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RESONANT REPULSION OF KEPLER PLANET PAIRS

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ABSTRACT

Planetary systems discovered by the Kepler space telescope exhibit an intriguing feature. While the period ratios of adjacent low-mass planets appear largely random, there is a significant excess of pairs that lie just wide of resonances and a deficit on the near side. We demonstrate that this feature naturally arises when two near-resonant planets interact in the presence of weak dissipation that damps eccentricities. The two planets repel each other as orbital energy is lost to heat. This moves near-resonant pairs just beyond resonance, by a distance that reflects the integrated dissipation they experienced over their lifetimes. We find that the observed distances may be explained by tidal dissipation if tides are efficient (tidal quality factor ~ 10). Once the effect of resonant repulsion is accounted for, the initial orbits of these low mass planets show little preference for resonances. This is a strong constraint on their origin.

1. INTRODUCTION

NASA's Kepler mission is revolutionizing our knowledge of planetary systems. It has already discovered thousands of transiting planetary candidates, including hundreds of systems with two or more planets (Batalha et al. 2012). Most of these are Neptune- or Earth-sized planets. To date, one of the most intriguing Kepler discoveries is that, while the spacing between planets appears to be roughly random, there is a distinct excess of planetary pairs just wide of certain resonances, and a nearly empty gap just narrow of them (Lissauer et al. 2011; Fabrycky et al. 2012). These features are particularly prominent near the 3:2 and 2:1 resonances, and affects planets that fall within a few percent of resonances (Fabrycky et al. 2012).

Is this resonance asymmetry a feature planetary systems are born with, or one they acquire much later on? Many studies have reported that planets become trapped into first-order resonances when they migrate in protoplanetary disks (e.g., Lee & Peale 2002; Snellgrove et al. 2001; Papaloizou & Szuszkiewicz 2005). In fact, the presence of resonances among giant planets detected by radial velocity has been regarded as strong evidence for disk migration (e.g., Marcy et al. 2001; Tinney et al. 2006). However, Kepler's low-mass planets appear to be less influenced by resonances, and the pile-ups just outside resonances are partly counterbalanced by the gaps inside them.

In this paper, we identify a process that can modify the pair separation and give rise to the observed resonance asymmetry. But first, let us consider a commonly invoked mechanism, tidal circularization. If the inner planet is eccentric, tides raised on it would damp its eccentricity, decrease its semi-major axis, and hence increase the period ratio of the pair (Novak et al. 2003; Terquem & Papaloizou 2007)[3]. Adopting the equilibrium tide expression from Hut (1981), the damping rate for a psudo-synchronized planet is

$$\gamma_e = \frac{1}{e}\frac{de}{dt} = -\frac{9}{2}\frac{k_2}{T_1}q(1+q)\left(\frac{R_1}{a}\right)^8 , \qquad (1)$$

where $q = M_*/m_1$ is the mass ratio of the star to planet, k_2 the tidal love number, R_1 the inner planet's radius and a its orbital separation. In this tidal model, $T_1 = R_1^3/(Gm_1\tau_1)$ where τ_1 is the assumed constant tidal lag time which we take to be $\tau_1 = P_1/(2Q_1)$, with Q_1 the inner planet's tidal quality factor (Goldreich & Soter 1966) and P_1 its orbital period. Numerically,

$$\gamma_e \sim (6.5\times 10^7\,\mathrm{yrs})^{-1}\left(\frac{Q_1}{10}\right)^{-1}\left(\frac{k_2}{0.1}\right)\left(\frac{M_*}{M_\odot}\right)^{-2/3} \times\left(\frac{m_1}{10M_\oplus}\right)^{-1}\left(\frac{R_1}{2R_\oplus}\right)^{5}\left(\frac{P_1}{5\mathrm{day}}\right)^{-13/3} . \qquad (2)$$

The orbital decay rate is $\dot a/a = 2e\dot e$ because orbital angular momentum is largely conserved. So tidal evolution could have potentially circularized orbits inward of ~ 10 days. As it does so, it moves the inner planet inward by $\Delta a/a \sim -e_1^2$. This increases the period ratio for a planet pair by a fractional amount of $3e_1^2/2 = 1.5\%(e_1/0.1)^2$. However, tidal circularization alone can not reproduce the observed asymmetry: assuming all near-resonant pairs were initially uniformly distributed in their period ratios, all systems march to larger period ratios by a comparable amount. This produces neither gap nor peak.

A more selective mechanism is required. In this paper, we show that for a pair of planets that happen to lie near a mean-motion resonance, dissipation causes the planets to repel each other. The rate of repulsion is greatest at exact resonance and falls off steeply away from resonance. Planets that are initially slightly closer than resonance are pushed wide of the resonance; those that are initially wider are pushed even further apart. And planet pairs far away from the resonance are not affected. So the combined action of resonant interaction and damping naturally give rise to the observed resonance asymmetry. This effect, which we term "resonant repulsion",

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[3] Tides raised on the star play little role.

was first investigated by Lithwick & Wu (2008) to account for the orbits of Pluto's minor moons. Though it likely fails quantitatively in that particular case, we show here that it can account for the observed asymmetry in Kepler planets. A similar effect plays a role in the migration of the moons of Jupiter and Saturn (e.g., Peale 1986). But in that case, the dominant dissipative effect (tides raised by the moons on the central bodies, Jupiter or Saturn) pushes the moons outwards which often locks them into resonances. Papaloizou (2011) also investigated the effect of tidal circularization on multiple planet systems.

2. RESONANT REPULSION

We consider the evolution of two planets orbiting a star and assume that the interaction between the planets is predominantly due to the 2:1 resonance. We will also include weak external eccentricity-damping forces. The energy (or Hamiltonian) of the two planets is, to leading order in eccentricity,

$$H = -\frac{GM_*m_1}{2a_1} - \frac{GM_*m_2}{2a_2} - \frac{Gm_1m_2}{a_2} \times \left(f_1e_1\cos(2\lambda_2-\lambda_1-\varpi_1) + f_2e_2\cos(2\lambda_2-\lambda_1-\varpi_2)\right) \quad (3)$$

where we follow standard notation (e.g., Murray & Dermott 2000), with the orbital parameters for the inner planet denoted by $\{a_1, e_1, \lambda_1, \varpi_1\}$, and those for the outer planet subscripted by 2. The mass of the star and planets are M_*, m_1, m_2, and the Laplace coefficients are $f_1 = -(2+\alpha D/2)b^2_{1/2}$ and $f_2 = (3/2+\alpha D/2)b^1_{1/2} - 2\alpha$ (Murray & Dermott 2000). Near 2:1 resonance ($\alpha = 2^{-2/3}$), the Laplace coefficients are $f_1 = -1.19$ and $f_2 = 0.428$.

We choose units such that

$$GM_* = 1\ , \quad (4)$$

and assumes that the eccentricities are small. In terms of the complex eccentricity

$$z_j \equiv e_j e^{i\varpi_j}\ , \quad (5)$$

the equations of motion for planet j are (e.g. Murray & Dermott 2000; Lithwick & Wu 2008)

$$\frac{d\lambda_j}{dt} = \frac{2\sqrt{a_j}}{m_j}\frac{\partial H}{\partial a_j} \quad (6)$$

$$\frac{dz_j}{dt} = -\frac{2i}{m_j\sqrt{a_j}}\frac{\partial H}{\partial z_j^*} \quad (7)$$

$$\frac{da_j}{dt} = -\frac{2\sqrt{a_j}}{m_j}\frac{\partial H}{\partial \lambda_j} \quad (8)$$

To leading order in m_j/M_*, the semi-major axes are constant, and the equations for λ_j are

$$\frac{d\lambda_j}{dt} = n_j\ , \quad (9)$$

where

$$n_j \equiv a_j^{-3/2}\ . \quad (10)$$

Hence

$$\lambda_j \approx n_j t\ , \quad (11)$$

The eccentricity equations become, after adding damping terms,

$$\frac{dz_1}{dt} = i\mu_2 n_2\sqrt{\frac{a_2}{a_1}}f_1e^{i\phi} - \gamma_{e1}z_1 \quad (12)$$

$$\frac{dz_2}{dt} = i\mu_1 n_2 f_2 e^{i\phi} - \gamma_{e2}z_2\ , \quad (13)$$

where

$$\mu_j \equiv m_j/M_* \quad (14)$$

$$\phi \equiv 2\lambda_2 - \lambda_1 \approx -2\delta\cdot n_2 t\ . \quad (15)$$

Here

$$\delta \equiv \frac{n_1 - 2n_2}{2n_2}\ . \quad (16)$$

is the fractional distance to nominal resonance. When $\delta < 0$ the pair is on the near side of resonance, otherwise it is on the far side.[4] The γ_{ej} in Equations (12)–(13) denote the eccentricity damping rates on each of the two planets due to some external force (e.g., tides or a dissipative disk). We assume that $\gamma_{ej} \ll |\delta n_2|$.

We discard the free solutions to Equations (12)–(13) because they decay to zero at the rates γ_{ej}, much faster than the rate of semi-major axis evolution, as we shall see below. The forced eccentricities are, to first order in $\gamma_{ej}/(\delta n_2) \ll 1$:

$$z_1 = -\frac{\mu_2}{2\delta}f_1\sqrt{\frac{a_2}{a_1}}e^{i\phi}\left(1 - i\frac{\gamma_{e1}}{2\delta n_2}\right) \quad (17)$$

$$z_2 = -\frac{\mu_1}{2\delta}f_2e^{i\phi}\left(1 - i\frac{\gamma_{e2}}{2\delta n_2}\right)\ . \quad (18)$$

The small phase shift, $O(\gamma_{ej})$, relative to the undamped forced eccentricities plays a crucial role in resonant repulsion.

Inserting the above forced eccentricities into the semi-major axis equations yields, as in Lithwick & Wu (2008),

$$\frac{d\ln a_1}{dt} = -\frac{\beta}{2}\frac{\mu_1^2}{\delta^2}(\gamma_{e1}f_1^2\beta + \gamma_{e2}f_2^2) - \gamma_{a1}|z_1|^2. \quad (19)$$

$$\frac{d\ln a_2}{dt} = \frac{\mu_1^2}{\delta^2}(\gamma_{e1}f_1^2\beta + \gamma_{e2}f_2^2)\ - \gamma_{a2}|z_2|^2 \quad (20)$$

where

$$\beta \equiv \frac{\mu_2\sqrt{a_2}}{\mu_1\sqrt{a_1}} \quad (21)$$

and we have included additional damping terms with rates $\gamma_{aj}e_j^2$. The form of our damping rates for a assume that it is the damping of the planets' eccentricities that lead to a evolution, as is true for instance with tides (see below). By contrast, if the planet is migrated in a disk, or pushed by tides raised on the central body (as for Jupiter's moons), the induced rate of change of a would be independent of eccentricity. We shall not consider those kinds of forces.

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[4] For brevity, we often refer to nominal resonance ($\delta = 0$) as simply resonance. This should not be confused with a pair being locked in resonance, i.e. in a state where the resonant angles librate.

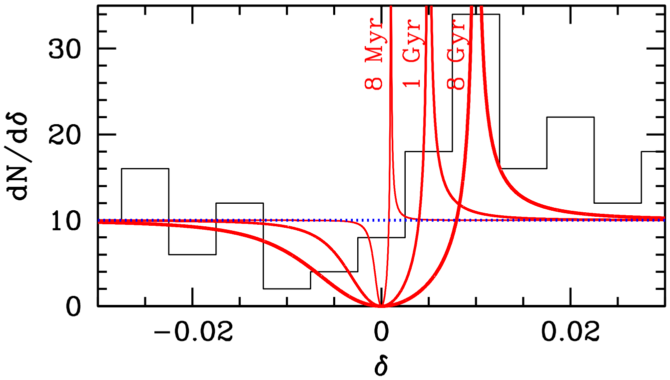

FIG. 1.— Effect of resonant repulsion on the period distribution of planet pairs. The dotted blue line shows the assumed initial condition: a flat differential number distribution of planet pairs. δ is the fractional distance to resonance. The three red curves show the effect of resonant repulsion at three later times (via Eq. 24). Planets are evacuated from the near side of resonance ($\delta < 0$) and piled up on the far side ($\delta > 0$). The parameters are chosen such that $\delta_{\rm mig} = .005(t/{\rm Gyr})^{1/3}$ in Equation (25), similar to our fiducial values for tidal damping (Eq. (26)). The pileup occurs at $\sim \delta_{\rm mig}$ and the evacuated region extends to $\sim -\delta_{\rm mig}$. The black histogram shows Kepler data for planet pairs near the 2:1 and 3:2 resonances (data obtained from Kepler website; see Batalha et al. 2012).

We conclude that the distance to resonance changes at the rate

$$\frac{d\delta}{dt} = \frac{3}{4}\frac{{\mu_1}^2}{\delta^2}\Gamma \ , \tag{22}$$

where

$$\Gamma \equiv (2+\beta)(\gamma_{e1} f_1^2 \beta + \gamma_{e2} f_2^2) + \frac{\gamma_{a1} f_1^2 \beta^2 - \gamma_{a2} f_2^2}{2} \tag{23}$$

for $|\delta| \ll 1$. We verify this rate with an N-body simulation below.

As long as $\Gamma > 0$, as we shall argue is the case, then δ always increases, independent of the sign of δ. A pair of planets that is initially spaced closer than nominal resonance ($\delta < 0$) will tend to be pushed outside of resonance, i.e. to $\delta > 0$. And a pair initially outside of resonance will be pushed even further apart. We term this effect *resonant repulsion.* Furthermore, since the speed of migration is slowest far from resonance, the region near nominal resonance ($\delta = 0$) should be unoccupied, and resonant pairs should evacuate the resonance region and pile up outside. This will lead to an asymmetry, with more planets outside of nominal resonance than inside.

Two planets that initially have $\delta = \delta_0$ repel each other to $\delta > \delta_0$, and at time t they migrate to

$$\delta(t) = \left(\delta_{\rm mig}^3 + \delta_0^3\right)^{1/3} \ , \tag{24}$$

where

$$\delta_{\rm mig}(t) = \left(\frac{9}{4}\mu_1^2 \Gamma t\right)^{1/3} . \tag{25}$$

Figure 1 illustrates the effect on the distribution of period ratios.

The sign of Γ is always positive due to eccentricity damping alone, i.e. to the γ_{ej} terms in Equation (23). Furthermore, if tides are the source of damping, then $\gamma_{aj} = 2\gamma_{ej}$ by angular momentum conservation, leaving $\Gamma > 0$; this is also true for any form of damping that conserves angular momentum. Other forms of damping could in principle result in values of γ_{aj} that make Γ negative. However, the fact that Kepler pairs are piled up outside of resonances argue that this did not happen.

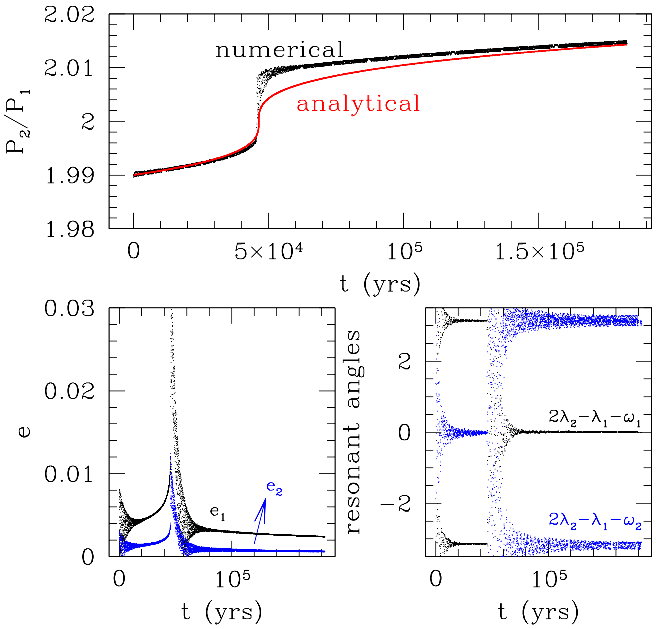

FIG. 2.— An N-body simulation of resonant repulsion, where the dissipation is provided by tidal damping on the inner planet. A pair of planets initially on the near side of the 2:1 resonance (i.e., with period ratio $P_2/P_1 < 2$) is pushed to the far side (black points, upper panel). The red curve is the analytic solution (Equations (24) and (26)) with the same parameters as the simulation. The lower panels show the evolution of eccentricity and the two resonant angles. The two planets both have mass $10M_\oplus$ and orbit a solar mass star, with $P_1 = 5$ days. To speed up the simulation, we artificially enhance the tidal effect by assuming a radius of $12R_E$ for the inner planet, while $Q_1 = 10$ and $k_2 = 0.1$. The simulation was performed with the SWIFT package (Levison & Duncan 1994), modified to include routines for tidal damping and relativistic precession.

3. RESONANT REPULSION BY TIDES

In this section we focus on the case when the dissipation is provided by tidal damping. The rate of eccentricity damping γ_{e1} is given by Equation (2). In addition, $\gamma_{a1} = 2\gamma_{e1}$ by angular momentum conservation, and we may ignore tides on the outer planet ($\gamma_{e2} = \gamma_{a2} = 0$) because tidal damping rates are steep functions of orbital period. Therefore Equation (25) becomes

$$\delta_{\rm mig} \approx 0.006 \left(\frac{Q_1}{10}\right)^{-1/3} \left(\frac{k_2}{0.1}\right)^{1/3} \left(\frac{m_1}{10M_\oplus}\right)^{1/3} \left(\frac{R_1}{2R_\oplus}\right)^{5/3} \times \left(\frac{M_*}{M_\odot}\right)^{-8/3} \left(\frac{P_1}{5{\rm day}}\right)^{-13/9} \left(\frac{t}{5{\rm Gyrs}}\right)^{1/3} \times (2\beta + 2\beta^2)^{1/3}. \tag{26}$$

Figure 2 shows an N-body simulation with tides of two planets initially on the near side of resonance. Resonant repulsion pushes them to the far side, in agreement with the analytic solution (Equations (24) and (26)). There is modest disagreement when the pair crosses through nominal resonance when the expansion in small e becomes invalid. The free eccentricities damp away after a

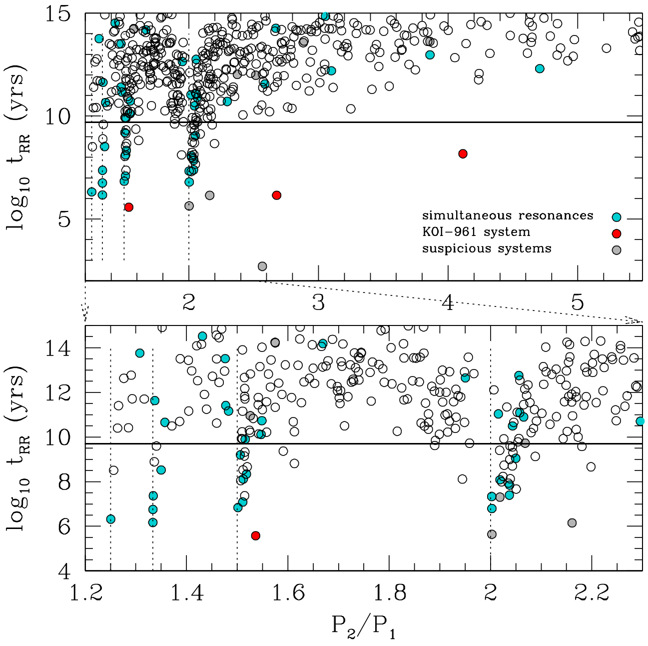

FIG. 3.— The timescale for resonant repulsion to move the period ratio of Kepler planet pairs by a distance $|\delta|$, where δ is the observed fractional distance to the closest first order resonance. We adopt KIC system parameters, with updated values for KOI-961 (red dots) from Muirhead et al. (2012). For tidal dissipation, $Q_1 = 10$ and $k_2 = 0.1$. The lower panel zooms in to the resonant region. If $t_{\rm RR} \gg$ system age (the horizontal line is the age of the Sun), the period ratios should have evolved little since birth; while for $t_{\rm RR} \ll$ age, we do not expect the systems to linger at the observed ratios. The fact that most pairs lie at or above the horizontal line is consistent with resonant repulsion by tides. Although a number of systems very near resonances have very small $t_{\rm RR}$, close inspection reveals that many of these (if not all) are related to 3-body effects: the turquoise circles indicate pairs where one or both planets are engaged in at least two resonances simultaneously (defined as $|\delta| < 3\%$). Our simple picture of resonant repulsion may break down in these cases. 'Suspicious systems' refer to those where the nominal total mass $\geq 1000 M_\oplus$ (assuming Earth density) and we discard them from consideration for fear of contamination.

brief initial period ($\lesssim 2 \times 10^4$ yr). On crossing nominal resonance, they are regenerated, but then quickly damp away again. Damping locks the system into libration (of both resonant angles), but this has little dynamical significance, as it is merely a consequence of the eccentricities taking on their purely forced values.

Figure 3 shows the "resonant repulsion time" ($t_{\rm RR}$) for all reported Kepler pairs. This is the timescale over which resonant repulsion by tides moves a pair towards or away from the nearest first order resonance. Mathematically, $t_{\rm RR} \equiv |\delta/\dot{\delta}|$, where δ is the observed fractional distance and $\dot{\delta}$ is the rate predicted by resonant repulsion (Equation (22)) assuming tidal damping is operating with $Q_1 = 10$, $k_2 = 0.1$, and using the observed planet and stellar parameters. On this plot, systems that have $t_{\rm RR}$ longer than their age have not experienced significant resonant repulsion, while all those with shorter $t_{\rm RR}$ should have moved to the right.

A number of inferences may be drawn. First, most systems far from resonances ($|\delta| \geq 10\%$) have experienced negligible resonant repulsion and were most likely born with the period ratio they have today.

Second, systems within $1 - 10\%$ of resonance exhibit $t_{\rm RR}$ that are as long as, or longer than, the typical age of systems (a few Gyrs). This is consistent with resonant repulsion by tides: systems with shorter $t_{\rm RR}$ would have been moved to the right until $t_{\rm RR}$ was comparable to the age of the system. Near the 2:1 resonance, it appears that pairs as far left as 1.8 and as far right as 2.2 could have been affected by the repulsion.

Last, many systems very near resonances ($|\delta| \lesssim 1\%$) exhibit such short $t_{\rm RR}$ that they should have migrated to much larger δ values. At first sight, their presence is troubling. However, an inspection of the Kepler catalogue reveals that many of these are in triples or higher multiple systems, and these planets are engaged simultaneously in two or more 2-body resonances. The worst-off cases are in simultaneous resonances, reminiscent of the Laplace resonance of Jupiter's moons (Yoder & Peale 1981). Moreover, the fraction of multiples is much higher amongst systems with $t_{\rm RR}$ falling below the solar age line than for other random pairs. Our simple picture of resonant repulsion fails when the planet is subject to two or more resonances. In this case, exact resonance may be maintained for a much longer time because the planets form a heavy ladder with an effectively large inertia. The prevalence of simultaneous resonances in these short $t_{\rm RR}$ systems spurs us to hypothesize that all pairs with short $t_{\rm RR}$ in Fig. 3 are results of 3-body effects; and that these resonances are not primordial, but a combined effect of resonant repulsion and 3-body effects.

Removing the colored circles in Fig. 3, we see a relatively clear picture that most pairs stay where they were born with, while pairs very close to resonances experience repulsion and are shifted by a few percent to larger period ratios.

4. DISCUSSION

In this work, we investigate the peculiar fact that there is an excess of Kepler planet pairs just wide of resonance, and a deficit just inward of resonance. We propose that dissipation is responsible for this asymmetry. Two nearly resonant planets whose eccentricities are weakly damped repel each other, as shown previously in Lithwick & Wu (2008) and Papaloizou (2011). This is because dissipation damps away the planets' free eccentricities, but the eccentricities that are forced by the resonance persist despite dissipation. Planets are typically repelled when dissipation acts on these forced eccentricities. As such, resonant interaction allows dissipation to continuously extract energy from the orbits. Resonant repulsion pushes pairs from the near side to the far side of resonance, and naturally explains the Kepler result. Pairs accumulate at a fractional distance $\delta_{\rm mig}$ wide of each resonance, with $\delta_{\rm mig} \sim (\mu^2 t/t_{\rm damp})^{1/3}$, where $t_{\rm damp}$ is the typical eccentricity damping time and t the system age (Equations (24)–(25)).

For the source of dissipation, we focused on tidal damping in the inner planet. The typical distance planets can repel each other is of order a few percent or less for Kepler parameters if the tidal damping is efficient. The deficit of pairs immediately inward of resonance may be explained by this repulsion. And the distances outward of resonance where planet pairs are found are consistent with the theoretically estimated repulsion distance.

However, a number of inconsistencies between theory and data require further investigation. For instance, many pairs remain very close to resonance despite a short resonant repulsion time. These are often found in systems with more than two planets where the planet pairs are engaged simultaneously in more than one resonance. We therefore speculate that in fact all systems with short resonant repulsion time are consequences of 3-body effects. This may be confirmed using transit-timing variation or other tools.

If resonant repulsion is the reason behind the resonance asymmetry, its signature should be observable in future studies. The planets should currently have nearly zero free eccentricities, and as a result both of the resonant angles should be locked at their center-of-resonance values, with very small libration amplitude. This could be tested if these planets are accessible to radial velocity studies, or if their transit-timing variation can be well characterized using data such as in Ford et al. (2012). Furthermore, if tidal damping is the dominant dissipation mechanism, we expect that the resonance asymmetry should vanish for planets at orbital periods greater than $10 - 20$ days. Long-term Kepler monitoring will decide between tides or alternative damping mechanisms, e.g., damping by a gaseous or planetesimal disk.

Our study suggests that the initial period distribution of Kepler planets was relatively flat, without major pileups at or near resonances.[5] This is in contrast to jovian mass planets and places a strong constraint on the origin of these low-mass planets. If disk migration is responsible for their current location, it must somehow have avoided pushing the planets into resonances, perhaps because the migration rate was very fast—faster than the resonant libration rate. Alternatively, planets may be formed *in-situ* (Hansen & Murray 2011) and have therefore avoided convergent migration.

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[5] The chain of resonances observed in systems like KOI-500, KOI-730 (Lissauer et al. 2011), KOI-2038 (Fabrycky et al. 2012) might also not be primordial, but a combined result of resonant repulsion (a 2-body effect) and 3-body interactions.