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A WIND-DRIVEN WARPING INSTABILITY IN ACCRETION DISKS

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ABSTRACT

A wind passing over a surface may cause an instability in the surface, such as the flapping seen when wind blows across a flag or waves when wind blows across water. We show that when a radially outflowing wind blows across a dense thin rotating disk, an initially flat disk is unstable to warping. When the wind is subsonic, the growth rate is dependent on the lift generated by the wind and the phase lag between the pressure perturbation and the vertical displacement in the disk caused by drag. When the wind is supersonic, the growth rate is primarily dependent on the form drag caused by the surface. While the radiative warping instability proposed by Pringle is promising for generating warps near luminous accreting objects, we expect that the wind-driven instability introduced here would dominate in objects that generate energetic outflows.

Subject heading: accretion, accretion disks — hydrodynamics — instabilities

1. INTRODUCTION

For a wind blowing over a surface, the velocity and pressure are related by Bernoulli's equation; $P + \frac{1}{2}\rho v^2$ is conserved along streamlines. Because the velocity increases as the wind passes over a protrusion of the surface, the pressure must decrease. This causes a force pulling the higher regions of the surface upward. This force also causes lift on airplane wings. The Kelvin-Helmholtz instability occurs for the same physical reason at the boundary of two fluids of different densities moving with respect to one another (e.g., Chandrasekhar 1961). A related instability exists for wind passing over water or for wind passing over fabric (e.g., Thwaites 1961; Mahon & Williams 1999).

Near accreting compact objects such as active galactic nuclei (AGNs), substantial amounts of kinetic energy can be present in a wind that may pass near denser colder material in the outer parts of an accretion disk. In this paper we consider the possibility that a wind passing over a dense disk can result in a warp instability similar to that caused by radiative forces (Pringle 1996; Maloney, Begelman, & Pringle 1996; Maloney, Begelman, & Nowak 1998). Previous work (Schandl & Meyer 1994; Porter 1998) has considered the torque on a disk that would be caused by ram pressure or the reaction force from a wind but has not explored the possibility that the wind/disk interaction could lift the disk and so cause a warping instability.

2. PERTURBATIVE RESPONSE OF A RADIAL WIND TO A CORRUGATED SURFACE

We divide the problem into two parts, a diffuse outflowing wind and a dense infinitely thin disk. We first compute the effect of a vertical perturbation or ripple in the disk on the wind. The flow has a perturbation in the pressure along the surface of the disk that pushes on the disk. We then incorporate this force into the equations of motion in the disk. This approach is similar to that used to investigate wind/water-wave interactions or wind/fabric interactions. We follow the perturbative approach of Chandrasekhar (1961).

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We describe a warped surface by a displacement in the direction normal to its undisturbed plane,

$$h(r, \theta, t) = \mathrm{Re}[S e^{i(m\theta - k_r r - \omega t)}] , \quad (1)$$

where $S \ll r$. We assume a primarily radial flow with velocity $\boldsymbol{u} = u_0 \hat{\boldsymbol{e}}_r + \boldsymbol{v}$, where $\boldsymbol{v}$ is a perturbation, $|\boldsymbol{v}| \ll u_0$. We constrain the velocity so the component normal to the surface is zero; $dh/dt = [\partial/\partial t + u_0(\partial/\partial r)]h = v_z$. This constraint implies that on the surface

$$v_z = \mathrm{Re}[-i(\omega + u_0 k_r) S e^{i(m\theta - k_r r - \omega t)}] . \quad (2)$$

The continuity equation, $\partial\rho/\partial t + \nabla \cdot \rho \boldsymbol{v} = 0$, in cylindrical coordinates is

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) + \frac{\partial \rho}{\partial t} = 0 , \quad (3)$$

where ρ is the density in the wind. When the wind is subsonic, we search for a solution for the velocity and pressure perturbations in the wind that decays exponentially with increasing distance from the displaced surface. When the wind is supersonic, we search for a solution for the velocity and pressure perturbations that vary in phase with distance from the displaced surface:

$$\begin{aligned} P_1 &= \mathrm{Re}[p_1 g(r, \theta, z, t)] , \\ v_\theta &= \mathrm{Re}[v_{\theta,1} g(r, \theta, z, t)] , \\ v_r &= \mathrm{Re}[v_{r,1} g(r, \theta, z, t)] , \\ v_z &= \mathrm{Re}[v_{z,1} g(r, \theta, z, t)] , \end{aligned} \quad (4)$$

where

$$g(r, \theta, z, t) = \begin{cases} e^{i(m\theta - k_r r - \omega t) - k_z|z|} & M < 1 , \\ e^{i(m\theta - k_r r - \omega t + k_z|z|)} & M > 1 , \end{cases} \quad (5)$$

and the Mach number $M \equiv u_0/c_s$. The pressure $P = P_0 + P_1$, $P_1 \ll P_0$, and we adopt an equation of state $P \propto \rho^\Gamma$ with sound speed in the wind c_s. For the Kelvin-Helmholtz instability in an incompressible fluid, k_z is directly related to k_x where the one fluid is moving with respect to the other in the x-direction. For incompressible potential flow solutions, the vertical forms of the variables are solved exactly and they decay exponentially with height when the flow is subsonic and vary in phase with height when the flow is supersonic (e.g., Shivamoggi 1998).

In the tight winding limit ($k_r \gg 1/r$) and to first order the continuity equation becomes

$$\frac{1}{\Gamma}\frac{p_1}{P_0}(\omega + u_0 k_r) + k_r v_{r,1} - \frac{m v_{\theta,1}}{r} = \begin{cases} i k_z v_{z,1} \text{ sgn } z & M < 1 \,, \\ k_z v_{z,1} \text{ sgn } z & M > 1 \,. \end{cases} \tag{6}$$

Euler's equation to first order in the same coordinate system is

$$v_{r,t} + u_0 v_{r,r} + u_{0,r} v_r = -\frac{P_{1,r}}{\rho} \,,$$

$$v_{\theta,t} + u_0 v_{\theta,r} + \frac{u_0 v_\theta}{r} = -\frac{P_{1,\theta}}{r\rho} \,,$$

$$v_{z,t} + u_0 v_{z,r} = -\frac{P_{1,z}}{\rho} \,. \tag{7}$$

Again in the tight winding limit,

$$(\omega + u_0 k_r) v_{r,1} = -k_r \frac{c_s^2}{\Gamma}\frac{p_1}{P_0} \,,$$

$$(\omega + u_0 k_r) v_{\theta,1} = \frac{m}{r}\frac{c_s^2}{\Gamma}\frac{p_1}{P_0} \,,$$

$$(\omega + u_0 k_r) v_{z,1} = \begin{cases} i k_z \dfrac{c_s^2}{\Gamma}\dfrac{p_1}{P_0} \text{ sgn } z & M < 1 \,, \\ k_z \dfrac{c_s^2}{\Gamma}\dfrac{p_1}{P_0} \text{ sgn } z & M > 1 \,. \end{cases} \tag{8}$$

From equations (6) and (8) we find that

$$(\omega + u_0 k_r)^2 + c_s^2\left(\pm k_z^2 - k_r^2 - \frac{m^2}{r^2}\right) = 0 \,, \tag{9}$$

where the sign of the k_z^2 term is positive for $M < 1$ and negative for $M > 1$. Neglecting the m^2/r^2 term in the tight winding limit,

$$k_r^2 - \frac{1}{c_s^2}(\omega + u_0 k_r)^2 = \begin{cases} k_z^2 & M < 1 \,, \\ -k_z^2 & M > 1 \,, \end{cases} \tag{10}$$

which relates the vertical exponential scale length or wavenumber to the radial wavenumber.

Using equations (2) and (8), we can see that

$$\frac{p_1}{P_0} = \begin{cases} -\dfrac{\Gamma}{c_s^2}\dfrac{S}{k_z}(\omega + u_0 k_r)^2 \text{ sgn } z & M < 1 \,, \\ -\dfrac{i\Gamma}{c_s^2}\dfrac{S}{k_z}(\omega + u_0 k_r)^2 & M > 1 \,. \end{cases} \tag{11}$$

When the wind is subsonic, the frequencies and wavevectors are real, and $k_z > 0$, then $p_1 < 0$ for $z > 0$. Where the surface is high, above the surface the pressure is lower than the mean and below the surface the pressure is higher. This causes a destabilizing force, which we also call "lift" in analogy to the aerodynamics of wings. Because the sign of the pressure perturbation is opposite on either side of the disk ($\propto$sgn z), the pressure differential across the surface exerts a force on the surface with magnitude twice P_1 in the direction normal to the surface (see Fig. 1). The pressure differential across the surface caused by the wind should exert a force on the surface. An instability should exit when $m = 0$, which is a direct analogy for the Kelvin-Helmholtz instability or for the instability of a wind passing over fabric.

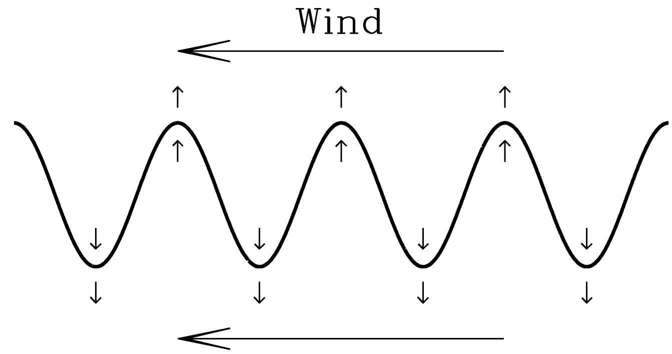

FIG. 1.—As a subsonic wind passes over corrugations, pressure changes cause lift at the high points of the surface.

When the wind is supersonic, the pressure perturbations are 90° out of phase with the surface perturbations. The phase of the pressure perturbations should increase with both r and $|z|$, which implies that the sign of k_z is the same as the sign of k_r. Because the sign of the pressure perturbation is opposite on either side of the disk, there is a drag force on the surface that is not present when the flow is subsonic and approximated as a potential flow solution. Because shocks should form, we expect the actual flow to be more complicated than given by the above equations (see Fig. 2 for an illustration). When the shocks and expansion waves lag the high points of the surface, we expect a vertical force on the high points of the surface that pushes the surface toward the midplane. When $m = 0$, instead of a destabilizing lift force, we expect a stabilizing vertical force. But since there is drag, energy from the wind is transferred to the surface, and the amplitude of the surface perturbation should increase (e.g., Shivamoggi 1998, § 5.5, on the Kelvin-Helmholtz instability with a supersonic flow).

When $m = 1$ for both subsonic and supersonic flows, we must consider the torque on an annulus, which is the cross product of the radial vector and the pressure differential, integrated azimuthally about the ring.

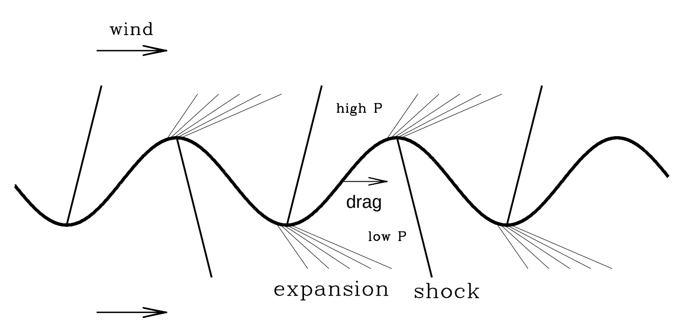

FIG. 2.—When the wind is supersonic, the maxima of the pressure perturbations vary as a function of distance from the surface. Shocks cause pressure increase, and expansion fans cause pressure decreases. The pressure differences across the surface result in a drag on the surface. When the expansion waves and shocks lag the high points of the surface, there is a vertical force on the surface that pushes the high points toward the midplane.

3. THE TORQUE ON DISK ANNULI

To integrate the torque azimuthally we now shift notation and follow that used by Pringle (1996) to describe the warped disk. The tilt vector for a ring at radius r

$$\hat{\boldsymbol{l}} = (\cos\gamma \sin\beta, \sin\gamma \sin\beta, \cos\beta)\,, \tag{12}$$

where $\beta(r, t)$ is the local angle of tilt of the disk with respect to the z-axis and the descending node of the disk material is at an angle $\gamma(r, t) - \pi/2$ to the x-axis. The unit vector toward points on the surface

$$\begin{aligned}\hat{\boldsymbol{x}}(r, \phi) = (&\cos\phi \sin\gamma + \sin\phi \cos\gamma \cos\beta\,,\\ &\times \sin\phi \sin\gamma \cos\beta - \cos\phi \cos\gamma,\\ &- \sin\phi \sin\beta)\,,\end{aligned} \tag{13}$$

where $\phi = 0$ at the descending node and γ and β are both functions of r. The external coordinate system, $\hat{\boldsymbol{x}} = (x, y, z)$, which in cylindrical coordinates is (r, θ, z) where $\theta = \tan^{-1}(y/x)$, can be related to that described by equation (13). When $\beta \ll 1$,

$$\theta = \phi + \gamma - \pi/2\,, \tag{14}$$

and the vertical displacement of the surface or z-component of $\hat{\boldsymbol{x}}$

$$h(r, \theta, t) = -\beta r \sin\phi = -\beta r \cos(\theta - \gamma)\,. \tag{15}$$

To relate this formalism to that used in the previous section, we set $\gamma = k_r r + \omega t$, $m = 1$, and $S = -\beta r$ so that

$$h(r, \theta, t) = -\beta r \cos(\theta - k_r r - \omega t)\,, \tag{16}$$

which is in the same form as equation (1).

Pringle (1996) defines an elemental area vector $d\boldsymbol{S} = \boldsymbol{s}_r \times \boldsymbol{s}_\phi\, dr\, d\phi$, where $\boldsymbol{s}_r = \partial\hat{\boldsymbol{x}}/\partial r$ and $\boldsymbol{s}_\phi = \partial\hat{\boldsymbol{x}}/\partial\phi$. The normal to the disk surface $\hat{\boldsymbol{n}} = d\boldsymbol{S}/|d\boldsymbol{S}|$. To first order in β,

$$\boldsymbol{s}_\phi = \hat{\boldsymbol{l}} \times \hat{\boldsymbol{x}} = [\cos(\phi + \gamma), \sin(\phi + \gamma), -\beta\cos\phi]\,, \tag{17}$$

$$\hat{\boldsymbol{n}} = \hat{\boldsymbol{l}} - \hat{\boldsymbol{x}}(\beta r \gamma' \cos\phi - r\beta' \sin\phi) \tag{18}$$

(see Pringle 1996, eqs. [2.11] and [2.17]), where β' and γ' refer to the derivatives of β and γ with respect to r.

When the wind is subsonic, in the previous section we found to first order that the pressure differential across the surface was in phase with the corrugations of the surface. However, the pressure in the region of laminar flow should actually be somewhat lower on the leeward side of each trough, and so there would be a lag between the pressure and the surface variations (Jeffreys 1924). Kendall (1970) measured sinusoidal pressure variations in response to a wind passing over a sinusoidally varying rubber surface and showed that the pressure was indeed offset in phase with the surface. We assume that the pressure difference across the disk surface can be described

$$\Delta P = 2p_1 \sin(\phi + \delta) \text{ for } M < 1\,, \tag{19}$$

where the phase lag between the pressure and the surface is given by δ. For $k_r > 0$ we expect $\delta > 0$, and $\delta < 0$ for $k_r < 0$.

When the wind is supersonic, shocks should cause an effective phase shift in the pressure perturbation on the surface. In this case

$$\Delta P = 2p_1' \cos(\phi + \delta) \text{ for } M > 1\,, \tag{20}$$

where

$$p_1' \equiv \frac{P_0 \Gamma}{c_s^2} \frac{S}{k_z} (\omega^2 + u_0 k_r)^2 \tag{21}$$

(see eq. [11]). The sign of p_1' depends on the sign of k_r, since the sign of k_z depends on the sign of k_r.

The pressure differential across the surface exerts a force on the disk parallel to the normal to the surface, $\hat{\boldsymbol{n}}$. The resulting torque per unit mass per unit mass for an annulus is the integral

$$\tau_w = \frac{1}{2\pi} \int_0^{2\pi} r\hat{\boldsymbol{x}} \times \hat{\boldsymbol{n}} \frac{\Delta P}{\Sigma}\, d\phi\,, \tag{22}$$

where Σ is the disk surface density (mass per unit area). Using the previous four equations, we perform this integral for small β and δ:

$$\tau_w = \begin{cases} \dfrac{p_1 r}{\Sigma}\,[(\sin\gamma, -\cos\gamma, 0) \\ \qquad - \delta(\cos\gamma, \sin\gamma, 0) + (0, 0, \delta\beta)] \quad M < 1\,, \\ \dfrac{p_1' r}{\Sigma}\,[-\delta(\sin\gamma, -\cos\gamma, 0) \\ \qquad - (\cos\gamma, \sin\gamma, 0) + (0, 0, \beta)] \quad M > 1\,. \end{cases} \tag{23}$$

To first order in β, the angular momentum axis of an annulus is $\hat{\boldsymbol{l}} = (\beta\cos\gamma, \beta\sin\gamma, 1)$. From comparing equation (23) to $d\hat{\boldsymbol{l}}/dt$, we can see that the first term in this equation causes the disk to precess and the second term causes the tilt to increase or decrease. The tilt increases when $\delta < 0$ or $k_r < 0$, for a subsonic wind, and when $p_1' < 0$ or $k_r < 0$ for a supersonic wind (see Figs. 3 and 4). The third term corresponds to a rate of change in the total angular momentum of the annulus that would cause a small amount of outflow in the disk.

3.1. *Equation of Motion for the Annulus*

The angular momentum per unit mass of an annulus of radius r is $\sim r^2\Omega$ where Ω is the angular rotation rate. When

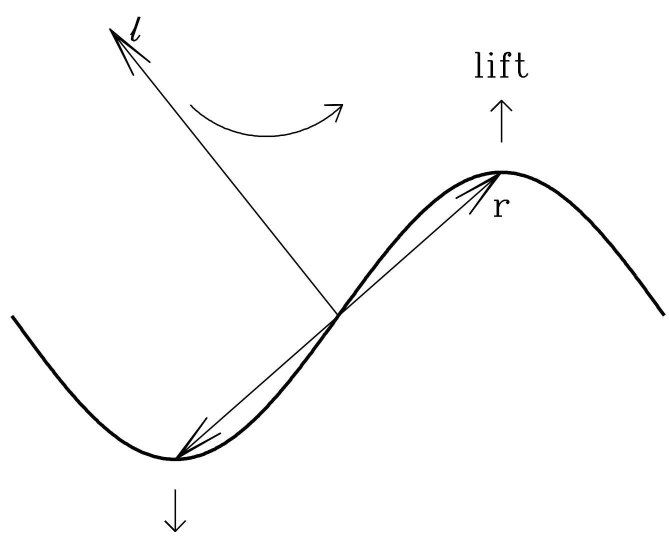

FIG. 3.—The lift on an annulus causes the annulus to precess. When the flow is supersonic, the force on the high points of the surface is toward the plane, and the annulus should precess in the opposite direction.

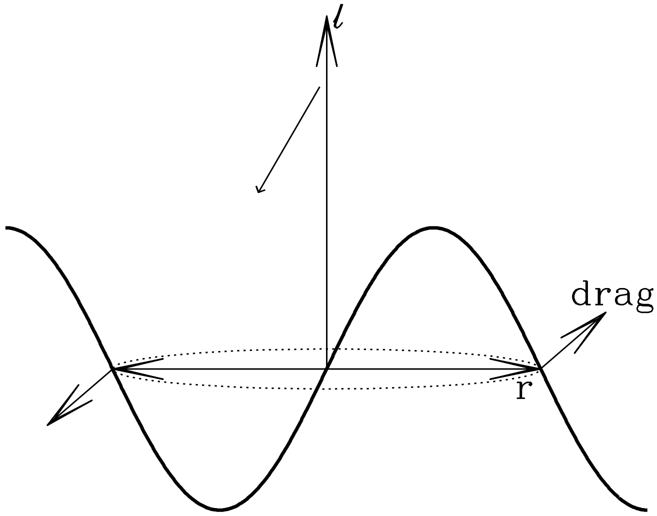

FIG. 4.—The drag on the surface causes a vertical force on the disk. This results in a torque that can either increase or decrease the tilt. A ring is shown tilted toward the line of sight. The slope of the warp with respect to the wind depends on whether the warp is leading or trailing. Consequently, the torque either increases or decreases the tilt depending on whether the warp is leading or trailing.

radial motions in the disk are small,

$$r^2\Omega \frac{\partial \hat{\boldsymbol{l}}}{\partial t} = \tau_g + \tau_v + \tau_w \,, \tag{24}$$

where τ_g is the torque from gravity (equal to zero when the potential is spherical) and τ_v is the torque due to viscous forces. As done by Pringle (1996), we set $W = \beta e^{i\gamma}$ so that $\hat{\boldsymbol{l}} = [\mathrm{Re}(W), \mathrm{Im}(W), 1]$. Ignoring the z-component of the torque and using equation (23), we find

$$\frac{\partial W}{\partial t} = \begin{cases} -\dfrac{p_1}{\Sigma r\Omega}(i+\delta)\dfrac{W}{\beta} & M<1\,, \\ \dfrac{p_1'}{\Sigma r\Omega}(1+\delta i)\dfrac{W}{\beta} & M>1\,, \end{cases} \tag{25}$$

where we have ignored gravitation and viscous terms. Using equations (10), (11), and (21) for p_1 and p_1',

$$\frac{\partial W}{\partial t} \approx \begin{cases} -\dfrac{\Gamma P_0 |k_r| M^2}{\Sigma\Omega\sqrt{1-M^2}}(i+\delta)W & M<1\,, \\ \dfrac{\Gamma P_0 k_r M^2}{\Sigma\Omega\sqrt{M^2-1}}(1+i\delta)W & M>1\,, \end{cases} \tag{26}$$

where we have assumed slowly varying modes, $\omega \ll u_0 k_r$. We define a parameter Q that describes the ratio of the kinetic energy in the wind to the rotational energy in the disk, $Q \equiv P_0 M^2/\Sigma\Omega^2 r \sim \rho_w r u_0^2/\Sigma v_c^2$, where ρ_w is the density of the wind and $v_c \equiv \Omega r$ is the velocity of a particle in a circular orbit. We can write

$$\frac{\partial W}{\partial t} \approx \begin{cases} -\dfrac{Q\Gamma |k_r r|\Omega}{\sqrt{1-M^2}}(i+\delta)W & M<1\,, \\ \dfrac{Q\Gamma k_r r\Omega}{\sqrt{M^2-1}}(1+i\delta)W & M>1\,. \end{cases} \tag{27}$$

The precession rate

$$\frac{\partial \gamma}{\partial t} \approx \omega \approx \begin{cases} -Q|k_r r|\Omega & M \ll 1\,, \\ \delta Q |k_r r|\Omega/M & M \gg 1\,, \end{cases} \tag{28}$$

where to simplify the expression we have assumed $M \ll 1$ or $M \gg 1$ and ignored the dependence on the adiabatic index.

For a subsonic wind, growing modes occur when $\delta < 0$. We expect that the phase lag $\delta < 0$ when $k_r < 0$ so that growing modes will occur for $\gamma' < 0$. The growing mode will have lines of nodes following a trailing spiral instead of a leading spiral as was the case for the radiatively driven warp instability (Pringle 1996). For a supersonic wind, growing modes occur when $k_r > 0$ and so will have a leading spiral shape.

We expect the growing mode to grow as $\beta \propto e^{t/t_w}$ with growth timescale

$$t_w \sim \begin{cases} \dfrac{1}{\delta\omega} & M<1\,, \\ \dfrac{\delta}{\omega} & M>1\,. \end{cases} \tag{29}$$

In terms of the dynamical time, $t_d = 1/\Omega$,

$$\frac{t_w}{t_d} \sim \begin{cases} \dfrac{1}{Q|k_r r|\delta} & M<1\,, \\ \dfrac{M}{Q|k_r r|} & M>1\,. \end{cases} \tag{30}$$

When the wind is supersonic, the growth rate exceeds the precession rate, and the amplitude of the perturbation can grow quickly. We see that the precession and growth rates depend on Q, the ratio of the kinetic energy in the wind and the rotational energy in the disk. Though we have neglected the viscous forces (in eq. [22]), we expect them to damp the growth of unstable modes primarily for the short-wavelength modes. We expect the disk self-gravity to affect the precession frequency but not strongly affect the growth rate of the mode.

Since we have followed the notation by Pringle (1996), we can directly compare the importance of the torque caused by the wind to that caused by the absorption of radiation from a central source. Ignoring the viscous damping term, the precession rate caused by the radiative torque is given by $\omega_r = Lk_r/12\pi\Sigma R^2\Omega c$ (Pringle 1996, eqs. [3.6] and [3.8]). Comparing this to our equations (27) and (28), we find that the wind-driven warping instability is likely to dominate when

$$\frac{L}{12\pi c} \lesssim P_0 M^2 r^2 \sim \rho_w u_0^2 r^2 \,. \tag{31}$$

We can interpret this inequality in terms of the momentum flux. When the momentum flux from the wind dominates that from a central radiation source, a wind-driven instability may dominate. This suggests that accreting sources that impart more energy into driving winds than into radiation would be more likely to drive wind-driven instabilities in their accretion disks.

4. THE PHASE LAG AND THE CRITICAL ANGLE FOR BOUNDARY LAYER SEPARATION

The simplest description of subsonic wind flow near a corrugated surface such as a flag or water wave is an irrotational flow above the interface with wind velocity averaged over one wavelength that is constant with height. This is a "potential flow" (Lamb 1932), and since the pressure is related to the wind velocity via Bernoulli's equation, it is in exact antiphase with the surface (as derived perturbatively above). So the energy flux between the wind and wave is zero. For the wind to do work on the wave, the pressure must be shifted in phase relative to the potential solution. Jeffreys (1924, 1925) suggested that the downwind side of the wave is sheltered from the wind so that the pressure on this slope is reduced there but increased on the upwind side. This causes a drag force, and as a result, waves traveling with the wind increase in amplitude, and those traveling against the wind are damped. Scaling based on idealized theory for this (Miles 1957 and subsequent work) unfortunately does not predict the experimentally measured growth rates very well (e.g., Kendall 1970; Riley, Donelan, & Hui 1982; Donelan 1999).

In the theory outlined by Miles (1957), the phase lag is set by numerical constants determined from turbulent mean profiles and does not depend upon the amplitude of the surface variations. We can understand this by considering the difference in the rate of growth of the turbulent boundary layer between the leeward and windward sides of the wavy surface. Because the growth rate of the boundary layer depends on the pressure gradient along the direction of flow outside the boundary layer, we expect a difference in the boundary layer thickness on the leeward and windward sides that is proportional to the pressure difference across the wave, and this in turn is proportional to the amplitude of the wave. However, the thickness of the boundary layer may be set by a turbulent eddy viscosity and so should also be proportional to the amplitude of the surface perturbation. Thus, we predict that the phase angle will not be strongly dependent on the amplitude of the surface variations.

In most astrophysical situations, we expect high Reynolds number flows and so can draw on the theory of aerodynamics. A laminar or viscous boundary layer next to a surface is only stable when the pressure gradient along the surface in the direction of flow is negative. However, this condition is violated to the leeward of a corrugation. The same situation occurs along airplane wings, and the boundary layer becomes turbulent. Because of the Coanda effect, turbulent boundary layers are less likely to separate from the surface than viscous boundary layers. This allows the boundary layer to remain near the surface along airplane wings even though the pressure gradient in the laminar flow region is not favorable. For most airplane wings, the boundary layer does not separate from the surface until the angle of attack from the wing is $\gtrsim 15^\circ$, though this critical angle can increase to 45° for short thick wings. Past this critical angle, little or no lift is generated and the wing "stalls." We expect a similar effect in our astrophysical disk. When the amplitude of the corrugations reaches a critical slope (a critical value for $|Sk_r|$), we expect that the boundary layer will separate from the surface, the lift will decrease causing the precession rate to drop, and the drag will increase. At this point we expect the mode to saturate.

When the wind is supersonic, a smooth continuous flow is unlikely (see Fig. 2). A full calculation would require resolving shocks and expansion flows. However, the perturbative method we have used should allow us to at least estimate the magnitude of the instability. We do expect a major difference in the character of the wind flow near the surface when the slope of the surface exceeds the Mach angle or when $|Sk_r| > \arctan M^{-1}$. Past this angle a turbulent boundary layer should develop, and the instability may saturate. In this regime it may be possible to estimate the torque on the disk using an approximation (that of Newtonian flow) that primarily considers ram pressure or reactive force on the disk surface (e.g., Porter 1998; Schandl & Meyer 1994).

5. DISCUSSION AND SUMMARY

In this paper we have outlined a possible instability that could occur in accreting objects with energetic outflows. A wind passing over a rotating thin dense disk is likely to cause a warping instability to grow in the disk. This instability could occur in situations where an energetic outflow driven by a compact object passes over a dense disk, such as might happen in binary X-ray sources or active galactic nuclei. In active galactic nuclei, at radii of order a parsec from a black hole there is evidence for the existence of dense, warped disks from maser observations (e.g., Herrnstein, Greenhill, & Moran 1996). Large-scale outflows are predicted for a variety of types of accretion flows and seem to be an integral part of these flows. Observational evidence for disk winds is fairly ubiquitous and seen in both X-ray binaries (Brandt & Schulz 2000; Chiang 2001) and AGNs (e.g., Murray & Chiang 1995).

When a subsonic wind passes over a corrugated dense disk, the corrugations will cause pressure variations along the surface of the disk. These pressure variations cause lift, which cause the annuli in the disk to precess. Form drag caused by the corrugated surface will cause pressure variations in the disk to lag the vertical displacement of the disk. This causes a torque on the warped disk that can increase the amplitude of the perturbation. We expect this instability to saturate or cease growing at a critical slope when the lift decreases because of boundary layer separation.

When a supersonic wind passes over a corrugated disk, the disk primarily experiences form drag. The drag causes a torque on the warped disk that can increase the amplitude of the corrugations. When there is an effective lag between the pressure and the vertical velocity of the surface, there will be a vertical force on the disk that causes it to precess. We expect a major change in the nature of the flow when the slope of the corrugations exceeds the Mach angle. While we primarily expect accreting astrophysical objects to drive supersonic outflows, if a thick outflowing subsonic boundary layer is created, the subsonic theory outlined here may be appropriate inside this layer. The instability can dominate the radiative induced warping instability in sources where the energy released in an outflow (either sub- or supersonic) exceeds that emitted as radiation.

In this paper we have concentrated on a purely hydrodynamic flow. However, we expect the magnetic field to be dynamically important both in the disk and wind. If the wind and disk are magnetized, flux freezing would make the wind and disk act as mutually impenetrable bodies. The wind would be less likely to cause hydrodynamical instabilities that could destroy the disk. In this paper we have focused on the affect of a radially outflowing wind on the planarity of a disk in the absence of radiation. Future investigations could develop a prediction for the phase lag angle for subsonic wind flows (a historically difficult hydrodynamics problem) and calculate the details of the shocks that are likely to be present when the wind is supersonic. In this paper we have restricted our study to radially outflowing winds; however, astrophysical outflows typically have a nonzero, radius-dependent, azimuthal velocity. We have also neglected the role of gravitational, magnetic, and viscous forces. Future work can consider the role of these forces, include rotating winds, and explore the effects of both radiation and winds on the planarity of the disk, particularly in the regime where the winds themselves are radiatively driven.

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