2 orbits.
- At every other perihelion, Mercury presents the same face to the Sun.
- Mercury probably became captured into this resonance because of tides excited by the Sun.
Low eccentricity Hot Exo-Planets

The low eccentricities of extrasolar planets with small major axes is thought to be a result of tidal circularization. This happens quickly when the planet is close to the parent star.
Synchronous orbit

For the Earth: “geosynchronous”
Orbital period of satellite is equal to the rotation period of Earth

\[ P_{spin} = P_o = \frac{2\pi r^{3/2}}{\sqrt{GM}} \]

\[ r = P_{spin}^3 (GM)^{1/3} (2\pi)^{-2/3} \]

A satellite outside synchronous orbit is going slower than planet rotation, slows planet down. Conservation of angular momentum means orbit increases (Moon!)
A satellite inside synchronous orbit is going faster than planet rotation, speeds planet up. Orbit decreases.
Example of the size of the Hill Sphere

Hot Jupiter exoplanet TrES-4b
mass of 0.9MJ (Jupiter masses)
radius of 1.8 RJ (Jupiter Radii)
semi-major axis of 0.051 AU.

TrES-4b’s host star is more massive than the Sun at 1.4 $M_{\text{sol}}$.

Q: How large is TrES-4b’s Hill sphere in units of its radius?
Example of the size of the Hill Sphere

Hot Jupiter exoplanet TrES-4b

mass of $0.9M_J$ (Jupiter masses), radius of $1.8 \, R_J$ (Jupiter Radii)
semi-major axis of $0.051 \, \text{AU}$. TrES-4b’s host star $1.4 \, M_{\text{Sol}}$.

Q: How large is TrES-4b’s Hill sphere in units of its radius?

A:

$$r_H = 0.051\, \text{AU} \left( \frac{0.9 \times 10^{-3}}{3 \times 1.4} \right)^{\frac{1}{3}}$$

$$= 0.003\, \text{AU} = 4.5 \times 10^{10} \, \text{cm}$$

Radius of Jupiter $7.1 \times 10^9 \, \text{cm}$
Hill radius/planet radius for TrES-4b = 3.5
Hill radius is only a few times larger than planet radius!
Implication: no room for rings, satellites
Review

Tidal force \( \sim \frac{2GMmr}{d^3} \)

Hill sphere radius \( r_H = d \left( \frac{M_p}{3M_*} \right)^{1/3} \)

Vocabulary and Concepts:
- Tidal heating, torque, bulges, disruption, and locking
- Lagrange points
- Roche Lobe overflow
- Stability of orbits
- Trojan asteroids
- Synchronous orbit