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Identifying non-resonant *Kepler* planetary systems

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ABSTRACT

The *Kepler* mission has discovered a plethora of multiple transiting planet candidate exosystems, many of which feature putative pairs of planets near mean motion resonance commensurabilities. Identifying potentially resonant systems could help guide future observations and enhance our understanding of planetary formation scenarios. We develop and apply an algebraic method to determine which *Kepler* two-planet systems *cannot* be in a first–fourth order resonance, given the current, publicly available data. This method identifies when any potentially resonant angle of a system must circulate. We identify and list 70 near-resonant systems which cannot actually reside in resonance, assuming a widely used formulation for deriving planetary masses from their observed radii and that these systems do not contain unseen bodies that affect the interactions of the observed planets. This work strengthens the argument that a high fraction of exoplanetary systems may be near resonance but not actually in resonance.

Key words: planets and satellites: dynamical evolution and stability.

1 INTRODUCTION

The *Kepler* mission has identified more planetary candidates than the total number of extrasolar planets that were known before its launch (Borucki et al. 2011). This windfall of mission results enables investigators to perform better statistical analyses to more accurately estimate properties of the exoplanet population. A dynamical question of particular interest is, ‘what is the frequency of resonant extrasolar systems?’

The absence of resonance may be just as important, especially in systems with tightly packed but stable configurations (Lissauer et al. 2011a) and/or which might feature transit timing variations that span several orders of magnitude (Ford et al. 2011; Veras, Ford & Payne 2011). Resonances may be a signature of a particular mode of planetary formation; they are likely to be indicative of convergent migration in nascent protoplanetary discs (Thommes & Lissauer 2003; Kley, Peitz & Bryden 2004; Papaloizou & Szuszkiewicz 2005). Their absence might be indicative of a dynamical history dominated by gravitational planet–planet scattering (Raymond et al. 2008). Alternatively, resonant statistics might best constrain how disc evolution and gravitational scattering interface (Matsumura et al. 2010; Moeckel & Armitage 2011).

Lissauer et al. (2011b) provide a comprehensive accounting of all planetary period ratios in *Kepler* systems, and present distributions and statistics linking the periods to potentially resonant behaviour. Our focus here simply is to identify which systems cannot be resonant. A variety of mean motion resonances are known to exist in the Solar system and extrasolar systems. However, for most of these cases, the masses, eccentricities and arguments of pericenter have been measured directly. Contrastingly, for *Kepler* systems, these parameters are constrained poorly, if at all. Hence, analysing resonant *Kepler* systems poses a unique challenge.

In order to address this situation, we present an analysis that (i) treats the entire potential eccentricity phase space in most cases, (ii) considers a relevant limiting case of the unknown orbital angles and (iii) pinpoints the manner in which planetary masses can affect the possibility of resonance. The analysis is also entirely algebraic, allowing us to avoid numerical integrations and hence investigate an ensemble of systems. The results produce definitive claims about which systems *cannot* be in resonance, given the current data. We consider the ensemble of *Kepler* two-planet systems which may be near first–fourth order eccentricity-based resonances, under the assumptions of coplanarity, non-crossing orbits and the non-existence of additional, as-yet-unidentified planets that would affect the interactions of the observed planets. In Section 2, we explain our analytical model, and then apply it in Section 3. We present our list of non-resonant *Kepler* systems in Table 1, and briefly discuss and summarize our results in Section 4.

2 ANALYTIC MODEL

2.1 Proving circulation

The behaviour of linear combinations of orbital angles from each of the two planets determines if a system is in resonance or not.

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Table 1. Non-resonant two-planet *Kepler* systems.

KOI number	Number of near-resonant instances	Closest $j_1:\lvert j_2\rvert$	Maximum β
116	1	3:1	0.078
123	1	3:1	0.018
150	1	3:1	0.027
153	4	9:5	0.28
209	2	3:1	0.22
220	6	9:5	0.039
232	3	7:3	0.016
244	3	2:1	0.67
270	3	3:1	0.0043
279	4	9:5	0.17
282	1	3:1	0.016
284	2	3:1	0.033
291	1	4:1	0.010
313	3	7:3	0.043
339	1	3:1	0.0043
343	2	7:3	0.12
386	2	5:2	0.16
401	1	5:1	0.031
416	1	5:1	0.024
431	2	5:2	0.60
440	1	3:1	0.040
446	6	9:5	0.18
448	2	4:1	0.038
456	1	3:1	0.048
459	2	3:1	0.042
474	3	5:2	0.020
475	4	9:5	0.23
497	1	3:1	0.31
508	2	2:1	0.84
509	2	3:1	0.042
510	3	7:3	0.035
518	1	3:1	0.028
534	2	7:3	0.37
551	3	2:1	0.25
573	1	3:1	0.11
584	2	2:1	0.083
590	2	4:1	0.0049
612	3	7:3	0.21
638	2	3:1	0.12
645	2	3:1	0.031
658	6	9:5	0.10
672	3	5:2	0.12
676	1	3:1	0.12
691	5	9:5	0.11
693	5	9:5	0.17
700	1	3:1	0.027
708	3	7:3	0.036
736	2	3:1	0.063
752	1	5:1	0.0049
775	2	2:1	0.78
800	3	3:1	0.023
837	3	9:5	0.035
842	2	3:1	0.090
853	6	9:5	0.19
869	1	5:1	0.051
896	3	5:2	0.14
954	2	5:1	0.0059
1015	2	7:3	0.085
1060	2	5:2	0.028
1113	1	3:1	0.030
1151	15	9:7	0.044
1163	2	3:1	0.015
1203	3	7:3	0.059
1215	4	9:5	0.084
1278	1	4:1	0.0081
1301	1	3:1	0.13
1307	3	7:3	0.049
1360	2	5:2	0.27
1364	1	3:1	0.19
1396	6	9:5	0.89

'Libration' is a term often used to denote oscillation of a resonant angle, and 'circulation' often refers to the absence of oscillation. Resonant systems must have a librating angle; for mean motion resonances, the focus of this study, the librating angle incorporates the positions of both planets. This seemingly simple criterion, however, belies complex behaviour often seen in real systems.

For example, Brasser, Heggie & Mikkola (2004) illustrate how the resonant angle of a Neptunian Trojan can switch irregularly between libration and circulation, while Farmer & Goldreich (2006) illustrate how long periods of circulation can be punctuated by short periods of libration. Characterizing libration versus circulation in extrasolar systems – often with two massive extrasolar planets – sometimes necessitates careful statistical measures (Veras & Ford 2010).

The gravitational potential between two coplanar bodies orbiting a star can be described by two 'disturbing functions', denoted by R_w, with $w = 1, 2$, which are infinite linear sums of cosine terms with arguments of the form:

$$\phi(t) = j_1\lambda_1(t) + j_2\lambda_2(t) + j_3\varpi_1(t) + j_4\varpi_2(t), \tag{1}$$

where λ represents mean longitude, ϖ represents longitude of pericentre, the 'j' values represent integer constants, and the subscript '1' refers to the outer body while the subscript '2' refers to the inner body. We choose this subscripting convention so our equations conform with those in Murray & Dermott (1999) and Veras (2007). The mean longitude is directly proportional to mean longitude at epoch, denoted by ϵ_w, such that $\lambda_w \equiv \varpi_w + M_w = \pi_w + \Omega_w + M_{w0} + \int_{t_0}^{t} n_w(t')\,\mathrm{d}t' = \epsilon_w + \int_{t_0}^{t} \mu_w^{(1/2)} a_w(t')^{(-3/2)}\mathrm{d}t'$, where M_w denotes mean anomaly, π_w denotes argument of pericentre, and n_w denotes mean motion. The mean motion is related to its semimajor axis, a_w, and mass through Kepler's third law by $n_w^2 a_w^3 = \mu_w$, where $\mu_w = G(m_0 + m_w)$, with m_w representing the planet's mass, m_0 the central body's mass, and G the universal gravitational constant.

The time derivative of equation (1) yields

$$\dot{\phi}(t) = j_1\mu_1^{1/2}a_1^{-(3/2)}(t) + j_1\dot{\epsilon}_1(t) + j_2\mu_2^{1/2}a_2^{-(3/2)}(t) + j_2\dot{\epsilon}_2(t) + j_3\dot{\varpi}_1(t) + j_4\dot{\varpi}_2(t). \tag{2}$$

Lagrange's planetary equations (e.g. Murray & Dermott 1999) relevant to equation (2) are

$$\frac{\mathrm{d}\epsilon_w}{\mathrm{d}t} = -a_w^{1/2}A_{w,1}\frac{\partial R_w}{\partial a_w} + a_w^{-(1/2)}A_{w,2}A_{w,3}\frac{\partial R_w}{\partial e_w}, \tag{3}$$

$$\frac{\mathrm{d}\varpi_w}{\mathrm{d}t} = a_w^{-(1/2)}A_{w,2}\frac{\partial R_w}{\partial e_w}, \tag{4}$$

where

$$A_{w,1} = 2\mu_w^{-(1/2)}, \tag{5}$$

$$A_{w,2} = \mu_w^{-(1/2)} e_w^{-1} \left(1 - e_w^2\right)^{1/2}, \tag{6}$$

$$A_{w,3} = 1 - \left(1 - e_w^2\right)^{1/2}. \tag{7}$$

The form of R_w used dictates how to proceed. Veras (2007) expresses Ellis & Murray's (2000) disturbing function as

$$R_w = a_1^{-1} \sum_{y=1}^{\infty} \left[\sum_{p=1}^{\infty} C_w^{(y,p)} X^{(y,p)}\right] \cos\phi^{(y)}, \tag{8}$$

where $C_w^{(i,p)}$ is a function of the masses and semimajor axes, and $X^{(y,p)}$ is a function of the eccentricities and inclinations. Both of these auxiliary variables contain the detailed functional forms needed to model individual resonances.

When inserted into equations (2)–(4), the disturbing function will yield

$$\dot{\phi}^{(u)} = \sum_{y} \left(D^{(y,u)} \cos\phi^{(y)}\right) + E^{(u)}, \tag{9}$$

where $\dot{\phi}^{(u)}$ is a potentially resonant angle, $D^{(y,u)}$ is an explicit algebraic function of the masses and orbital elements, and

$$E^{(u)} = \sum_{w=1}^{2} j_w^{(u)} \mu_w^{1/2} a_w^{-(3/2)}. \tag{10}$$

Hence, a particular angle $\phi^{(u)}$ cannot librate if

$$\sum |D^{(y,u)}| < |E^{(u)}|. \tag{11}$$

Equation (11) represents the criterion for an angle to circulate. Note that no integrations are required to evaluate the criterion. For systems which cannot be in resonance, the maximum value of $\sum |D^{(y,u)}|/|E^{(u)}| \equiv \beta_u$ provides an estimate of the proximity to a potential resonance.

The dependence these variables have on planetary mass is important to understand for the implications for *Kepler* systems: μ_1 or μ_2 appears in each of the terms in $E^{(u)}$ and $D^{(y,u)}$, and are insensitive to the planetary masses, as long as these masses are negligible compared to the star's mass. Further, each term in $D^{(y,u)}$ is linearly proportional to either m_1 or m_2. Therefore, if m_1 and m_2 are both scaled by the same factor of their radii in a common mass–radius relationship, then the relative strength of the terms in $D^{(y,u)}$ will vary by a factor proportional to the planetary radii ratio. The signs of these terms depend on the resonance being studied. Hence, for a particular resonance, one may determine which terms are additive, and obtain an explicit dependence on planetary mass.

2.2 Hill stability

Unless two bodies are resonantly locked in a configuration that allows for crossing orbits (such as Neptune and Pluto), the bodies will undergo dynamical instability if their eccentricities are too great and their semimajor axis difference is too small. Planets whose orbits never cross are said to be Hill stable and obey (Gladman 1993)

$$\begin{aligned} &\frac{1+\eta_1+\eta_2}{(\eta_1+\eta_1\eta_2+\eta_2)^3}\left[\eta_1+\frac{\eta_2}{\alpha}\right] \\ &\times \left[\eta_2\sqrt{\alpha\left(1-e_2^2\right)}+\eta_1\sqrt{1-e_1^2}\right]^2 \\ &> 1+\frac{3^{4/3}\eta_1\eta_2}{(\eta_1+\eta_2)^{\frac{4}{3}}}-\frac{\eta_1\eta_2\left(11\eta_2+7\eta_1\right)}{3\left(\eta_1+\eta_2\right)^2}, \end{aligned} \tag{12}$$

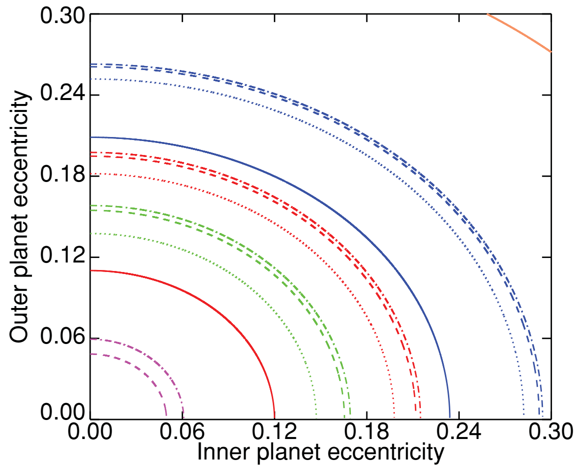

Figure 1. Hill stability eccentricity portrait. Each curve (for the following planet/star mass ratios: solid lines – $10^{-3}\,\mathrm{M}_{\odot}$; dotted lines – $10^{-4}\,\mathrm{M}_{\odot}$; dashed lines – $10^{-5}\,\mathrm{M}_{\odot}$; dot–dashed lines – $10^{-6}\,\mathrm{M}_{\odot}$) bounds eccentricities at which two planets may be Hill stable, for semimajor axes ratios corresponding to the following commensurabilities (in order of dashed lines going outwards from the origin): 7:6 (magenta), 3:2 (green), 5:3 (red), 2:1 (blue) and 3:1 (salmon). Stable systems lie below the curves, and observed *Kepler* systems are assumed to be stable. Note that the curves are not symmetrical about the origin. The solid salmon curve indicates that nearly all planets whose semimajor axis ratio is within a factor of ≈2 are subject to stability constraints for $e_1, e_2 < 0.3$.

where $\eta_1 \equiv m_1/m_0$, $\eta_2 \equiv m_2/m_0$ and $\alpha = a_2/a_1$. In principle, systems which do not satisfy equation (12) may be stable, but generally this is true only for resonant systems. Thus, we focus our efforts only on those systems which are provably stable. Fig. 1 displays level curves of equation (12) for different values of η_1 and η_2 [$10^{-3}\,\mathrm{M}_{\odot}$ (solid), $10^{-4}\,\mathrm{M}_{\odot}$ (dotted), $10^{-5}\,\mathrm{M}_{\odot}$ (dashed), $10^{-6}\,\mathrm{M}_{\odot}$ (dot–dashed)] and at different commensurabilities of interest [7:6 (magenta), 3:2 (green), 5:3 (red), 2:1 (blue), 3:1 (salmon)], which each correspond to the appropriate value of α. The range of eccentricities plotted and considered in this study is well within the absolute convergence limits of this disturbing function (Ferraz-Mello 1994; Sidlichovsky & Nesvorny 1994) and corresponds roughly to a regime where a fourth-order treatment (as in Veras 2007) is accurate to within $\approx 0.3^4 < 1$ per cent.

3 APPLICATION TO *KEPLER* SYSTEMS

3.1 Method

We use the publicly available *Kepler* data base,[1] which provides directly measured values for planetary periods and estimated values of the stellar mass. With these, we estimate the planetary semimajor axes by assuming values for the planetary masses based on a mass–radius relationship. Lissauer et al. (2011b) estimate these values using

$$m_1 = \left(\frac{R_1}{\mathrm{R}_{\oplus}}\right)^b \mathrm{m}_{\oplus}, \tag{13}$$

where $b = 2.06$. A similar formula holds for the other planet in the system.

[1] http://archive.stsci.edu/kepler/planet_candidates.html

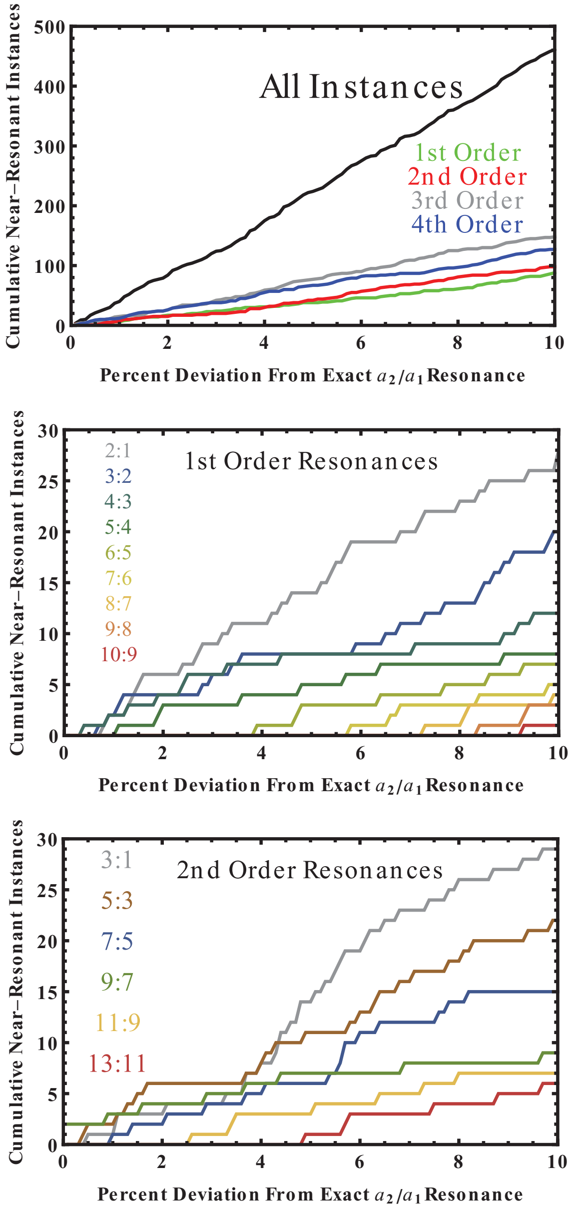

Figure 2. Number of near-resonant instances in two-planet *Kepler* systems. Plotted is the cumulative number of triplets (KOI, j_1, j_2) versus x, where x is defined in equation (14) and KOI refers to the *Kepler* object of interest or candidate exoplanet host star. The upper panel plots all instances for first–fourth order resonances, and the other panels plot selected resonances of interest.

We assume equation (13) to be true, and then determine the 'nearness' of all two-planet systems to a mean motion commensurability through

$$(1 - x/100)\,(j_1/j_2)^{2/3} < a_1/a_2 < (1 + x/100)\,(j_1/j_2)^{2/3}\,, \tag{14}$$

where x effectively measures the per cent offset from resonance in semimajor axis space. Because we consider all potential resonances up to fourth order, the same planetary system may be close to several resonances. We refer to a 'near-resonant instance' as a case where equation (14) is satisfied. We plot the cumulative frequency of near-resonant instances as a function of x in Fig. 2. Not included in the figure are (j_1, j_2) multiplicities: higher order harmonic angles which can be expressed as a linear combination of the angles with the lowest order relatively prime values of j_1 and j_2, although these higher order terms are included in the computation of $D^{(y,u)}$ (see e.g. table 4 of Veras 2007). The figure demonstrates the potentially wide variety of resonant configurations which may be possible depending on one's definition of 'near-resonant'. We choose to be conservative and include all 'near-resonant instances' on the plots in our analysis.

Having identified a potential resonance with a given j_1 and j_2, and having adopted m_1, m_2, a_1 and a_2, we then sample the entire eccentricity phase space for which the planets are Hill stable. In the few cases where Hill stable systems admit $e_1 > 0.3$ and/or $e_2 > 0.3$, we limit the eccentricities to these values for accuracy and convergence considerations, as described earlier. At each of 31 evenly spaced values of e_2 from 0 to 0.3, we compute the maximum Hill stable value of e_1, and then sample 31 evenly spaced values of e_1 from 0 to that Hill maximum value. Finally, we compute $\beta_u = \sum_y |D^{(y,u)}|/|E^{(u)}|$ for every possible disturbing function argument for j_1, $|j_2| > 0$ which satisfy the d'Alembert relations.[2] If no value of β_u exceeds unity, then we flag the system as non-resonant and record the highest value of β_u as a proxy for closest proximity to resonance.

3.2 Results

Of the 116 *Kepler* systems with two transiting candidates for which public data are available, 94 have planets with periods similar enough to each other to be considered 'near-resonant' for at least one first–fourth order resonance with $x \leq 10$. These 94 systems registered a total of 465 near-resonant instances for $j_1 < 20$. Of those 465, 313 cannot be resonant. Only four systems (KOI #: 89, 523, 657, 738) have planets which may be resonant for at least one instance and cannot be resonant for at least one other, thereby restricting the system's potential membership in some resonances. For 20 systems (KOI #: 82, 111, 115, 124, 222, 271, 314, 341, 543, 749, 787, 841, 870, 877, 945, 1102, 1198, 1221, 1241, 1589), mean motion resonance could not be ruled out for any of the near-resonant instances sampled. Quantifying the likelihood of resonance in these systems would require future detailed individual analysis.

We claim that the remaining 70 systems *cannot* be in resonance, despite their close proximity to a commensurability. We list these systems in Table 1, along with the number of their near-resonant instances, their maximum value of β and the mean motion resonance this value corresponds to. The larger the value of β, the closer the system is to having parameters which could admit resonance. In most cases, $\beta < 0.5$, meaning that the systems are well outside of resonance. Notable exceptions are KOI 244, KOI 431, KOI 508, KOI 775 and KOI 1396, three of which are close to the strong 2:1 commensurability. KOI 1151 is so close to so many commensurabilities because for that system, $1/\alpha = 1.26$, meaning that the planets are tightly packed and on the edge of stability.

We do not expect variations in the masses of the *Kepler* planets to greatly affect the composition of Table 1, assuming that the value of b from equation (13) does not vary by more than a few tenths from 2.06. The value of β for a particular system can hint at the potential implications of mass variation. In particular, values of β close to unity indicate that the system is on the border of potentially resonant behaviour. For example, for KOI 1396 ($\beta = 0.89$), if we

[2] The secular arguments $j_1 = j_2 = 0$ up to fourth order are included in the computation of $D^{(y,u)}$ even though we test only for circulation of angles which can lead to mean motion resonance.

set $b = 1.7$, then the resulting planetary masses could allow the system to harbour a (weak) 9:5 resonance.

4 CONCLUSION AND IMPLICATIONS

We have identified 70 *Kepler* two-planet near-commensurate systems which cannot be in an eccentricity-based mean motion resonance of up to fourth order. These systems, may, in principle, achieve resonance with crossing orbits or high ($e > 0.3$) eccentricities that could remain stable. The criterion of equation (11) is generally applicable to any three-body system suspected of harbouring resonant behaviour. We caution that these results could be affected by the presence of additional planets which have yet to be detected.

Systems which are provably non-resonant may be incorporated in formation studies and detailed analyses of *Kepler* data. *Kepler* multiplanet systems may be preferentially clustered around particular resonances (Lissauer et al. 2011b). One possible explanation may be that convergent migration locks planets in a mean motion resonance which later gets broken by some additional perturbation. We find that *Kepler* multiplanet systems preferentially cluster around commensurabilities where $|j_2|$ is low and rarely do so when $|j_2|$ is high. In particular, the number of *Kepler* systems near the 2:1, 3:2, 3:1 and 5:3 commensurabilities is higher than what would be expected from a random distribution of *Kepler* planet candidate semimajor axes. Provably non-resonant planets may also complement transit timing variation statistics, as these variations take on distinctly different characteristics for near-resonant and resonant systems (Veras et al. 2011).

The analysis in this work cannot be performed with *Kepler* systems that contain more than two-planet candidates because (i) more disturbing functions must be incorporated, and hence the criteria for circulation becomes decidedly more complex, and (ii) analytical formulae for Hill stability no longer hold. Fig. 29 of Chatterjee et al. (2008) demonstrates that widely separated pairs of planets in three-planet systems whose orbits would be Hill stable in the two-planet-only case may eventually become unstable. Even if a multiplanet system was assumed to be stable over a specified period of time and more disturbing functions were introduced, the resulting expansion of the phase space might render the computational cost of a similar algebraic analysis prohibitive compared to numerical integrations. However, the investigation of the non-planar two-planet case with inclination-based resonances might be a fruitful avenue to explore, especially because multiple transits detected by *Kepler* may constrain the planets' mutual inclination, albeit weakly.

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REFERENCES

Borucki W. J. et al., 2011, ApJ, 736, 19
Brasser R., Heggie D. C., Mikkola S., 2004, Celest. Mech. Dynamical Astron., 88, 123
Chatterjee S., Ford E. B., Matsumura S., Rasio F. A., 2008, ApJ, 686, 580
Ellis K. M., Murray C. D., 2000, Icarus, 147, 129
Farmer A. J., Goldreich P., 2006, Icarus, 180, 403
Ferraz-Mello S., 1994, Celest. Mech. Dynamical Astron., 58, 37
Ford E. B. et al., 2011, ApJS, 197, 2
Gladman B., 1993, Icarus, 106, 247
Kley W., Peitz J., Bryden G., 2004, A&A, 414, 735
Lissauer J. J., Fabrycky D. C., Ford E. B. et al., 2011a, Nat, 470, 53
Lissauer J. J. et al., 2011b, ApJS, 197, 8L
Matsumura S., Thommes E. W., Chatterjee S., Rasio F. A., 2010, ApJ, 714, 194
Moeckel N., Armitage P. J., 2011, MNRAS, preprint (arXiv:1108.5382)
Murray C. D., Dermott S. F., 1999, Solar System Dynamics. Cambridge Univ. Press, Cambridge
Papaloizou J. C. B., Szuszkiewicz E., 2005, MNRAS, 363, 153
Raymond S. N., Barnes R., Armitage P. J., Gorelick N., 2008, ApJ, 687, L107
Sidlichovsky M., Nesvorny D., 1994, A&A, 289, 972
Thommes E. W., Lissauer J. J., 2003, ApJ, 597, 566
Veras D., 2007, Celest. Mech. Dynamical Astron., 99, 197
Veras D., Ford E. B., 2010, ApJ, 715, 803
Veras D., Ford E. B., Payne M. J., 2011, ApJ, 727, 74

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